Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/32-1.2.1.1-a+b-x+c-x^2-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [143]. This is test number [32].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (143)	0.00 (0)
Mathematica	100.00 (143)	0.00 (0)
Mupad	92.31 (132)	7.69 (11)
Maple	79.02 (113)	20.98 (30)
Fricas	79.02 (113)	20.98 (30)
Giac	77.62 (111)	22.38 (32)
Maxima	75.52 (108)	24.48 (35)
Sympy	70.63 (101)	29.37 (42)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

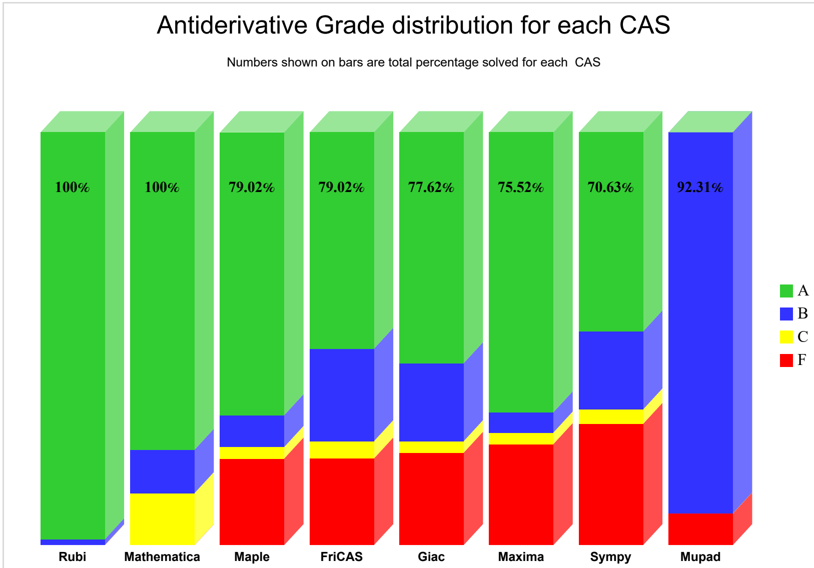
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

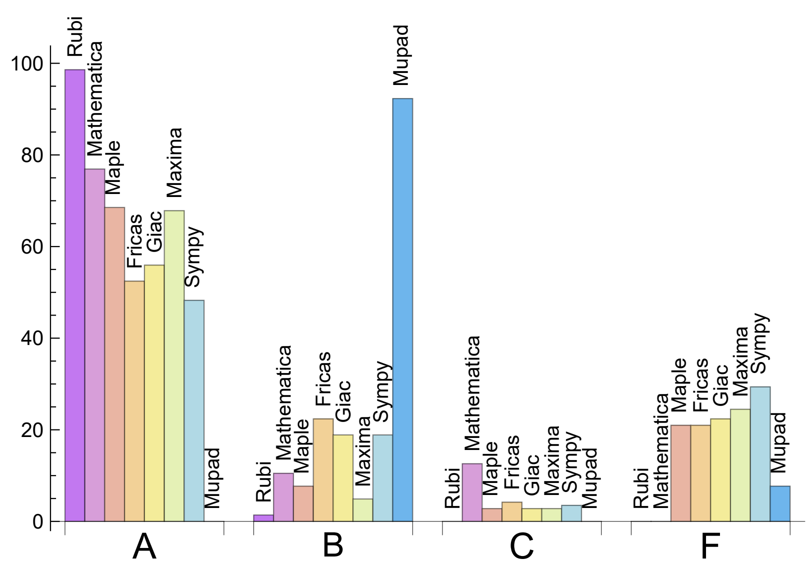
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.902	2.098	0.000	0.000
Mathematica	76.923	10.490	12.587	0.000
Maple	68.531	7.692	2.797	20.979
Maxima	67.832	4.895	2.797	24.476
Giac	55.944	18.881	2.797	22.378
Fricas	52.448	22.378	4.196	20.979
Sympy	48.252	18.881	3.497	29.371
Mupad	0.000	92.308	0.000	7.692

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Mupad	11	0.00	100.00	0.00
Fricas	30	100.00	0.00	0.00
Maple	30	100.00	0.00	0.00
Giac	32	100.00	0.00	0.00
Maxima	35	85.71	0.00	14.29
Sympy	42	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.19
Maxima	0.24
Giac	0.27
Fricas	0.32
Sympy	0.33
Mathematica	1.34
Maple	2.21
Mupad	6.26

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	39.27	0.84	33.50	0.79
Maple	41.17	0.96	31.00	0.83
Maxima	45.49	1.05	33.50	1.03
Mathematica	50.11	1.24	44.00	1.00
Giac	54.18	1.40	37.00	1.05
Fricas	63.38	1.57	43.00	1.21
Rubi	80.74	1.09	40.00	1.00
Sympy	87.38	1.83	41.00	1.06

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

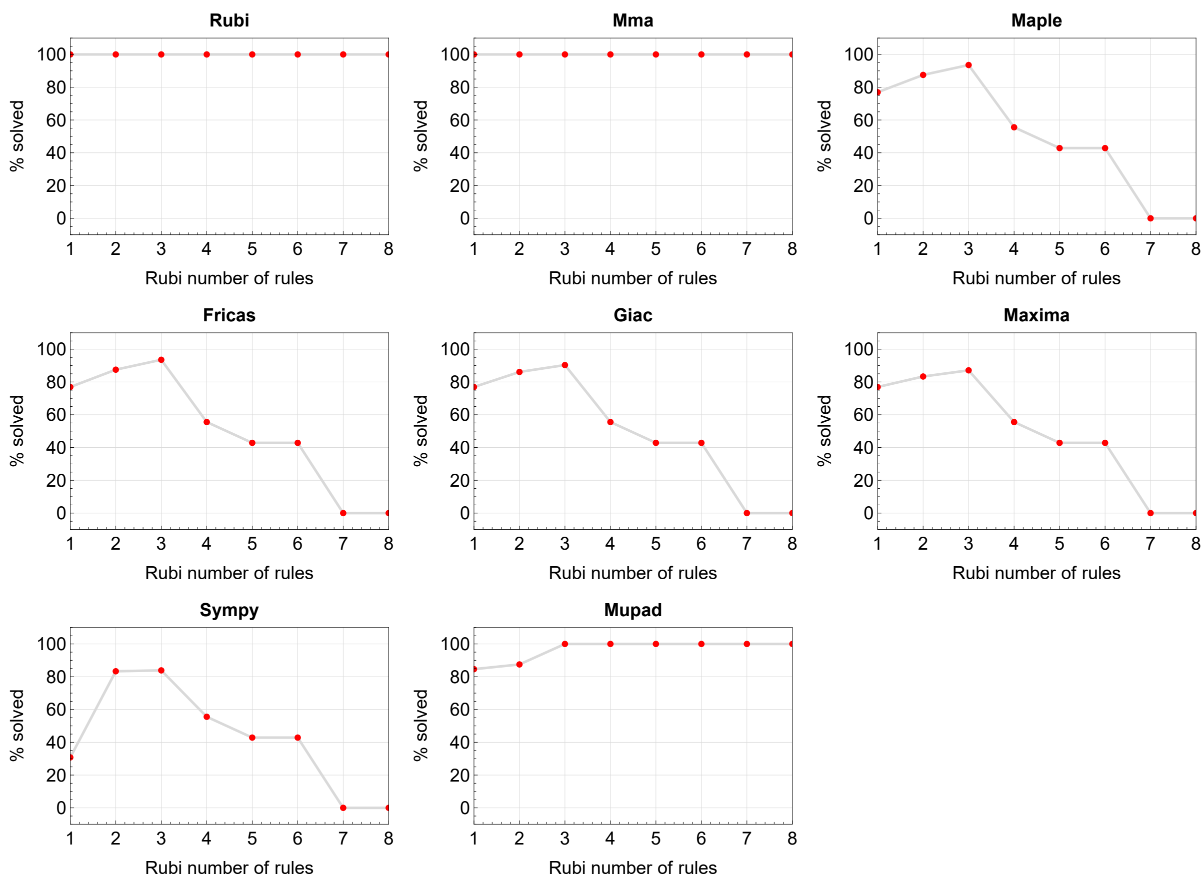


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

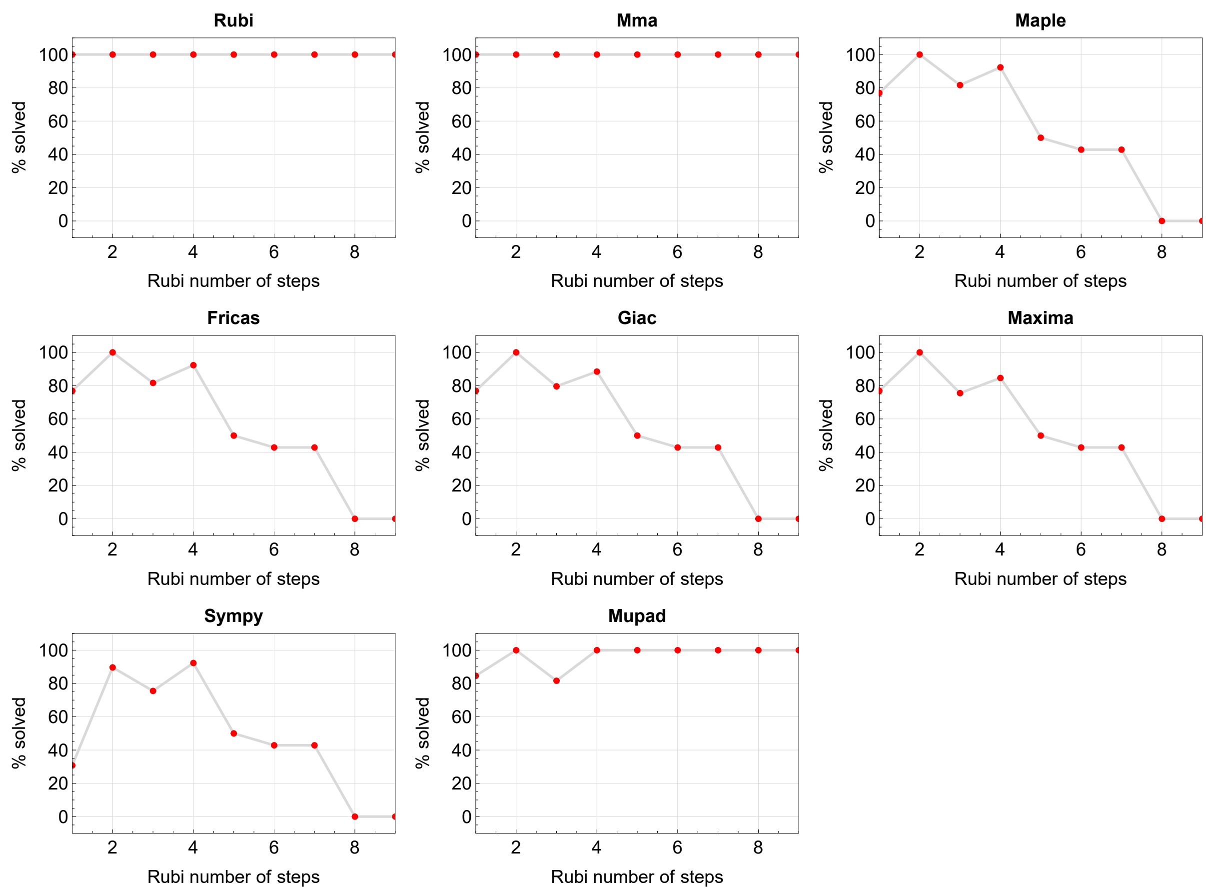


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

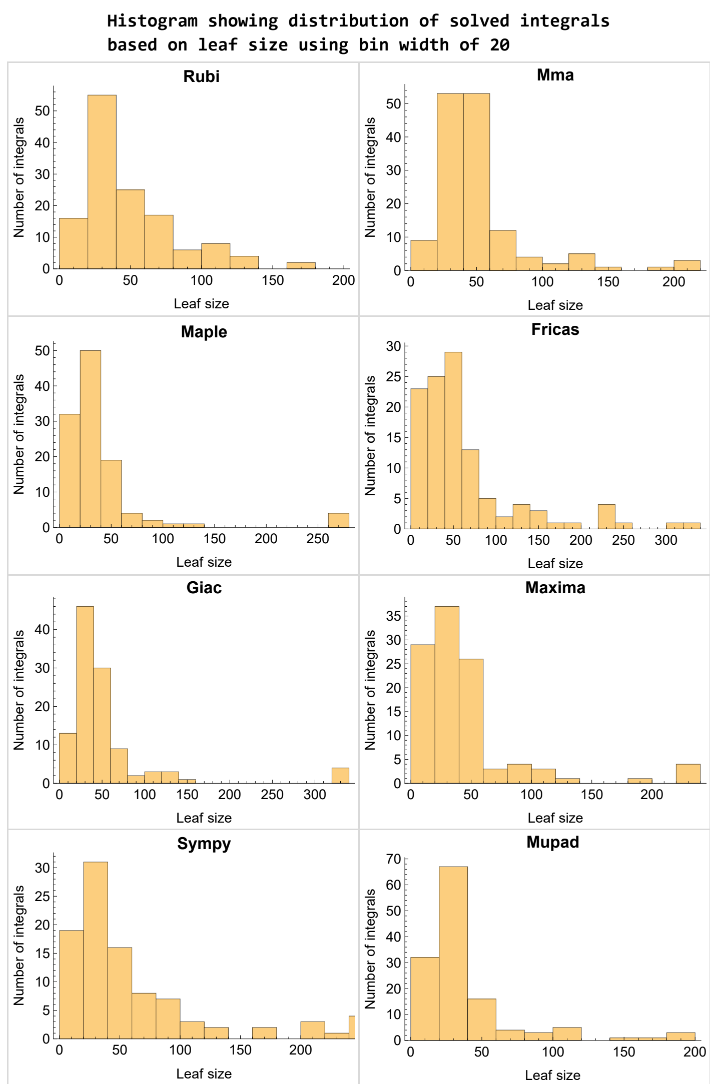


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

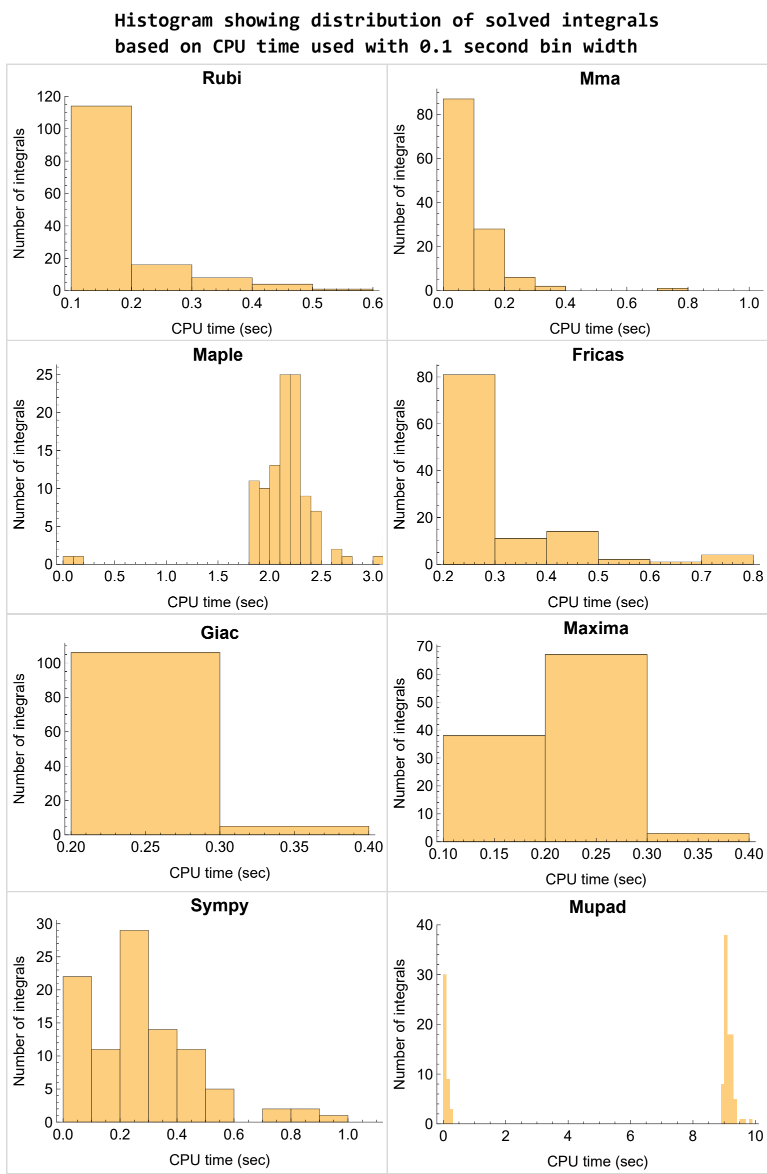


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

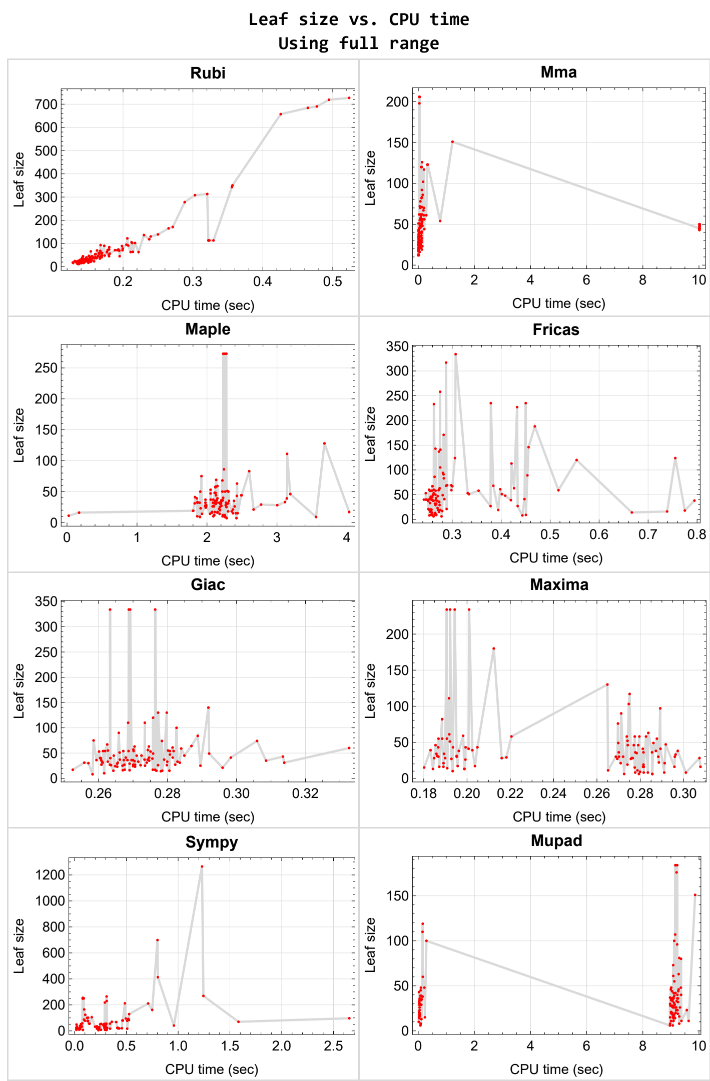


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{30, 31, 32, 33, 34, 35, 36, 37, 38, 39\}$

Mathematica {}

Maple $\{65, 68\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

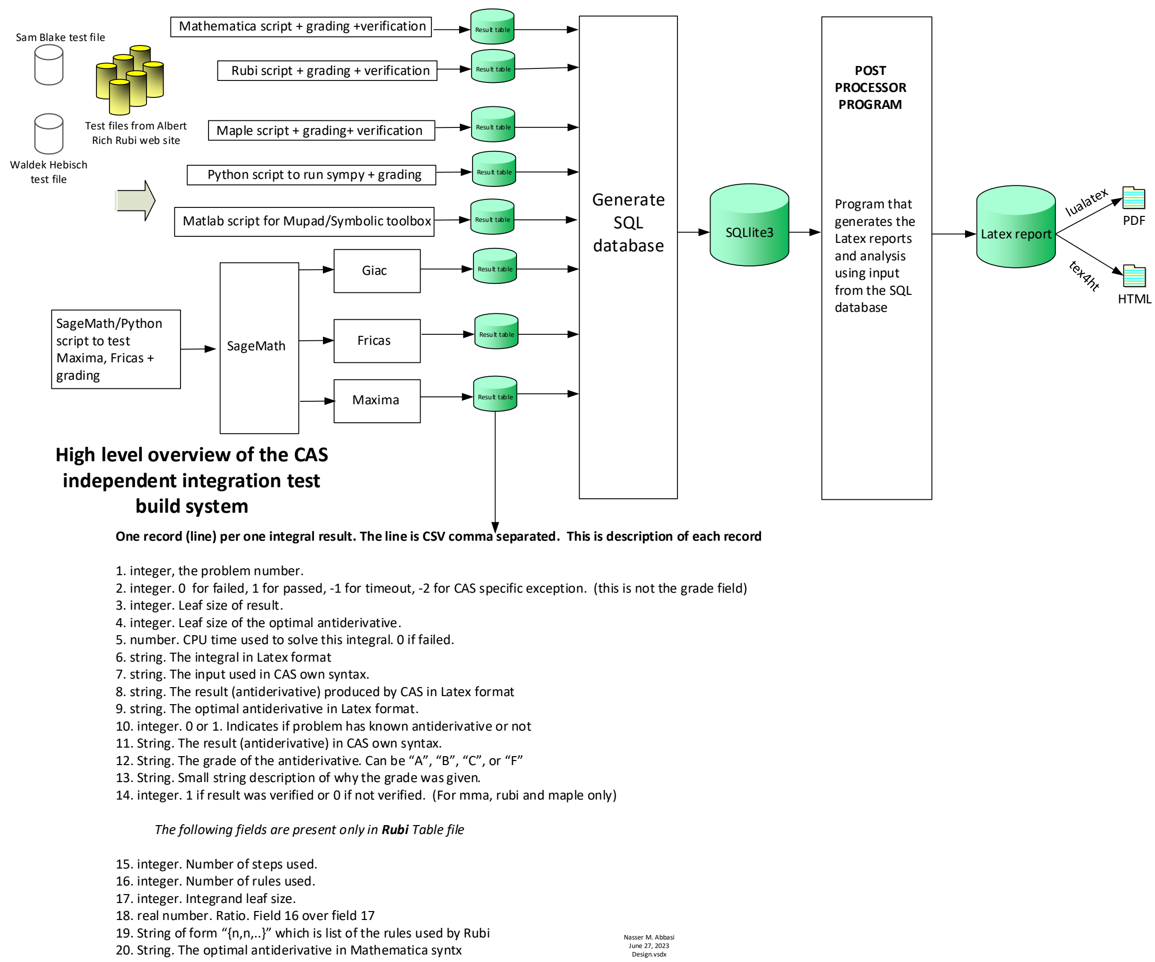
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade { 80, 83, 100 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 119, 120, 121, 122, 123, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade { 17, 21, 25, 26, 27, 28, 29, 83, 99, 100, 115, 118, 124, 126, 127 }

C grade { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 69, 70, 72, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 129, 130, 131, 138 }

B grade { 25, 74, 75, 76, 77, 83, 101, 102, 125, 126, 127 }

C grade { 65, 68, 71, 73 }

F normal fail { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 22, 23, 24, 26, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 78, 79, 81, 82, 86, 88, 89, 91, 92, 93, 94, 98, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 116, 119, 121, 123, 130, 131, 138 }

B grade { 17, 18, 21, 25, 27, 74, 75, 76, 77, 80, 83, 84, 85, 87, 90, 95, 96, 97, 99, 100, 104, 114, 115, 117, 118, 120, 124, 125, 126, 127, 128, 129 }

C grade { 70, 71, 72, 73, 111, 122 }

F normal fail { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 78, 79, 80, 81, 82, 84, 85, 86, 87, 90, 91, 92, 93, 94, 97, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 138 }

B grade { 17, 74, 75, 76, 77, 83, 101 }

C grade { 71, 73, 111, 122 }

F normal fail { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { 88, 89, 95, 96, 98 }

2.1.6 Giac

A grade { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 22, 23, 24, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 116, 119, 121, 123, 128, 129, 130, 131, 138 }

B grade { 3, 4, 5, 17, 18, 21, 25, 26, 27, 28, 29, 64, 74, 75, 76, 77, 83, 101, 114, 115, 117, 118, 120, 124, 125, 126, 127 }

C grade { 70, 71, 72, 73 }

F normal fail { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 111, 122, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 138 }

C grade { }

F normal fail { }

F(-1) timedout fail { 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 21, 27, 28, 29, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 65, 66, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 91, 92, 93, 94, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 138 }

B grade { 2, 6, 25, 26, 53, 54, 61, 62, 63, 64, 74, 75, 76, 77, 83, 84, 85, 87, 88, 89, 90, 95, 96, 97, 98, 99, 100 }

C grade { 86, 101, 102, 111, 122 }

F normal fail { 16, 18, 19, 20, 22, 23, 24, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 67, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	171	151	128	180	258	699	130	151
N.S.	1	1.16	1.03	0.87	1.22	1.76	4.76	0.88	1.03
time (sec)	N/A	0.268	1.219	3.678	0.212	0.276	0.801	0.280	9.864

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	136	117	69	130	69	269	140	100
N.S.	1	1.12	0.97	0.57	1.07	0.57	2.22	1.16	0.83
time (sec)	N/A	0.232	0.202	2.131	0.265	0.298	1.244	0.292	0.293

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	105	86	58	103	59	162	130	80
N.S.	1	1.11	0.91	0.61	1.08	0.62	1.71	1.37	0.84
time (sec)	N/A	0.210	0.156	2.121	0.275	0.298	0.749	0.277	9.359

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	83	47	76	49	92	120	60
N.S.	1	1.07	1.20	0.68	1.10	0.71	1.33	1.74	0.87
time (sec)	N/A	0.180	0.126	2.051	0.270	0.281	0.519	0.276	0.162

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	31	49	39	32	110	39
N.S.	1	1.00	1.44	0.72	1.14	0.91	0.74	2.56	0.91
time (sec)	N/A	0.156	0.084	2.129	0.270	0.420	0.286	0.269	0.085

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	118	102	63	117	68	211	57	81
N.S.	1	1.17	1.01	0.62	1.16	0.67	2.09	0.56	0.80
time (sec)	N/A	0.236	0.170	2.429	0.275	0.384	0.711	0.270	9.299

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	91	92	53	90	58	129	47	63
N.S.	1	1.15	1.16	0.67	1.14	0.73	1.63	0.59	0.80
time (sec)	N/A	0.209	0.126	2.051	0.271	0.354	0.527	0.281	9.276

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	64	82	43	63	48	68	37	45
N.S.	1	1.12	1.44	0.75	1.11	0.84	1.19	0.65	0.79
time (sec)	N/A	0.181	0.113	2.426	0.284	0.408	0.403	0.279	0.086

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	37	55	28	36	38	29	27	26
N.S.	1	1.06	1.57	0.80	1.03	1.09	0.83	0.77	0.74
time (sec)	N/A	0.164	0.076	1.989	0.281	0.795	0.240	0.263	9.078

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	37	47	27	36	35	26	25	26
N.S.	1	1.06	1.34	0.77	1.03	1.00	0.74	0.71	0.74
time (sec)	N/A	0.158	0.074	2.206	0.278	0.274	0.233	0.267	0.048

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	37	55	28	36	38	29	27	26
N.S.	1	1.06	1.57	0.80	1.03	1.09	0.83	0.77	0.74
time (sec)	N/A	0.162	0.082	2.267	0.270	0.268	0.238	0.270	9.101

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	56	67	39	55	43	56	35	39
N.S.	1	1.10	1.31	0.76	1.08	0.84	1.10	0.69	0.76
time (sec)	N/A	0.178	0.181	2.149	0.288	0.267	0.342	0.275	9.121

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	32	41	32	34	33	29
N.S.	1	1.00	1.31	0.91	1.17	0.91	0.97	0.94	0.83
time (sec)	N/A	0.154	0.070	1.895	0.201	0.265	0.213	0.261	0.110

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	46	32	43	32	34	33	29
N.S.	1	1.00	1.24	0.86	1.16	0.86	0.92	0.89	0.78
time (sec)	N/A	0.160	0.074	1.863	0.187	0.267	0.217	0.272	9.139

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	33	43	36	32	37	29
N.S.	1	1.00	1.28	0.85	1.10	0.92	0.82	0.95	0.74
time (sec)	N/A	0.161	0.108	1.864	0.199	0.273	0.198	0.266	0.080

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	89	70	75	111	105	0	74	96
N.S.	1	1.07	0.84	0.90	1.34	1.27	0.00	0.89	1.16
time (sec)	N/A	0.193	0.200	1.921	0.192	0.276	0.000	0.278	9.223

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	51	10	21	19	8	110	19
N.S.	1	1.00	3.19	0.62	1.31	1.19	0.50	6.88	1.19
time (sec)	N/A	0.142	0.050	1.861	0.271	0.271	0.294	0.273	9.247

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	19	28	39	0	64	20
N.S.	1	1.00	0.92	0.73	1.08	1.50	0.00	2.46	0.77
time (sec)	N/A	0.135	0.078	1.801	0.190	0.267	0.000	0.287	0.045

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	36	39	55	63	0	74	31
N.S.	1	1.00	0.68	0.74	1.04	1.19	0.00	1.40	0.58
time (sec)	N/A	0.155	0.110	1.854	0.187	0.274	0.000	0.306	9.112

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	84	48	50	82	83	0	84	40
N.S.	1	1.06	0.61	0.63	1.04	1.05	0.00	1.06	0.51
time (sec)	N/A	0.182	0.146	1.904	0.188	0.283	0.000	0.289	9.231

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	14	46	9	8	19	8	27	8
N.S.	1	1.17	3.83	0.75	0.67	1.58	0.67	2.25	0.67
time (sec)	N/A	0.137	0.051	1.899	0.291	0.276	0.256	0.271	8.972

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	19	28	29	0	29	18
N.S.	1	1.00	0.95	0.86	1.27	1.32	0.00	1.32	0.82
time (sec)	N/A	0.138	0.079	1.803	0.216	0.266	0.000	0.269	0.043

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	31	55	46	0	39	28
N.S.	1	1.00	0.69	0.69	1.22	1.02	0.00	0.87	0.62
time (sec)	N/A	0.158	0.106	1.824	0.189	0.256	0.000	0.276	9.005

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	41	41	82	61	0	49	73
N.S.	1	1.07	0.61	0.61	1.22	0.91	0.00	0.73	1.09
time (sec)	N/A	0.180	0.129	1.829	0.189	0.398	0.000	0.276	9.082

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	18	57	25	21	27	54	41	42
N.S.	1	1.50	4.75	2.08	1.75	2.25	4.50	3.42	3.50
time (sec)	N/A	0.142	0.070	2.184	0.291	0.378	0.258	0.281	9.098

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	58	37	29	27	51	59	36
N.S.	1	1.00	2.42	1.54	1.21	1.12	2.12	2.46	1.50
time (sec)	N/A	0.142	0.092	2.089	0.218	0.434	0.247	0.284	9.074

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	12	40	7	8	18	5	25	6
N.S.	1	1.20	4.00	0.70	0.80	1.80	0.50	2.50	0.60
time (sec)	N/A	0.134	0.049	2.281	0.301	0.775	0.231	0.290	9.067

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	39	9	17	17	17	33	11
N.S.	1	1.00	2.44	0.56	1.06	1.06	1.06	2.06	0.69
time (sec)	N/A	0.135	0.039	1.899	0.204	0.257	0.238	0.280	9.374

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	39	14	17	17	17	33	11
N.S.	1	1.00	2.44	0.88	1.06	1.06	1.06	2.06	0.69
time (sec)	N/A	0.137	0.045	1.928	0.196	0.280	0.216	0.283	9.631

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	448	350	48	0	0	0	0	0	36
N.S.	1	0.78	0.11	0.00	0.00	0.00	0.00	0.00	0.08
time (sec)	N/A	0.353	10.021	0.000	0.000	0.000	0.000	0.000	9.248

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	387	313	45	0	0	0	0	0	36
N.S.	1	0.81	0.12	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.321	10.014	0.000	0.000	0.000	0.000	0.000	9.077

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	278	43	0	0	0	0	0	36
N.S.	1	0.86	0.13	0.00	0.00	0.00	0.00	0.00	0.11
time (sec)	N/A	0.292	10.016	0.000	0.000	0.000	0.000	0.000	9.044

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	384	308	47	0	0	0	0	0	36
N.S.	1	0.80	0.12	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.327	10.020	0.000	0.000	0.000	0.000	0.000	9.068

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	448	343	50	0	0	0	0	0	36
N.S.	1	0.77	0.11	0.00	0.00	0.00	0.00	0.00	0.08
time (sec)	N/A	0.353	10.019	0.000	0.000	0.000	0.000	0.000	9.063

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	842	727	48	0	0	0	0	0	36
N.S.	1	0.86	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	0.545	10.020	0.000	0.000	0.000	0.000	0.000	9.088

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	781	690	45	0	0	0	0	0	36
N.S.	1	0.88	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.478	10.015	0.000	0.000	0.000	0.000	0.000	9.058

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	715	657	45	0	0	0	0	0	36
N.S.	1	0.92	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.438	10.015	0.000	0.000	0.000	0.000	0.000	9.089

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	773	684	45	0	0	0	0	0	36
N.S.	1	0.88	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.478	10.017	0.000	0.000	0.000	0.000	0.000	9.072

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	838	719	50	0	0	0	0	0	36
N.S.	1	0.86	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	0.527	10.019	0.000	0.000	0.000	0.000	0.000	9.053

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	139	48	0	0	0	0	0	36
N.S.	1	1.17	0.40	0.00	0.00	0.00	0.00	0.00	0.30
time (sec)	N/A	0.261	10.021	0.000	0.000	0.000	0.000	0.000	9.065

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	102	45	0	0	0	0	0	36
N.S.	1	1.13	0.50	0.00	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.224	10.018	0.000	0.000	0.000	0.000	0.000	9.028

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	102	45	0	0	0	0	0	36
N.S.	1	1.13	0.50	0.00	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.220	10.014	0.000	0.000	0.000	0.000	0.000	8.998

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	71	45	0	0	0	0	0	36
N.S.	1	1.22	0.78	0.00	0.00	0.00	0.00	0.00	0.62
time (sec)	N/A	0.196	10.016	0.000	0.000	0.000	0.000	0.000	9.018

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	71	43	0	0	0	0	0	36
N.S.	1	1.20	0.73	0.00	0.00	0.00	0.00	0.00	0.61
time (sec)	N/A	0.195	10.015	0.000	0.000	0.000	0.000	0.000	9.040

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	95	45	0	0	0	0	0	36
N.S.	1	1.14	0.54	0.00	0.00	0.00	0.00	0.00	0.43
time (sec)	N/A	0.218	10.019	0.000	0.000	0.000	0.000	0.000	9.091

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	130	50	0	0	0	0	0	36
N.S.	1	1.13	0.43	0.00	0.00	0.00	0.00	0.00	0.31
time (sec)	N/A	0.251	10.023	0.000	0.000	0.000	0.000	0.000	9.249

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	165	50	0	0	0	0	0	36
N.S.	1	1.13	0.34	0.00	0.00	0.00	0.00	0.00	0.25
time (sec)	N/A	0.286	10.019	0.000	0.000	0.000	0.000	0.000	9.225

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	45	0	0	0	0	0	48
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.166	0.038	0.000	0.000	0.000	0.000	0.000	9.355

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	43	43	49	43	43
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84
time (sec)	N/A	0.184	0.004	2.128	0.194	0.256	0.019	0.313	0.025

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.167	0.003	2.136	0.195	0.267	0.023	0.314	0.039

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.158	0.002	2.173	0.189	0.289	0.016	0.296	0.030

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.141	0.000	0.024	0.193	0.266	0.017	0.262	0.017

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.138	0.005	2.069	0.277	0.300	0.063	0.278	0.055

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.153	0.032	2.140	0.278	0.554	0.101	0.309	9.204

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	70	55	57	58	188	105	45	55
N.S.	1	1.13	0.89	0.92	0.94	3.03	1.69	0.73	0.89
time (sec)	N/A	0.166	0.050	2.121	0.277	0.469	0.166	0.285	9.121

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	90	71	59	58	146	97	63	37
N.S.	1	1.07	0.85	0.70	0.69	1.74	1.15	0.75	0.44
time (sec)	N/A	0.182	0.134	2.149	0.221	0.456	2.653	0.270	9.090

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	68	60	48	43	124	70	49	37
N.S.	1	1.05	0.92	0.74	0.66	1.91	1.08	0.75	0.57
time (sec)	N/A	0.177	0.093	2.109	0.205	0.756	1.583	0.292	8.969

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	36	28	94	41	37	35
N.S.	1	1.00	1.04	0.78	0.61	2.04	0.89	0.80	0.76
time (sec)	N/A	0.158	0.054	2.205	0.185	0.281	0.959	0.274	0.126

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	0.80
time (sec)	N/A	0.146	0.008	2.371	0.199	0.272	0.509	0.273	9.004

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88
time (sec)	N/A	0.135	0.046	2.438	0.191	0.270	0.343	0.278	0.034

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	28
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.149	0.079	2.098	0.186	0.271	0.495	0.282	8.981

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	66	40	37	46	69	413	41	44
N.S.	1	1.14	0.69	0.64	0.79	1.19	7.12	0.71	0.76
time (sec)	N/A	0.162	0.097	1.978	0.185	0.290	0.804	0.298	9.041

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	93	51	48	61	91	1265	55	61
N.S.	1	1.21	0.66	0.62	0.79	1.18	16.43	0.71	0.79
time (sec)	N/A	0.176	0.115	2.123	0.192	0.282	1.230	0.280	9.085

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	25	20	17	30	19	80	45	19
N.S.	1	1.09	0.87	0.74	1.30	0.83	3.48	1.96	0.83
time (sec)	N/A	0.141	0.012	2.072	0.288	0.394	0.462	0.272	0.050

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	23	23	25	16	30	9	19	26	19
N.S.	1	1.00	1.09	0.70	1.30	0.39	0.83	1.13	0.83
time (sec)	N/A	0.145	0.006	2.067	0.270	0.451	0.294	0.280	9.027

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	9	6	8	22	25	14
N.S.	1	1.00	0.90	0.31	0.21	0.28	0.76	0.86	0.48
time (sec)	N/A	0.144	0.007	2.253	0.279	0.443	0.315	0.273	9.091

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	20	16	9	14	0	17	21
N.S.	1	1.00	0.80	0.64	0.36	0.56	0.00	0.68	0.84
time (sec)	N/A	0.146	0.006	2.147	0.280	0.667	0.000	0.268	9.005

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	23	23	25	16	30	9	19	26	13
N.S.	1	1.00	1.09	0.70	1.30	0.39	0.83	1.13	0.57
time (sec)	N/A	0.144	0.014	0.172	0.272	0.257	0.422	0.277	0.114

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	6	8	22	15	14
N.S.	1	1.00	0.90	0.79	0.21	0.28	0.76	0.52	0.48
time (sec)	N/A	0.146	0.011	1.909	0.286	0.254	0.414	0.281	9.148

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	30	9	19	26	18
N.S.	1	1.00	1.17	1.17	1.30	0.39	0.83	1.13	0.78
time (sec)	N/A	0.143	0.008	1.970	0.269	0.268	0.432	0.272	9.115

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	10	6	6	24	23	15
N.S.	1	1.00	0.97	0.34	0.21	0.21	0.83	0.79	0.52
time (sec)	N/A	0.148	0.007	2.091	0.273	0.278	0.417	0.271	9.080

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	30	9	20	26	18
N.S.	1	1.00	1.17	1.17	1.30	0.39	0.87	1.13	0.78
time (sec)	N/A	0.147	0.007	2.093	0.296	0.258	0.458	0.264	0.062

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	10	6	6	24	23	15
N.S.	1	1.00	0.97	0.34	0.21	0.21	0.83	0.79	0.52
time (sec)	N/A	0.152	0.004	2.223	0.286	0.262	0.414	0.265	9.119

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	206	273	234	233	253	334	184
N.S.	1	1.04	1.89	2.50	2.15	2.14	2.32	3.06	1.69
time (sec)	N/A	0.335	0.033	2.276	0.192	0.263	0.086	0.269	9.160

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	206	273	234	235	250	334	184
N.S.	1	1.04	1.89	2.50	2.15	2.16	2.29	3.06	1.69
time (sec)	N/A	0.323	0.047	2.232	0.194	0.450	0.089	0.269	9.165

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	198	273	234	227	253	334	176
N.S.	1	1.04	1.82	2.50	2.15	2.08	2.32	3.06	1.61
time (sec)	N/A	0.335	0.038	2.256	0.201	0.433	0.078	0.263	9.202

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	206	273	234	235	248	334	184
N.S.	1	1.04	1.89	2.50	2.15	2.16	2.28	3.06	1.69
time (sec)	N/A	0.332	0.044	2.272	0.191	0.379	0.079	0.276	9.226

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	21	18	17	16	16	22	16	16
N.S.	1	1.17	1.00	0.94	0.89	0.89	1.22	0.89	0.89
time (sec)	N/A	0.144	0.006	4.031	0.282	0.739	0.049	0.269	0.031

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	16	12	9	8	10	7	8	8
N.S.	1	1.33	1.00	0.75	0.67	0.83	0.58	0.67	0.67
time (sec)	N/A	0.148	0.018	3.558	0.284	0.264	0.081	0.258	9.296

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	45	34	17	27	39	39	31	15
N.S.	1	2.37	1.79	0.89	1.42	2.05	2.05	1.63	0.79
time (sec)	N/A	0.196	0.026	2.151	0.276	0.259	0.053	0.256	0.236

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	21	13	12	13	13	10	15	8
N.S.	1	1.62	1.00	0.92	1.00	1.00	0.77	1.15	0.62
time (sec)	N/A	0.159	0.005	2.104	0.184	0.259	0.044	0.277	0.084

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	15	15	14	17	8
N.S.	1	1.10	1.00	0.67	0.71	0.71	0.67	0.81	0.38
time (sec)	N/A	0.162	0.004	2.180	0.180	0.273	0.054	0.253	0.096

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	17	17	14	13	13	12	15	6
N.S.	1	2.83	2.83	2.33	2.17	2.17	2.00	2.50	1.00
time (sec)	N/A	0.158	0.005	2.043	0.199	0.265	0.050	0.264	0.079

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	38	50	76	40	23
N.S.	1	1.00	1.00	0.89	1.41	1.85	2.81	1.48	0.85
time (sec)	N/A	0.163	0.012	1.982	0.196	0.266	0.127	0.263	9.566

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	39	51	76	45	23
N.S.	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	0.85
time (sec)	N/A	0.160	0.013	2.080	0.183	0.334	0.130	0.264	9.219

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	41	87	27	23
N.S.	1	1.00	1.00	0.90	0.87	1.32	2.81	0.87	0.74
time (sec)	N/A	0.160	0.012	3.004	0.193	0.448	0.091	0.261	9.187

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	39	51	76	45	23
N.S.	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	0.85
time (sec)	N/A	0.159	0.015	1.988	0.202	0.401	0.126	0.271	9.236

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	0	113	124	34	46
N.S.	1	1.00	1.00	0.92	0.00	2.97	3.26	0.89	1.21
time (sec)	N/A	0.169	0.014	2.115	0.000	0.421	0.103	0.270	8.994

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	41	34	35	0	124	100	30	33
N.S.	1	1.17	0.97	1.00	0.00	3.54	2.86	0.86	0.94
time (sec)	N/A	0.168	0.011	2.313	0.000	0.306	0.122	0.257	9.062

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	37	41	32	49	67	102	55	28
N.S.	1	1.16	1.28	1.00	1.53	2.09	3.19	1.72	0.88
time (sec)	N/A	0.165	0.012	2.106	0.286	0.260	0.131	0.263	9.066

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	46	43	34	36	45	39	36	33
N.S.	1	1.07	1.00	0.79	0.84	1.05	0.91	0.84	0.77
time (sec)	N/A	0.170	0.027	2.199	0.285	0.259	0.066	0.261	0.038

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	69	62	37	47	68	58	51	34
N.S.	1	1.60	1.44	0.86	1.09	1.58	1.35	1.19	0.79
time (sec)	N/A	0.201	0.029	2.053	0.292	0.288	0.074	0.266	9.009

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	41	33	32	34	53	31	36	34
N.S.	1	1.21	0.97	0.94	1.00	1.56	0.91	1.06	1.00
time (sec)	N/A	0.177	0.013	1.978	0.189	0.332	0.061	0.267	0.090

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	45	42	32	34	53	32	36	34
N.S.	1	1.07	1.00	0.76	0.81	1.26	0.76	0.86	0.81
time (sec)	N/A	0.184	0.017	2.084	0.186	0.246	0.071	0.259	0.081

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	68	0	334	265	67	119
N.S.	1	1.00	0.99	0.96	0.00	4.70	3.73	0.94	1.68
time (sec)	N/A	0.206	0.064	2.220	0.000	0.307	0.309	0.263	0.160

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	78	72	86	0	317	230	75	107
N.S.	1	1.08	1.00	1.19	0.00	4.40	3.19	1.04	1.49
time (sec)	N/A	0.208	0.053	2.242	0.000	0.288	0.311	0.259	9.153

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	78	83	97	171	218	90	100
N.S.	1	1.07	1.13	1.20	1.41	2.48	3.16	1.30	1.45
time (sec)	N/A	0.201	0.071	2.603	0.289	0.283	0.292	0.266	9.118

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	76	62	111	0	89	212	100	110
N.S.	1	1.23	1.00	1.79	0.00	1.44	3.42	1.61	1.77
time (sec)	N/A	0.246	0.134	3.144	0.000	0.454	0.486	0.283	0.153

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	63	120	31	55	63	56	56	38
N.S.	1	1.91	3.64	0.94	1.67	1.91	1.70	1.70	1.15
time (sec)	N/A	0.227	0.118	2.302	0.191	0.426	0.140	0.274	0.154

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	63	120	31	51	59	56	54	38
N.S.	1	2.03	3.87	1.00	1.65	1.90	1.81	1.74	1.23
time (sec)	N/A	0.219	0.099	2.200	0.192	0.517	0.146	0.269	0.127

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	24	17	33	27	15	165	27	27
N.S.	1	1.41	1.00	1.94	1.59	0.88	9.71	1.59	1.59
time (sec)	N/A	0.156	0.022	3.114	0.269	0.256	0.093	0.267	9.039

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	30	23	39	33	21	70	33	42
N.S.	1	1.30	1.00	1.70	1.43	0.91	3.04	1.43	1.83
time (sec)	N/A	0.160	0.035	3.141	0.296	0.251	0.359	0.263	9.164

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	48	29	38	40	32	40	39
N.S.	1	1.00	1.26	0.76	1.00	1.05	0.84	1.05	1.03
time (sec)	N/A	0.169	0.080	2.773	0.297	0.246	0.294	0.261	0.076

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	36	49	25	38	53	26	24	23
N.S.	1	1.20	1.63	0.83	1.27	1.77	0.87	0.80	0.77
time (sec)	N/A	0.166	0.100	2.278	0.287	0.262	0.307	0.265	0.049

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	52	40	44	41	39
N.S.	1	1.00	1.00	0.82	1.06	0.82	0.90	0.84	0.80
time (sec)	N/A	0.174	0.107	2.231	0.274	0.242	0.278	0.275	0.111

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	48	53	35	46	58	41	53	48
N.S.	1	1.07	1.18	0.78	1.02	1.29	0.91	1.18	1.07
time (sec)	N/A	0.172	0.138	2.385	0.277	0.255	0.285	0.260	9.009

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	48	61	35	46	60	42	36	35
N.S.	1	1.07	1.36	0.78	1.02	1.33	0.93	0.80	0.78
time (sec)	N/A	0.178	0.198	2.216	0.274	0.262	0.292	0.270	0.051

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	50	58	58	53	54	48
N.S.	1	1.00	0.90	0.81	0.94	0.94	0.85	0.87	0.77
time (sec)	N/A	0.175	0.201	2.260	0.280	0.266	0.300	0.267	0.095

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	56	32	41	60	37	31	30
N.S.	1	1.00	1.30	0.74	0.95	1.40	0.86	0.72	0.70
time (sec)	N/A	0.168	0.147	2.175	0.289	0.253	0.300	0.284	8.971

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	50	58	58	54	54	48
N.S.	1	1.00	1.03	0.85	0.98	0.98	0.92	0.92	0.81
time (sec)	N/A	0.176	0.283	2.220	0.283	0.255	0.307	0.270	9.013

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	46	46	86	56	0	36
N.S.	1	1.00	0.92	0.78	0.78	1.46	0.95	0.00	0.61
time (sec)	N/A	0.178	0.777	3.190	0.279	0.264	0.305	0.000	0.052

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	50	58	58	54	54	48
N.S.	1	1.00	0.90	0.81	0.94	0.94	0.87	0.87	0.77
time (sec)	N/A	0.176	0.213	2.305	0.274	0.259	0.287	0.262	0.218

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	56	32	41	60	32	31	30
N.S.	1	1.00	1.44	0.82	1.05	1.54	0.82	0.79	0.77
time (sec)	N/A	0.166	0.142	2.272	0.281	0.273	0.302	0.266	0.050

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	24	9	8	20	10	40	20
N.S.	1	1.00	1.71	0.64	0.57	1.43	0.71	2.86	1.43
time (sec)	N/A	0.143	0.069	2.263	0.282	0.258	0.297	0.260	9.051

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	14	25	7	8	33	7	24	6
N.S.	1	1.40	2.50	0.70	0.80	3.30	0.70	2.40	0.60
time (sec)	N/A	0.144	0.083	2.421	0.278	0.255	0.310	0.270	8.963

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	21	22	20	22	41	20
N.S.	1	1.00	0.96	0.84	0.88	0.80	0.88	1.64	0.80
time (sec)	N/A	0.152	0.001	2.666	0.289	0.256	0.305	0.268	9.166

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	21	27	15	16	38	20	53	26
N.S.	1	1.17	1.50	0.83	0.89	2.11	1.11	2.94	1.44
time (sec)	N/A	0.149	0.091	2.411	0.296	0.262	0.292	0.275	9.164

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	22	39	15	16	40	20	36	16
N.S.	1	1.16	2.05	0.79	0.84	2.11	1.05	1.89	0.84
time (sec)	N/A	0.144	0.110	2.370	0.308	0.248	0.289	0.268	9.298

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	30	28	38	32	54	26
N.S.	1	1.00	0.86	0.86	0.80	1.09	0.91	1.54	0.74
time (sec)	N/A	0.150	0.108	2.192	0.277	0.251	0.292	0.261	9.324

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	30	12	11	40	15	31	11
N.S.	1	1.00	1.76	0.71	0.65	2.35	0.88	1.82	0.65
time (sec)	N/A	0.136	0.089	2.255	0.265	0.258	0.309	0.274	9.007

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	27	30	28	37	32	54	26
N.S.	1	1.00	0.84	0.94	0.88	1.16	1.00	1.69	0.81
time (sec)	N/A	0.149	0.085	2.285	0.307	0.251	0.298	0.282	9.108

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	28	26	16	67	37	0	17
N.S.	1	1.00	0.85	0.79	0.48	2.03	1.12	0.00	0.52
time (sec)	N/A	0.150	0.113	2.395	0.281	0.260	0.288	0.000	9.072

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	30	28	38	32	54	26
N.S.	1	1.00	0.86	0.86	0.80	1.09	0.91	1.54	0.74
time (sec)	N/A	0.150	0.101	2.123	0.276	0.261	0.298	0.281	9.191

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	30	12	11	40	12	31	11
N.S.	1	1.00	2.31	0.92	0.85	3.08	0.92	2.38	0.85
time (sec)	N/A	0.141	0.088	2.228	0.279	0.252	0.294	0.261	9.094

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	44	51	16	137	80	60	40
N.S.	1	1.00	2.00	2.32	0.73	6.23	3.64	2.73	1.82
time (sec)	N/A	0.147	0.213	2.292	0.189	0.273	0.467	0.333	9.364

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	123	44	19	141	83	63	46
N.S.	1	1.00	5.35	1.91	0.83	6.13	3.61	2.74	2.00
time (sec)	N/A	0.149	0.337	2.489	0.274	0.276	0.526	0.275	9.274

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	123	44	19	143	78	61	44
N.S.	1	1.00	6.15	2.20	0.95	7.15	3.90	3.05	2.20
time (sec)	N/A	0.145	0.320	2.495	0.281	0.266	0.508	0.281	9.261

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	26	38	0	17	15
N.S.	1	1.00	1.00	0.95	1.37	2.00	0.00	0.89	0.79
time (sec)	N/A	0.136	0.128	2.353	0.199	0.261	0.000	0.276	8.971

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	30	41	0	17	17
N.S.	1	1.00	1.00	0.78	1.30	1.78	0.00	0.74	0.74
time (sec)	N/A	0.139	0.143	2.302	0.195	0.284	0.000	0.276	0.058

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	33	20	30	29	0	29	19
N.S.	1	1.00	1.43	0.87	1.30	1.26	0.00	1.26	0.83
time (sec)	N/A	0.139	0.123	2.186	0.187	0.274	0.000	0.265	0.055

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	59	49	0	36	29
N.S.	1	1.00	1.00	0.84	1.37	1.14	0.00	0.84	0.67
time (sec)	N/A	0.160	0.007	2.224	0.198	0.261	0.000	0.270	0.034

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	40	37	0	0	0	0	0	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	40	37	0	0	0	0	0	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	21	30	21	0	0	0	0	0	0
N.S.	1	1.43	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	48	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	0.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	19	20	16	32
N.S.	1	1.00	1.00	0.94	0.89	1.06	1.11	0.89	1.78
time (sec)	N/A	0.134	0.004	2.112	0.187	0.253	0.029	0.267	9.255

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	33	26	0	0	0	0	0	0
N.S.	1	1.06	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.019	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	33	26	0	0	0	0	0	0
N.S.	1	1.06	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	40	37	0	0	0	0	0	0
N.S.	1	1.05	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	0.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	40	37	0	0	0	0	0	0
N.S.	1	1.05	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.157	0.022	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [35] had the largest ratio of [.6153849999999999960]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.16	13	0.462
2	A	7	6	1.12	15	0.400
3	A	6	5	1.11	15	0.333
4	A	5	4	1.07	15	0.267
5	A	4	3	1.00	15	0.200
6	A	7	6	1.17	13	0.462
7	A	6	5	1.15	13	0.385
8	A	5	4	1.12	13	0.308
9	A	4	3	1.06	13	0.231
10	A	4	3	1.06	13	0.231
11	A	4	3	1.06	13	0.231
12	A	5	4	1.10	11	0.364
13	A	4	3	1.00	11	0.273
14	A	4	3	1.00	11	0.273
15	A	4	3	1.00	11	0.273
16	A	3	3	1.07	13	0.231
17	A	3	2	1.00	15	0.133
18	A	1	1	1.00	15	0.067
19	A	2	2	1.00	15	0.133
20	A	3	3	1.06	15	0.200
21	A	3	2	1.17	13	0.154
22	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	13	0.154
24	A	3	3	1.07	13	0.231
25	A	3	2	1.50	16	0.125
26	A	3	2	1.00	15	0.133
27	A	3	2	1.20	13	0.154
28	A	3	2	1.00	11	0.182
29	A	3	2	1.00	11	0.182
30	A	7	6	0.78	13	0.462
31	A	6	5	0.81	13	0.385
32	A	5	4	0.86	13	0.308
33	A	6	5	0.80	13	0.385
34	A	7	6	0.77	13	0.462
35	A	9	8	0.86	13	0.615
36	A	8	7	0.88	13	0.538
37	A	7	6	0.92	13	0.462
38	A	8	7	0.88	13	0.538
39	A	9	8	0.86	13	0.615
40	A	6	5	1.17	13	0.385
41	A	5	4	1.13	13	0.308
42	A	5	4	1.13	13	0.308
43	A	4	3	1.22	13	0.231
44	A	4	3	1.20	13	0.231
45	A	5	4	1.14	13	0.308
46	A	6	5	1.13	13	0.385
47	A	7	6	1.13	13	0.462
48	A	1	1	1.00	11	0.091
49	A	2	2	1.00	9	0.222
50	A	2	2	1.00	9	0.222
51	A	2	2	1.00	9	0.222
52	A	1	1	1.00	7	0.143
53	A	1	1	1.00	9	0.111
54	A	2	2	1.00	9	0.222
55	A	3	3	1.13	9	0.333
56	A	6	5	1.07	11	0.455

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	5	4	1.05	11	0.364
58	A	4	3	1.00	11	0.273
59	A	3	2	1.00	11	0.182
60	A	1	1	1.00	11	0.091
61	A	2	2	1.00	11	0.182
62	A	3	3	1.14	11	0.273
63	A	4	4	1.21	11	0.364
64	A	2	2	1.09	14	0.143
65	A	2	2	1.00	14	0.143
66	A	2	2	1.00	14	0.143
67	A	1	1	1.00	14	0.071
68	A	2	2	1.00	14	0.143
69	A	2	2	1.00	14	0.143
70	A	2	2	1.00	14	0.143
71	A	2	2	1.00	14	0.143
72	A	2	2	1.00	14	0.143
73	A	2	2	1.00	14	0.143
74	A	2	2	1.04	23	0.087
75	A	2	2	1.04	23	0.087
76	A	2	2	1.04	23	0.087
77	A	2	2	1.04	23	0.087
78	A	3	2	1.17	12	0.167
79	A	3	2	1.33	15	0.133
80	B	2	2	2.37	12	0.167
81	A	2	2	1.62	12	0.167
82	A	2	2	1.10	12	0.167
83	B	2	2	2.83	10	0.200
84	A	3	2	1.00	12	0.167
85	A	3	2	1.00	12	0.167
86	A	3	2	1.00	12	0.167
87	A	3	2	1.00	12	0.167
88	A	3	2	1.00	12	0.167
89	A	3	2	1.17	13	0.154
90	A	3	2	1.16	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	4	3	1.07	12	0.250
92	A	3	3	1.60	12	0.250
93	A	2	2	1.21	12	0.167
94	A	2	2	1.07	12	0.167
95	A	4	3	1.00	12	0.250
96	A	4	3	1.08	13	0.231
97	A	4	3	1.07	14	0.214
98	A	3	2	1.23	40	0.050
99	A	2	2	1.91	30	0.067
100	B	2	2	2.03	31	0.065
101	A	3	2	1.41	14	0.143
102	A	3	2	1.30	16	0.125
103	A	4	3	1.00	14	0.214
104	A	4	3	1.20	14	0.214
105	A	4	3	1.00	14	0.214
106	A	4	3	1.07	14	0.214
107	A	4	3	1.07	14	0.214
108	A	4	3	1.00	14	0.214
109	A	4	3	1.00	14	0.214
110	A	4	3	1.00	14	0.214
111	A	4	3	1.00	14	0.214
112	A	4	3	1.00	14	0.214
113	A	4	3	1.00	14	0.214
114	A	3	2	1.00	14	0.143
115	A	3	2	1.40	14	0.143
116	A	3	2	1.00	14	0.143
117	A	3	2	1.17	14	0.143
118	A	3	2	1.16	14	0.143
119	A	3	2	1.00	14	0.143
120	A	3	2	1.00	14	0.143
121	A	3	2	1.00	14	0.143
122	A	3	2	1.00	14	0.143
123	A	3	2	1.00	14	0.143
124	A	3	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	3	2	1.00	27	0.074
126	A	3	2	1.00	30	0.067
127	A	3	2	1.00	28	0.071
128	A	1	1	1.00	12	0.083
129	A	1	1	1.00	14	0.071
130	A	1	1	1.00	16	0.062
131	A	2	2	1.00	14	0.143
132	A	1	1	1.00	12	0.083
133	A	3	2	1.08	12	0.167
134	A	3	2	1.00	12	0.167
135	A	3	2	1.08	12	0.167
136	A	3	2	1.43	12	0.167
137	A	1	1	1.00	10	0.100
138	A	1	1	1.00	7	0.143
139	A	3	2	1.06	12	0.167
140	A	3	2	1.06	12	0.167
141	A	3	2	1.05	12	0.167
142	A	3	2	1.00	12	0.167
143	A	3	2	1.05	12	0.167

CHAPTER 3

Listing of integrals

3.1 $\int (bx+cx^2)^{7/2}\,dx$

3.1.1 Optimal result

Integrand size = 13, antiderivative size = 147

$$\int (bx+cx^2)^{7/2}\,dx = -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)(bx+cx^2)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)(bx+cx^2)^{5/2}}{384c^2}+\frac{(b+2cx)(bx+cx^2)^{7/2}}{16c}+\frac{35b^8\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}}$$

output
```
35/6144*b^4*(2*c*x+b)*(c*x^2+b*x)^(3/2)/c^3-7/384*b^2*(2*c*x+b)*(c*x^2+b*x
)^(5/2)/c^2+1/16*(2*c*x+b)*(c*x^2+b*x)^(7/2)/c+35/16384*b^8*arctanh(x*c^(1
/2)/(c*x^2+b*x)^(1/2))/c^(9/2)-35/16384*b^6*(2*c*x+b)*(c*x^2+b*x)^(1/2)/c^
4
```

3.1.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03

$$\int (bx+cx^2)^{7/2}\,dx = \frac{\sqrt{x(b+cx)}\Big(\sqrt{c}(-105b^7+70b^6cx-56b^5c^2x^2+48b^4c^3x^3+10880b^3c^4x^4+25856b^2c^5x^5+21}{49152c^{9/2}}$$

input
```
Integrate[(b*x + c*x^2)^(7/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(-105*b^7 + 70*b^6*c*x - 56*b^5*c^2*x^2 + 48*b
^4*c^3*x^3 + 10880*b^3*c^4*x^4 + 25856*b^2*c^5*x^5 + 21504*b*c^6*x^6 + 614
4*c^7*x^7) + (210*b^8*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])
])/(Sqrt[x]*Sqrt[b + c*x])))/(49152*c^(9/2))
```

3.1.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1087, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{7/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}-\frac{7b^2\int \left(cx^2+bx\right)^{5/2}dx}{32c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}-\frac{7b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int \left(cx^2+bx\right)^{3/2}dx}{24c}\right)}{32c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}-\frac{7b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)}{24c}\right)}{32c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}-\frac{7b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c}\right)}{32c}$$

$$\Big\downarrow \; 1091$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}-\frac{7b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c}\right)}{32c}$$

$$\Big\downarrow \; 219$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}-\frac{7b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c}\right)}{32c}$$

input
```
Int[(b*x + c*x^2)^(7/2),x]
```

3.1. $\int \left(bx+cx^2\right)^{7/2} dx$

output
```
((b + 2*c*x)*(b*x + c*x^2)^(7/2))/(16*c) - (7*b^2*(((b + 2*c*x)*(b*x + c*x
^2)^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*b
^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[
b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24*c)))/(32*c)
```

3.1.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.1.4 Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\left(-6144c^7x^7-21504b\,c^6x^6-25856b^2c^5x^5-10880b^3c^4x^4-48b^4c^3x^3+56b^5c^2x^2-70b^6cx+105b^7\right)x(cx+b)}{49152c^4\sqrt{x(cx+b)}}+\frac{35b^8\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2}\right.}{32768c^{\frac{9}{2}}}$
default	$\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{7}{2}}}{16c}-\frac{7b^2\left(\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{5}{2}}}{12c}-\frac{5b^2\left(\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}\right)}{32c}$

input
```
int((c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/49152*(-6144*c^7*x^7-21504*b*c^6*x^6-25856*b^2*c^5*x^5-10880*b^3*c^4*x^
4-48*b^4*c^3*x^3+56*b^5*c^2*x^2-70*b^6*c*x+105*b^7)*x*(c*x+b)/c^4/(x*(c*x+
b))^(1/2)+35/32768*b^8/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.76

$$\int (bx+cx^2)^{7/2}\,dx = \left[\frac{105\,b^8\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(6144\,c^8x^7+21504\,bc^7x^6+25856\,b^2c^6x^5+10\ldots}{98304\,c^5}\right.$$

$$-\frac{105\,b^8\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)-(6144\,c^8x^7+21504\,bc^7x^6+25856\,b^2c^6x^5+10880\,b^3c^5x^4+48\,b^4c^4x^3-\ldots}{49152\,c^5}$$

input
```
integrate((c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
[1/98304*(105*b^8*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2
*(6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4 +
 48*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 +
b*x))/c^5, -1/49152*(105*b^8*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c
*x)) - (6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5
*x^4 + 48*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*
x^2 + b*x))/c^5]
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 699, normalized size of antiderivative = 4.76

$$\int (bx + cx^2)^{7/2}\, dx = b^3 \left(\begin{cases} \frac{7b^5 \left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{256c^4} + \sqrt{bx+cx^2}\left(-\frac{7b^4}{128c^4} + \frac{7b^3x}{192c^3} - \frac{7b^2x^2}{240c^2} + \frac{bx^3}{40c} + \cdots \\ \frac{2(bx)^{\frac{9}{2}}}{9b^4} \\ 0 \end{cases}\right)$$

$$+3b^2c \left(\begin{cases} -\frac{21b^6 \left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{1024c^5} + \sqrt{bx+cx^2}\cdot\left(\frac{21b^5}{512c^5} - \frac{7b^4x}{256c^4} + \frac{7b^3x^2}{320c^3} - \frac{3b^2x^3}{160c^2} + \frac{bx^4}{60c} + \cdots \\ \frac{2(bx)^{\frac{11}{2}}}{11b^5} \\ 0 \end{cases}\right)$$

$$+3bc^2 \left(\begin{cases} \frac{33b^7 \left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{2048c^6} + \sqrt{bx+cx^2}\left(-\frac{33b^6}{1024c^6} + \frac{11b^5x}{512c^5} - \frac{11b^4x^2}{640c^4} + \frac{33b^3x^3}{2240c^3} - \frac{11b^2x^4}{840c^2} \cdots \\ \frac{2(bx)^{\frac{13}{2}}}{13b^6} \\ 0 \end{cases}\right)$$

$$+c^3 \left(\begin{cases} -\frac{429b^8 \left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{32768c^7} + \sqrt{bx+cx^2}\cdot\left(\frac{429b^7}{16384c^7} - \frac{143b^6x}{8192c^6} + \frac{143b^5x^2}{10240c^5} - \frac{429b^4x^3}{35840c^4} + \cdots \\ \frac{2(bx)^{\frac{15}{2}}}{15b^7} \\ 0 \end{cases}\right)$$

input
```
integrate((c*x**2+b*x)**(7/2),x)
```

output
```
b**3*Piecewise((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2
*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(
2*c) + x)**2), True))/(256*c**4) + sqrt(b*x + c*x**2)*(-7*b**4/(128*c**4)
+ 7*b**3*x/(192*c**3) - 7*b**2*x**2/(240*c**2) + b*x**3/(40*c) + x**4/5),
Ne(c, 0)), (2*(b*x)**(9/2)/(9*b**4), Ne(b, 0)), (0, True)) + 3*b**2*c*Piec
ewise((-21*b**6*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/s
qrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) +
x)**2), True))/(1024*c**5) + sqrt(b*x + c*x**2)*(21*b**5/(512*c**5) - 7*b*
*4*x/(256*c**4) + 7*b**3*x**2/(320*c**3) - 3*b**2*x**3/(160*c**2) + b*x**4
/(60*c) + x**5/6), Ne(c, 0)), (2*(b*x)**(11/2)/(11*b**5), Ne(b, 0)), (0, T
rue)) + 3*b*c**2*Piecewise((33*b**7*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x
+ c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x
)/sqrt(c*(b/(2*c) + x)**2), True))/(2048*c**6) + sqrt(b*x + c*x**2)*(-33*b
**6/(1024*c**6) + 11*b**5*x/(512*c**5) - 11*b**4*x**2/(640*c**4) + 33*b**3
*x**3/(2240*c**3) - 11*b**2*x**4/(840*c**2) + b*x**5/(84*c) + x**6/7), Ne(
c, 0)), (2*(b*x)**(13/2)/(13*b**6), Ne(b, 0)), (0, True)) + c**3*Piecewise
((-429*b**8*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(
c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**
2), True))/(32768*c**7) + sqrt(b*x + c*x**2)*(429*b**7/(16384*c**7) - 143*
b**6*x/(8192*c**6) + 143*b**5*x**2/(10240*c**5) - 429*b**4*x**3/(35840*...
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.22

$$\int \left(bx+cx^2\right)^{7/2}\,dx=\frac{1}{8}\left(cx^2+bx\right)^{\frac{7}{2}}x-\frac{35\sqrt{cx^2+bx}b^6x}{8192\,c^3}+\frac{35\left(cx^2+bx\right)^{\frac{3}{2}}b^4x}{3072\,c^2}-\frac{7\left(cx^2+bx\right)^{\frac{5}{2}}b^2x}{192\,c}+\frac{35\,b^8\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{32768\,c^{\frac{9}{2}}}-\frac{35\sqrt{cx^2+bx}b^7}{16384\,c^4}+\frac{35\left(cx^2+bx\right)^{\frac{3}{2}}b^5}{6144\,c^3}-\frac{7\left(cx^2+bx\right)^{\frac{5}{2}}b^3}{384\,c^2}+\frac{\left(cx^2+bx\right)^{\frac{7}{2}}b}{16\,c}$$

input
```
integrate((c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
1/8*(c*x^2 + b*x)^(7/2)*x - 35/8192*sqrt(c*x^2 + b*x)*b^6*x/c^3 + 35/3072*
(c*x^2 + b*x)^(3/2)*b^4*x/c^2 - 7/192*(c*x^2 + b*x)^(5/2)*b^2*x/c + 35/327
68*b^8*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(9/2) - 35/16384*sqr
t(c*x^2 + b*x)*b^7/c^4 + 35/6144*(c*x^2 + b*x)^(3/2)*b^5/c^3 - 7/384*(c*x^
2 + b*x)^(5/2)*b^3/c^2 + 1/16*(c*x^2 + b*x)^(7/2)*b/c
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int (bx+cx^2)^{7/2}\,dx = -\frac{35\,b^8\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{32768\,c^{\frac{9}{2}}} - \frac{1}{49152}\left(\frac{105\,b^7}{c^4}-2\left(\frac{35\,b^6}{c^3}-4\left(\frac{7\,b^5}{c^2}-2\left(\frac{3\,b^4}{c}+8\left(85\,b^3+2\left(101\,b^2c+12\left(2\,c^3x+7\,bc^2\right)x\right)x\right)x\right)x\right)x\right)x$$

input
```
integrate((c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-35/32768*b^8*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(9
/2) - 1/49152*(105*b^7/c^4 - 2*(35*b^6/c^3 - 4*(7*b^5/c^2 - 2*(3*b^4/c + 8
*(85*b^3 + 2*(101*b^2*c + 12*(2*c^3*x + 7*b*c^2)*x)*x)*x)*x)*x)*x)*sqrt(c*
x^2 + b*x)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03

$$\int (bx+cx^2)^{7/2}\,dx = \frac{(c\,x^2+b\,x)^{7/2}\left(\frac{b}{2}+c\,x\right)}{8\,c} - \frac{7\,b^2\left(\frac{(c\,x^2+b\,x)^{5/2}\left(\frac{b}{2}+c\,x\right)}{6\,c}-\frac{5\,b^2\left(\frac{(c\,x^2+b\,x)^{3/2}\left(\frac{b}{2}+c\,x\right)}{4\,c}-\frac{3\,b^2\left(\sqrt{c\,x^2+b\,x}\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}\right)}{24\,c}\right)}{32\,c}$$

input
```
int((b*x + c*x^2)^(7/2),x)
```

output
```
((b*x + c*x^2)^(7/2)*(b/2 + c*x))/(8*c) - (7*b^2*(((b*x + c*x^2)^(5/2)*(b/
2 + c*x))/(6*c) - (5*b^2*(((b*x + c*x^2)^(3/2)*(b/2 + c*x))/(4*c) - (3*b^2
*((b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*
x + c*x^2)^(1/2)))/(8*c^(3/2))))/(16*c)))/(24*c)))/(32*c)
```

3.2 $\int (3ix+4x^2)^{7/2}\,dx$

3.2.1 Optimal result

Integrand size = 15, antiderivative size = 121

$$\int (3ix+4x^2)^{7/2}\,dx = \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304} + \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} + \frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2} + \frac{229635i\arcsin\left(1-\frac{8ix}{3}\right)}{16777216}$$

output
```
945/131072*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+21/2048*(3*I+8*x)*(3*I*x+4*x^2)^(
5/2)+1/64*(3*I+8*x)*(3*I*x+4*x^2)^(7/2)-229635/16777216*I*arcsin(-1+8/3*I*
x)+25515/4194304*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$\int (3ix+4x^2)^{7/2}\,dx = \frac{\sqrt{x}\sqrt{3i+4x}\left(2\sqrt{x}\sqrt{3i+4x}(76545i-68040x-72576ix^2+82944x^3-25067520ix^4-79429\right.}{8388608\sqrt{x(3i+4x}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(7/2),x]
```

output
```
(Sqrt[x]*Sqrt[3*I + 4*x]*(2*Sqrt[x]*Sqrt[3*I + 4*x]*(76545*I - 68040*x - (
72576*I)*x^2 + 82944*x^3 - (25067520*I)*x^4 - 79429632*x^5 + (88080384*I)*
x^6 + 33554432*x^7) - 229635*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]]))/(8388608*
Sqrt[x*(3*I + 4*x)])
```

3.2.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1087, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^2+3ix\right)^{7/2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{63}{128}\int \left(4x^2+3ix\right)^{5/2}dx+\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{63}{128}\left(\frac{15}{32}\int \left(4x^2+3ix\right)^{3/2}dx+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}\right)+\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{63}{128}\left(\frac{15}{32}\left(\frac{27}{64}\int \sqrt{4x^2+3ix}dx+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}\right)+\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{63}{128}\left(\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{4x^2+3ix}}dx+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4\right.\right. \frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{63}{128}\left(\frac{15}{32}\left(\frac{27}{64}\left(\frac{3}{64}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right) -$$
$$\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}$$

$\downarrow$ 222

$$\frac{63}{128}\left(\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{64}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3$$
$$\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}$$

input
```
Int[((3*I)*x + 4*x^2)^(7/2),x]
```

output
```
((3*I + 8*x)*((3*I)*x + 4*x^2)^(7/2))/64 + (63*(((3*I + 8*x)*((3*I)*x + 4*
x^2)^(5/2))/48 + (15*(((3*I + 8*x)*((3*I)*x + 4*x^2)^(3/2))/32 + (27*(((3*
I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + (9*ArcSinh[(3*I + 8*x)/3])/64))/64))/
32))/128
```

3.2.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.2.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

method	result
risch	$\frac{(33554432x^7+88080384ix^6-79429632x^5-25067520ix^4+82944x^3-72576ix^2-68040x+76545i)(3i+4x)x}{4194304\sqrt{x(3i+4x)}}+\frac{229635 \operatorname{arcsinh}(i+\frac{8x}{3}}{16777216}$
default	$\frac{(3i+8x)(4x^2+3ix)^{\frac{7}{2}}}{64}+\frac{21(3i+8x)(4x^2+3ix)^{\frac{5}{2}}}{2048}+\frac{945(3i+8x)(4x^2+3ix)^{\frac{3}{2}}}{131072}+\frac{25515(3i+8x)\sqrt{4x^2+3ix}}{4194304}+\frac{229635 \operatorname{arcsinh}(i+}{16777216}$
trager	$\left(21ix^6+8x^7-\frac{765}{128}ix^4-\frac{303}{16}x^5-\frac{567}{32768}ix^2+\frac{81}{4096}x^3+\frac{76545}{4194304}i-\frac{8505}{524288}x\right)\sqrt{4x^2+3ix}+\frac{229635 \ln\left(4}{}$

input
```
int((3*I*x+4*x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/4194304*(76545*I-68040*x-72576*I*x^2+82944*x^3-25067520*I*x^4-79429632*x
^5+88080384*I*x^6+33554432*x^7)*(3*I+4*x)*x/(x*(3*I+4*x))^(1/2)+229635/167
77216*arcsinh(I+8/3*x)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int (3ix+4x^2)^{7/2}\,dx = \frac{1}{4194304}\left(33554432\,x^7+88080384i\,x^6-79429632\,x^5-25067520i\,x^4+82944\,x^3-72576i\,x^2\right.$$
$$-\frac{229635}{16777216}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{1165671}{268435456}$$

input
```
integrate((3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

output
```
1/4194304*(33554432*x^7 + 88080384*I*x^6 - 79429632*x^5 - 25067520*I*x^4 +
 82944*x^3 - 72576*I*x^2 - 68040*x + 76545*I)*sqrt(4*x^2 + 3*I*x) - 229635
/16777216*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 1165671/268435456
```

3.2.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(107) = 214.

Time = 1.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.22

$$\int \left(3ix+4x^2\right)^{7/2}\,dx =$$

$$-108\sqrt{4x^2+3ix}\left(\frac{x^5}{6}+\frac{ix^4}{80}+\frac{27x^3}{2560}-\frac{189ix^2}{20480}-\frac{567x}{65536}+\frac{5103i}{524288}\right)$$
$$+64\sqrt{4x^2+3ix}\left(\frac{x^7}{8}+\frac{3ix^6}{448}+\frac{39x^5}{7168}-\frac{1287ix^4}{286720}-\frac{34749x^3}{9175040}+\frac{34749ix^2}{10485760}+\frac{104247x}{33554432}-\frac{938223i}{268435456}\right)$$
$$-27i\left(\sqrt{4x^2+3ix}\left(\frac{x^4}{5}+\frac{3ix^3}{160}+\frac{21x^2}{1280}-\frac{63ix}{4096}-\frac{567}{32768}\right)+\frac{1701i\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{131072}\right)$$
$$+144i\left(\sqrt{4x^2+3ix}\left(\frac{x^6}{7}+\frac{ix^5}{112}+\frac{33x^4}{4480}-\frac{891ix^3}{143360}-\frac{891x^2}{163840}+\frac{2673ix}{524288}+\frac{24057}{4194304}\right)-\frac{72171i\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{16777216}\right)-\frac{16041645\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{16777216}$$

input

```
integrate((3*I*x+4*x**2)**(7/2),x)
```

output

```
-108*sqrt(4*x**2 + 3*I*x)*(x**5/6 + I*x**4/80 + 27*x**3/2560 - 189*I*x**2/
20480 - 567*x/65536 + 5103*I/524288) + 64*sqrt(4*x**2 + 3*I*x)*(x**7/8 + 3
*I*x**6/448 + 39*x**5/7168 - 1287*I*x**4/286720 - 34749*x**3/9175040 + 347
49*I*x**2/10485760 + 104247*x/33554432 - 938223*I/268435456) - 27*I*(sqrt(
4*x**2 + 3*I*x)*(x**4/5 + 3*I*x**3/160 + 21*x**2/1280 - 63*I*x/4096 - 567/
32768) + 1701*I*asinh(8*x/3 + I)/131072) + 144*I*(sqrt(4*x**2 + 3*I*x)*(x*
*6/7 + I*x**5/112 + 33*x**4/4480 - 891*I*x**3/143360 - 891*x**2/163840 + 2
673*I*x/524288 + 24057/4194304) - 72171*I*asinh(8*x/3 + I)/16777216) - 160
41645*asinh(8*x/3 + I)/16777216
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \left(3ix+4x^2\right)^{7/2}\,dx=\frac{1}{8}\left(4\,x^2+3i\,x\right)^{\frac{7}{2}}x+\frac{3}{64}i\left(4\,x^2+3i\,x\right)^{\frac{7}{2}}+\frac{21}{256}\left(4\,x^2+3i\,x\right)^{\frac{5}{2}}x+\frac{63}{2048}i\left(4\,x^2+3i\,x\right)^{\frac{5}{2}}+\frac{945}{16384}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x+\frac{2835}{131072}i\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}+\frac{25515}{524288}\sqrt{4\,x^2+3i\,x}x+\frac{76545}{4194304}i\sqrt{4\,x^2+3i\,x}+\frac{229635}{16777216}\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

output
```
1/8*(4*x^2 + 3*I*x)^(7/2)*x + 3/64*I*(4*x^2 + 3*I*x)^(7/2) + 21/256*(4*x^2
 + 3*I*x)^(5/2)*x + 63/2048*I*(4*x^2 + 3*I*x)^(5/2) + 945/16384*(4*x^2 + 3
*I*x)^(3/2)*x + 2835/131072*I*(4*x^2 + 3*I*x)^(3/2) + 25515/524288*sqrt(4*
x^2 + 3*I*x)*x + 76545/4194304*I*sqrt(4*x^2 + 3*I*x) + 229635/16777216*log
(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \left(3ix+4x^2\right)^{7/2}\,dx=\frac{1}{8388608}\left(8\left(16\left(8\left(32\left(8\left(16\left(8\,x+21i\right)x-303\right)x-765i\right)x+81\right)x-567i\right)x-8505\right)x+7654\right.$$
$$-\frac{229635}{16777216}\log\left(2\sqrt{8\,x^2+2\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-8\,x-3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

output
```
1/8388608*(8*(16*(8*(32*(8*(16*(8*x + 21*I)*x - 303)*x - 765*I)*x + 81)*x
- 567*I)*x - 8505)*x + 76545*I)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/
(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 229635/16777216*log(2*sqrt(8*x^2 + 2
*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*
I)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \left(3ix+4x^2\right)^{7/2}\,dx=\frac{229635\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{16777216}+\frac{945\left(4\,x+\frac{3}{2}\mathrm{i}\right)\left(4\,x^2+x\,3\mathrm{i}\right)^{3/2}}{65536}+\frac{21\left(4\,x+\frac{3}{2}\mathrm{i}\right)\left(4\,x^2+x\,3\mathrm{i}\right)^{5/2}}{1024}+\frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\left(4\,x^2+x\,3\mathrm{i}\right)^{7/2}}{32}+\frac{25515\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\sqrt{4\,x^2+x\,3\mathrm{i}}}{262144}$$

input
```
int((x*3i + 4*x^2)^(7/2),x)
```

output
```
(229635*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/16777216 + (945*(4*x + 3i/
2)*(x*3i + 4*x^2)^(3/2))/65536 + (21*(4*x + 3i/2)*(x*3i + 4*x^2)^(5/2))/10
24 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(7/2))/32 + (25515*(x/2 + 3i/16)*(x*3i +
 4*x^2)^(1/2))/262144
```

3.3 $\int (3ix+4x^2)^{5/2}\,dx$

3.3.1 Optimal result

Integrand size = 15, antiderivative size = 95

$$\int (3ix+4x^2)^{5/2}\,dx = \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768}+\frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2}+\frac{3645i\arcsin\left(1-\frac{8ix}{3}\right)}{131072}$$

output
```
15/1024*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+1/48*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)-3
645/131072*I*arcsin(-1+8/3*I*x)+405/32768*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int (3ix+4x^2)^{5/2}\,dx = \frac{\sqrt{x(3i+4x)}\Big(7290i-6480x-6912ix^2-497664x^3+983040ix^4+524288x^5-\frac{10935\log(-2\sqrt{x}}{\sqrt{x}\sqrt{3i+}}}{196608}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(5/2),x]
```

output
```
(Sqrt[x*(3*I + 4*x)]*(7290*I - 6480*x - (6912*I)*x^2 - 497664*x^3 + (98304
0*I)*x^4 + 524288*x^5 - (10935*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(Sqrt[x]
*Sqrt[3*I + 4*x])))/196608
```

3.3.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^2+3ix\right)^{5/2}dx$$

$$\Big\downarrow \ 1087$$

$$\frac{15}{32}\int \left(4x^2+3ix\right)^{3/2}dx+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

$$\Big\downarrow \ 1087$$

$$\frac{15}{32}\left(\frac{27}{64}\int \sqrt{4x^2+3ix}dx+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

$$\Big\downarrow \ 1087$$

$$\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{4x^2+3ix}}dx+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

$$\Big\downarrow \ 1090$$

$$\frac{15}{32}\left(\frac{27}{64}\left(\frac{3}{64}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

$$\Big\downarrow \ 222$$

$$\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{64}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

input
```
Int[((3*I)*x + 4*x^2)^(5/2),x]
```

output
```
((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/48 + (15*(((3*I + 8*x)*((3*I)*x + 4*
x^2)^(3/2))/32 + (27*(((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + (9*ArcSinh[
(3*I + 8*x)/3])/64))/64))/32
```

3.3.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.3.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61

method	result
risch	$\frac{(262144x^5+491520ix^4-248832x^3-3456ix^2-3240x+3645i)(3i+4x)x}{98304\sqrt{x(3i+4x)}}+\frac{3645\operatorname{arcsinh}\left(i+\frac{8x}{3}\right)}{131072}$
default	$\frac{(3i+8x)(4x^2+3ix)^{\frac{5}{2}}}{48}+\frac{15(3i+8x)(4x^2+3ix)^{\frac{3}{2}}}{1024}+\frac{405(3i+8x)\sqrt{4x^2+3ix}}{32768}+\frac{3645\operatorname{arcsinh}\left(i+\frac{8x}{3}\right)}{131072}$
trager	$\left(5ix^4+\frac{8}{3}x^5-\frac{9}{256}ix^2-\frac{81}{32}x^3+\frac{1215}{32768}i-\frac{135}{4096}x\right)\sqrt{4x^2+3ix}-\frac{3645\ln\left(-440x-144-165i-192i\sqrt{4x^2+3ix}+38\right.}{131072}$

input
```
int((3*I*x+4*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/98304*(3645*I-3240*x-3456*I*x^2-248832*x^3+491520*I*x^4+262144*x^5)*(3*I
+4*x)*x/(x*(3*I+4*x))^(1/2)+3645/131072*arcsinh(I+8/3*x)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

$$\int (3ix+4x^2)^{5/2}\,dx = \frac{1}{98304}\left(262144\,x^5+491520i\,x^4-248832\,x^3-3456i\,x^2-3240\,x+3645i\right)\sqrt{4\,x^2+3i\,x} - \frac{3645}{131072}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{8991}{1048576}$$

input
```
integrate((3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

output
```
1/98304*(262144*x^5 + 491520*I*x^4 - 248832*x^3 - 3456*I*x^2 - 3240*x + 36
45*I)*sqrt(4*x^2 + 3*I*x) - 3645/131072*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3
/4*I) - 8991/1048576
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71

$$\int (3ix+4x^2)^{5/2}\,dx = -9\sqrt{4x^2+3ix}\left(\frac{x^3}{4}+\frac{ix^2}{32}+\frac{15x}{512}-\frac{135i}{4096}\right) + 16\sqrt{4x^2+3ix}\left(\frac{x^5}{6}+\frac{ix^4}{80}+\frac{27x^3}{2560}-\frac{189ix^2}{20480}-\frac{567x}{65536}+\frac{5103i}{524288}\right) + 24i\left(\sqrt{4x^2+3ix}\left(\frac{x^4}{5}+\frac{3ix^3}{160}+\frac{21x^2}{1280}-\frac{63ix}{4096}-\frac{567}{32768}\right)+\frac{1701i\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{131072}\right) + \frac{44469\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{131072}$$

input
```
integrate((3*I*x+4*x**2)**(5/2),x)
```

output
```
-9*sqrt(4*x**2 + 3*I*x)*(x**3/4 + I*x**2/32 + 15*x/512 - 135*I/4096) + 16*
sqrt(4*x**2 + 3*I*x)*(x**5/6 + I*x**4/80 + 27*x**3/2560 - 189*I*x**2/20480
 - 567*x/65536 + 5103*I/524288) + 24*I*(sqrt(4*x**2 + 3*I*x)*(x**4/5 + 3*I
*x**3/160 + 21*x**2/1280 - 63*I*x/4096 - 567/32768) + 1701*I*asinh(8*x/3 +
 I)/131072) + 44469*asinh(8*x/3 + I)/131072
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08

$$\int \left(3ix+4x^2\right)^{5/2}\,dx = \frac{1}{6}\left(4\,x^2+3i\,x\right)^{\frac{5}{2}}x+\frac{1}{16}i\left(4\,x^2+3i\,x\right)^{\frac{5}{2}} + \frac{15}{128}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x+\frac{45}{1024}i\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{4\,x^2+3i\,x}x + \frac{1215}{32768}i\sqrt{4\,x^2+3i\,x}+\frac{3645}{131072}\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(4*x^2 + 3*I*x)^(5/2)*x + 1/16*I*(4*x^2 + 3*I*x)^(5/2) + 15/128*(4*x^2
 + 3*I*x)^(3/2)*x + 45/1024*I*(4*x^2 + 3*I*x)^(3/2) + 405/4096*sqrt(4*x^2
+ 3*I*x)*x + 1215/32768*I*sqrt(4*x^2 + 3*I*x) + 3645/131072*log(8*x + 4*sq
rt(4*x^2 + 3*I*x) + 3*I)
```

3.3.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(63) = 126.

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int (3ix +4x^2)^{5/2}\,dx = \frac{1}{196608}\left(8\left(16\left(8\left(32\left(8\,x+15i\right)x-243\right)x-27i\right)x-405\right)x+3645i\right)\sqrt{8\,x^2+2\sqrt{16\,x^2+9}x}\Bigg(- \frac{3645}{131072}\log\left(2\sqrt{8\,x^2+2\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-8\,x-3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/196608*(8*(16*(8*(32*(8*x + 15*I)*x - 243)*x - 27*I)*x - 405)*x + 3645*I
)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2))
 + 1) - 3645/131072*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2
 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \left(3ix+4x^2\right)^{5/2}\,dx=\frac{3645\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{131072}+\frac{15\,\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,\left(4\,x^2+x\,3\mathrm{i}\right)^{3/2}}{512}+\frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,\left(4\,x^2+x\,3\mathrm{i}\right)^{5/2}}{24}+\frac{405\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{2048}$$

input
```
int((x*3i + 4*x^2)^(5/2),x)
```

output
```
(3645*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/131072 + (15*(4*x + 3i/2)*(x
*3i + 4*x^2)^(3/2))/512 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(5/2))/24 + (405*(x
/2 + 3i/16)*(x*3i + 4*x^2)^(1/2))/2048
```

3.4 $\int (3ix+4x^2)^{3/2}\,dx$

3.4.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int (3ix+4x^2)^{3/2}\,dx=\frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{243i\arcsin\left(1-\frac{8ix}{3}\right)}{4096}$$

output
```
1/32*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)-243/4096*I*arcsin(-1+8/3*I*x)+27/1024*(
3*I+8*x)*(3*I*x+4*x^2)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int (3ix+4x^2)^{3/2}\,dx=\frac{2x(-243+108ix-3744x^2+7680ix^3+4096x^4)-243\sqrt{x}\sqrt{3i+4x}\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{2048\sqrt{x(3i+4x)}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(3/2),x]
```

output
```
(2*x*(-243 + (108*I)*x - 3744*x^2 + (7680*I)*x^3 + 4096*x^4) - 243*Sqrt[x]
*Sqrt[3*I + 4*x]*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(2048*Sqrt[x*(3*I + 4*
x)])
```

3.4.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^2+3ix\right)^{3/2} dx$$

$$\Big\downarrow \ 1087$$

$$\frac{27}{64}\int \sqrt{4x^2+3ix}dx+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}$$

$$\Big\downarrow \ 1087$$

$$\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{4x^2+3ix}}dx+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}$$

$$\Big\downarrow \ 1090$$

$$\frac{27}{64}\left(\frac{3}{64}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}$$

$$\Big\downarrow \ 222$$

$$\frac{27}{64}\left(\frac{9}{64}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}$$

input
```
Int[((3*I)*x + 4*x^2)^(3/2),x]
```

output
```
((3*I + 8*x)*((3*I)*x + 4*x^2)^(3/2))/32 + (27*(((3*I + 8*x)*Sqrt[(3*I)*x
+ 4*x^2])/16 + (9*ArcSinh[(3*I + 8*x)/3])/64))/64
```

3.4.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.4.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

method	result
risch	$\frac{(1024x^3+1152ix^2-72x+81i)(3i+4x)x}{1024\sqrt{x(3i+4x)}}+\frac{243\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{4096}$
default	$\frac{(3i+8x)(4x^2+3ix)^{\frac{3}{2}}}{32}+\frac{27(3i+8x)\sqrt{4x^2+3ix}}{1024}+\frac{243\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{4096}$
trager	$\left(\frac{9}{8}ix^2+x^3+\frac{81}{1024}i-\frac{9}{128}x\right)\sqrt{4x^2+3ix}-\frac{243\ln\left(-440x-144-165i-192i\sqrt{4x^2+3ix}+384ix+220\sqrt{4x^2+3ix}\right.}{4096}$
pseudoelliptic	$\frac{729\left(-\frac{27\ln\left(\frac{-2x+\sqrt{x(3i+4x)}}{x}\right)}{512}+\frac{27\ln\left(\frac{\sqrt{x(3i+4x)}+2x}{x}\right)}{512}+\left(ix^2+\frac{8}{9}x^3+\frac{9}{128}i-\frac{1}{16}x\right)\sqrt{x(3i+4x)}\right)x^4}{8\left(\sqrt{x(3i+4x)}+2x\right)^4\left(2x-\sqrt{x(3i+4x)}\right)^4}$

input
```
int((3*I*x+4*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1024*(81*I-72*x+1152*I*x^2+1024*x^3)*(3*I+4*x)*x/(x*(3*I+4*x))^(1/2)+243
/4096*arcsinh(I+8/3*x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.71

$$\int \left(3ix+4x^2\right)^{3/2}\,dx = \frac{1}{1024}\left(1024\,x^3+1152i\,x^2-72\,x+81i\right)\sqrt{4\,x^2+3i\,x} - \frac{243}{4096}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{567}{32768}$$

input
```
integrate((3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/1024*(1024*x^3 + 1152*I*x^2 - 72*x + 81*I)*sqrt(4*x^2 + 3*I*x) - 243/409
6*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 567/32768
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \left(3ix+4x^2\right)^{3/2}\,dx = 4\sqrt{4x^2+3ix}\left(\frac{x^3}{4}+\frac{ix^2}{32}+\frac{15x}{512}-\frac{135i}{4096}\right) + 3i\left(\sqrt{4x^2+3ix}\left(\frac{x^2}{3}+\frac{ix}{16}+\frac{9}{128}\right)-\frac{27i\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{512}\right)-\frac{405\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{4096}$$

input
```
integrate((3*I*x+4*x**2)**(3/2),x)
```

output
```
4*sqrt(4*x**2 + 3*I*x)*(x**3/4 + I*x**2/32 + 15*x/512 - 135*I/4096) + 3*I*
(sqrt(4*x**2 + 3*I*x)*(x**2/3 + I*x/16 + 9/128) - 27*I*asinh(8*x/3 + I)/51
2) - 405*asinh(8*x/3 + I)/4096
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \left(3ix+4x^2\right)^{3/2}\,dx = \frac{1}{4}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x+\frac{3}{32}i\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{4\,x^2+3i\,x}x + \frac{81}{1024}i\sqrt{4\,x^2+3i\,x}+\frac{243}{4096}\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*x^2 + 3*I*x)^(3/2)*x + 3/32*I*(4*x^2 + 3*I*x)^(3/2) + 27/128*sqrt(4
*x^2 + 3*I*x)*x + 81/1024*I*sqrt(4*x^2 + 3*I*x) + 243/4096*log(8*x + 4*sqr
t(4*x^2 + 3*I*x) + 3*I)
```

3.4.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int (3ix+4x^2)^{3/2}\,dx = \frac{1}{2048}\,(8\,(16\,(8\,x+9i)x-9)x+81i)\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right) - \frac{243}{4096}\,\log\left(2\,\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-8\,x-3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2048*(8*(16*(8*x + 9*I)*x - 9)*x + 81*I)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)
*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 243/4096*log(2*sqrt(8*x^2
 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x
- 3*I)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int (3ix+4x^2)^{3/2}\,dx = \frac{243\,\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{4096} + \frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,\left(4\,x^2+x\,3\mathrm{i}\right)^{3/2}}{16} + \frac{27\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{64}$$

input
```
int((x*3i + 4*x^2)^(3/2),x)
```

output
```
(243*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/4096 + ((4*x + 3i/2)*(x*3i +
4*x^2)^(3/2))/16 + (27*(x/2 + 3i/16)*(x*3i + 4*x^2)^(1/2))/64
```

3.4. $\int \left(3ix+4x^2\right)^{3/2}\,dx$

3.5 $\int \sqrt{3ix+4x^2}\,dx$

3.5.1 Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \sqrt{3ix+4x^2}\,dx=\frac{1}{16}(3i+8x)\sqrt{3ix+4x^2}+\frac{9}{64}i\arcsin\left(1-\frac{8ix}{3}\right)$$

output
```
-9/64*I*arcsin(-1+8/3*I*x)+1/16*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \sqrt{3ix+4x^2}\,dx=\frac{1}{32}\sqrt{x(3i+4x)}\left(6i+16x-\frac{9\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{\sqrt{x}\sqrt{3i+4x}}\right)$$

input
```
Integrate[Sqrt[(3*I)*x + 4*x^2],x]
```

output
```
(Sqrt[x*(3*I + 4*x)]*(6*I + 16*x - (9*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(
Sqrt[x]*Sqrt[3*I + 4*x])))/32
```

3.5.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4x^2+3ix}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{9}{32}\int \frac{1}{\sqrt{4x^2+3ix}}dx+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{3}{64}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{9}{64}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)$$

input
```
Int[Sqrt[(3*I)*x + 4*x^2],x]
```

output
```
((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + (9*ArcSinh[(3*I + 8*x)/3])/64
```

3.5.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.5.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{(3i+8x)\sqrt{4x^2+3ix}}{16}+\frac{9\,\operatorname{arcsinh}\left(i+\frac{8x}{3}\right)}{64}$	31
risch	$\frac{(3i+8x)(3i+4x)x}{16\sqrt{x(3i+4x)}}+\frac{9\,\operatorname{arcsinh}\left(i+\frac{8x}{3}\right)}{64}$	36
trager	$\left(\frac{3i}{16}+\frac{x}{2}\right)\sqrt{4x^2+3ix}-\frac{9\ln\left(-440x-144-165i-192i\sqrt{4x^2+3ix}+384ix+220\sqrt{4x^2+3ix}\right)}{64}$	64
pseudoelliptic	$-\frac{27\left(-\frac{3\ln\left(\frac{-2x+\sqrt{x(3i+4x)}}{x}\right)}{4}+\frac{3\ln\left(\frac{\sqrt{x(3i+4x)}+2x}{x}\right)}{4}+\left(i+\frac{8x}{3}\right)\sqrt{x(3i+4x)}\right)x^2}{16\left(\sqrt{x(3i+4x)}+2x\right)^2\left(2x-\sqrt{x(3i+4x)}\right)^2}$	100

input
```
int((3*I*x+4*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+9/64*arcsinh(I+8/3*x)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \sqrt{3ix+4x^2}\,dx=\frac{1}{16}\sqrt{4\,x^2+3i\,x}(8\,x+3i)-\frac{9}{64}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{9}{256}$$

input
```
integrate((3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(4*x^2 + 3*I*x)*(8*x + 3*I) - 9/64*log(-2*x + sqrt(4*x^2 + 3*I*x)
 - 3/4*I) - 9/256
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \sqrt{3ix+4x^2}\,dx = \left(\frac{x}{2}+\frac{3i}{16}\right)\sqrt{4x^2+3ix}+\frac{9\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{64}$$

input
```
integrate((3*I*x+4*x**2)**(1/2),x)
```

output
```
(x/2 + 3*I/16)*sqrt(4*x**2 + 3*I*x) + 9*asinh(8*x/3 + I)/64
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \sqrt{3ix+4x^2}\,dx = \frac{1}{2}\sqrt{4\,x^2+3i\,x}x+\frac{3}{16}i\sqrt{4\,x^2+3i\,x}+\frac{9}{64}\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(4*x^2 + 3*I*x)*x + 3/16*I*sqrt(4*x^2 + 3*I*x) + 9/64*log(8*x + 4*
sqrt(4*x^2 + 3*I*x) + 3*I)
```

3.5.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.56

$$\int \sqrt{3ix+4x^2}\,dx = \frac{1}{32}\sqrt{8\,x^2+2\sqrt{16\,x^2+9}x}(8\,x+3i)\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right) - \frac{9}{64}\log\left(2\sqrt{8\,x^2+2\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-8\,x-3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/32*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(1
6*x^4 + 9*x^2)) + 1) - 9/64*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*
x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \sqrt{3ix + 4x^2}\, dx = \frac{9\ln\left(x + \frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2} + \frac{3}{8}\mathrm{i}\right)}{64} + \left(\frac{x}{2} + \frac{3}{16}\mathrm{i}\right)\sqrt{4\,x^2 + x\,3\mathrm{i}}$$

input
```
int((x*3i + 4*x^2)^(1/2),x)
```

output
```
(9*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/64 + (x/2 + 3i/16)*(x*3i + 4*x^
2)^(1/2)
```

3.6 $\int (3x-4x^2)^{7/2}\,dx$

3.6.1 Optimal result

Integrand size = 13, antiderivative size = 101

$$\int (3x-4x^2)^{7/2}\,dx = -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072} -\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}-\frac{229635\arcsin\left(1-\frac{8x}{3}\right)}{16777216}$$

output
```
-945/131072*(3-8*x)*(-4*x^2+3*x)^(3/2)-21/2048*(3-8*x)*(-4*x^2+3*x)^(5/2)-
1/64*(3-8*x)*(-4*x^2+3*x)^(7/2)+229635/16777216*arcsin(-1+8/3*x)-25515/419
4304*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int (3x -4x^2)^{7/2}\,dx = \frac{\sqrt{-x(-3+4x)}\left(-2\sqrt{x}\sqrt{-3+4x}(76545+68040x+72576x^2+82944x^3-25067520x^4+7\right.}{8388608\sqrt{x}\sqrt{-3+}}$$

input
```
Integrate[(3*x - 4*x^2)^(7/2),x]
```

output
```
(Sqrt[-(x*(-3 + 4*x))]*(-2*Sqrt[x]*Sqrt[-3 + 4*x]*(76545 + 68040*x + 72576
*x^2 + 82944*x^3 - 25067520*x^4 + 79429632*x^5 - 88080384*x^6 + 33554432*x
^7) + 229635*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]]))/(8388608*Sqrt[x]*Sqrt[-3 +
 4*x])
```

3.6.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1087, 1087, 1087, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x-4x^2\right)^{7/2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{63}{128}\int \left(3x-4x^2\right)^{5/2}dx-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{63}{128}\left(\frac{15}{32}\int \left(3x-4x^2\right)^{3/2}dx-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}\right)-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{63}{128}\left(\frac{15}{32}\left(\frac{27}{64}\int \sqrt{3x-4x^2}dx-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}\right)-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{63}{128}\left(\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}dx-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4\right.\right.$$
$$\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{63}{128}\left(\frac{15}{32}\left(\frac{27}{64}\left(-\frac{3}{64}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{4}\right.$$
$$\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{63}{128}\left(\frac{15}{32}\left(\frac{27}{64}\left(-\frac{9}{64}\arcsin\left(\frac{1}{3}(3-8x)\right)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3\right.\right.$$
$$\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}$$

input
```
Int[(3*x - 4*x^2)^(7/2),x]
```

output
```
-1/64*((3 - 8*x)*(3*x - 4*x^2)^(7/2)) + (63*(-1/48*((3 - 8*x)*(3*x - 4*x^2
)^(5/2)) + (15*(-1/32*((3 - 8*x)*(3*x - 4*x^2)^(3/2)) + (27*(-1/16*((3 - 8
*x)*Sqrt[3*x - 4*x^2]) - (9*ArcSin[(3 - 8*x)/3])/64))/64))/32))/128
```

3.6.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.6.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

method	result
risch	$\frac{(33554432x^7-88080384x^6+79429632x^5-25067520x^4+82944x^3+72576x^2+68040x+76545)x(4x-3)}{4194304\sqrt{-x(4x-3)}}+\frac{229635\arcsin\left(-1+\frac{8x}{3}\right)}{16777216}$
meijerg	$-\frac{688905i\left(-\frac{i\sqrt{\pi}\sqrt{x}\sqrt{3}\left(\frac{33554432}{243}x^7-\frac{29360128}{81}x^6+\frac{26476544}{81}x^5-\frac{2785280}{27}x^4+\frac{1024}{3}x^3+\frac{896}{3}x^2+280x+315\right)\sqrt{-\frac{4x}{3}+1}}{1451520}+\frac{i\sqrt{\pi}\arcsin\left(\frac{2\sqrt{3}\sqrt{x}}{3}\right.}{3072}\right.}{8192\sqrt{\pi}}$
default	$-\frac{945(3-8x)\left(-4x^2+3x\right)^{\frac{3}{2}}}{131072}-\frac{21(3-8x)\left(-4x^2+3x\right)^{\frac{5}{2}}}{2048}-\frac{(3-8x)\left(-4x^2+3x\right)^{\frac{7}{2}}}{64}+\frac{229635\arcsin\left(-1+\frac{8x}{3}\right)}{16777216}-\frac{25515(3-8x)\sqrt{-4}}{4194304}$
trager	$\left(-8x^7+21x^6-\frac{303}{16}x^5+\frac{765}{128}x^4-\frac{81}{4096}x^3-\frac{567}{32768}x^2-\frac{8505}{524288}x-\frac{76545}{4194304}\right)\sqrt{-4x^2+3x}+\frac{229635\,\mathrm{Root}}{}$

input
```
int((-4*x^2+3*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/4194304*(33554432*x^7-88080384*x^6+79429632*x^5-25067520*x^4+82944*x^3+7
2576*x^2+68040*x+76545)*x*(4*x-3)/(-x*(4*x-3))^(1/2)+229635/16777216*arcsi
n(-1+8/3*x)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \left(3x-4x^2\right)^{7/2}\,dx =$$

$$-\frac{1}{4194304}\left(33554432\,x^7-88080384\,x^6+79429632\,x^5-25067520\,x^4+82944\,x^3+72576\,x^2+68040\,x+7\right.$$

$$-\frac{229635}{8388608}\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

input
```
integrate((-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

output
```
-1/4194304*(33554432*x^7 - 88080384*x^6 + 79429632*x^5 - 25067520*x^4 + 82
944*x^3 + 72576*x^2 + 68040*x + 76545)*sqrt(-4*x^2 + 3*x) - 229635/8388608
*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(90) = 180.

Time = 0.71 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.09

$$\int \left(3x-4x^2\right)^{7/2} dx = 27\sqrt{-4x^2+3x}\left(\frac{x^4}{5}-\frac{3x^3}{160}-\frac{21x^2}{1280}-\frac{63x}{4096}-\frac{567}{32768}\right) - 108\sqrt{-4x^2+3x}\left(\frac{x^5}{6}-\frac{x^4}{80}-\frac{27x^3}{2560}-\frac{189x^2}{20480}-\frac{567x}{65536}-\frac{5103}{524288}\right) + 144\sqrt{-4x^2+3x}\left(\frac{x^6}{7}-\frac{x^5}{112}-\frac{33x^4}{4480}-\frac{891x^3}{143360}-\frac{891x^2}{163840}-\frac{2673x}{524288}-\frac{24057}{4194304}\right) - 64\sqrt{-4x^2+3x}\left(\frac{x^7}{8}-\frac{3x^6}{448}-\frac{39x^5}{7168}-\frac{1287x^4}{286720}-\frac{34749x^3}{9175040}-\frac{34749x^2}{10485760}-\frac{104247x}{33554432}-\frac{938223}{268435456}\right) + \frac{229635\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{16777216}$$

input
```
integrate((-4*x**2+3*x)**(7/2),x)
```

output
```
27*sqrt(-4*x**2 + 3*x)*(x**4/5 - 3*x**3/160 - 21*x**2/1280 - 63*x/4096 - 5
67/32768) - 108*sqrt(-4*x**2 + 3*x)*(x**5/6 - x**4/80 - 27*x**3/2560 - 189
*x**2/20480 - 567*x/65536 - 5103/524288) + 144*sqrt(-4*x**2 + 3*x)*(x**6/7
 - x**5/112 - 33*x**4/4480 - 891*x**3/143360 - 891*x**2/163840 - 2673*x/52
4288 - 24057/4194304) - 64*sqrt(-4*x**2 + 3*x)*(x**7/8 - 3*x**6/448 - 39*x
**5/7168 - 1287*x**4/286720 - 34749*x**3/9175040 - 34749*x**2/10485760 - 1
04247*x/33554432 - 938223/268435456) + 229635*asin(8*x/3 - 1)/16777216
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16

$$\int \left(3x-4x^2\right)^{7/2} dx = \frac{1}{8}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}x - \frac{3}{64}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}} + \frac{21}{256}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}x - \frac{63}{2048}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}} + \frac{945}{16384}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x - \frac{2835}{131072}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}} + \frac{25515}{524288}\sqrt{-4\,x^2+3\,x}x - \frac{76545}{4194304}\sqrt{-4\,x^2+3\,x} - \frac{229635}{16777216}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

input
```
integrate((-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

output
```
1/8*(-4*x^2 + 3*x)^(7/2)*x - 3/64*(-4*x^2 + 3*x)^(7/2) + 21/256*(-4*x^2 +
3*x)^(5/2)*x - 63/2048*(-4*x^2 + 3*x)^(5/2) + 945/16384*(-4*x^2 + 3*x)^(3/
2)*x - 2835/131072*(-4*x^2 + 3*x)^(3/2) + 25515/524288*sqrt(-4*x^2 + 3*x)*
x - 76545/4194304*sqrt(-4*x^2 + 3*x) - 229635/16777216*arcsin(-8/3*x + 1)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.56

$$\int (3x-4x^2)^{7/2}\,dx = -\frac{1}{4194304}\,(8\,(16\,(8\,(32\,(8\,(16\,(8\,x-21)x+303)x-765)x+81)x+567)x+8505)x+76545)\sqrt{-4\,x^2+3\,x} + \frac{229635}{16777216}\arcsin\left(\frac{8}{3}\,x-1\right)$$

input
```
integrate((-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

output
```
-1/4194304*(8*(16*(8*(32*(8*(16*(8*x - 21)*x + 303)*x - 765)*x + 81)*x + 5
67)*x + 8505)*x + 76545)*sqrt(-4*x^2 + 3*x) + 229635/16777216*arcsin(8/3*x
 - 1)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int (3x-4x^2)^{7/2}\,dx = \frac{229635\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{16777216} + \frac{945\,\left(4\,x-\frac{3}{2}\right)\,\left(3\,x-4\,x^2\right)^{3/2}}{65536} + \frac{21\,\left(4\,x-\frac{3}{2}\right)\,\left(3\,x-4\,x^2\right)^{5/2}}{1024} + \frac{\left(4\,x-\frac{3}{2}\right)\,\left(3\,x-4\,x^2\right)^{7/2}}{32} + \frac{25515\,\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}}{262144}$$

input
```
int((3*x - 4*x^2)^(7/2),x)
```

output
```
(229635*asin((8*x)/3 - 1))/16777216 + (945*(4*x - 3/2)*(3*x - 4*x^2)^(3/2)
)/65536 + (21*(4*x - 3/2)*(3*x - 4*x^2)^(5/2))/1024 + ((4*x - 3/2)*(3*x -
4*x^2)^(7/2))/32 + (25515*(x/2 - 3/16)*(3*x - 4*x^2)^(1/2))/262144
```

3.7 $\int (3x-4x^2)^{5/2}\,dx$

3.7.1 Optimal result

Integrand size = 13, antiderivative size = 79

$$\int (3x-4x^2)^{5/2}\,dx = -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} -\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{3645\arcsin\left(1-\frac{8x}{3}\right)}{131072}$$

output
```
-15/1024*(3-8*x)*(-4*x^2+3*x)^(3/2)-1/48*(3-8*x)*(-4*x^2+3*x)^(5/2)+3645/1
31072*arcsin(-1+8/3*x)-405/32768*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int (3x-4x^2)^{5/2}\,dx = \frac{\sqrt{-x(-3+4x)}\left(2\sqrt{x}\sqrt{-3+4x}(-3645-3240x-3456x^2+248832x^3-491520x^4+26214\right.}{196608\sqrt{x}\sqrt{-3+4x}}$$

input
```
Integrate[(3*x - 4*x^2)^(5/2),x]
```

output
```
(Sqrt[-(x*(-3 + 4*x))]*(2*Sqrt[x]*Sqrt[-3 + 4*x]*(-3645 - 3240*x - 3456*x^
2 + 248832*x^3 - 491520*x^4 + 262144*x^5) + 10935*Log[-2*Sqrt[x] + Sqrt[-3
 + 4*x]]))/(196608*Sqrt[x]*Sqrt[-3 + 4*x])
```

3.7.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1087, 1087, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x-4x^2\right)^{5/2}dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{15}{32}\int \left(3x-4x^2\right)^{3/2}dx-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{15}{32}\left(\frac{27}{64}\int \sqrt{3x-4x^2}dx-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}dx-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(-\frac{3}{64}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(-\frac{9}{64}\arcsin\left(\frac{1}{3}(3-8x)\right)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

input
```
Int[(3*x - 4*x^2)^(5/2),x]
```

output
```
-1/48*((3 - 8*x)*(3*x - 4*x^2)^(5/2)) + (15*(-1/32*((3 - 8*x)*(3*x - 4*x^2
)^(3/2)) + (27*(-1/16*((3 - 8*x)*Sqrt[3*x - 4*x^2]) - (9*ArcSin[(3 - 8*x)/
3])/64))/64))/32
```

3.7.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.7.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{\left(262144 x^{5}-491520 x^{4}+248832 x^{3}-3456 x^{2}-3240 x -3645\right) x \left(4 x -3\right)}{98304 \sqrt{-x \left(4 x -3\right)}}+\frac{3645 \arcsin \left(-1+\frac{8 x}{3}\right)}{131072}$
default	$-\frac{15 \left(3-8 x \right) \left(-4 x^{2}+3 x \right)^{\frac{3}{2}}}{1024}-\frac{\left(3-8 x \right) \left(-4 x^{2}+3 x \right)^{\frac{5}{2}}}{48}+\frac{3645 \arcsin \left(-1+\frac{8 x}{3}\right)}{131072}-\frac{405 \left(3-8 x \right) \sqrt{-4 x^{2}+3 x}}{32768}$
meijerg	$-\frac{10935 i \left(-\frac{i \sqrt{\pi}\, \sqrt{x}\, \sqrt{3}\, \left(-\frac{1835008}{243} x^{5}+\frac{1146880}{81} x^{4}-7168 x^{3}+\frac{896}{9} x^{2}+\frac{280}{3} x +105\right) \sqrt{-\frac{4 x}{3}+1}}{30240}+\frac{i \sqrt{\pi}\, \arcsin \left(\frac{2 \sqrt{3}\, \sqrt{x}}{3}\right)}{192}\right)}{1024 \sqrt{\pi}}$
trager	$\left(\frac{8}{3} x^{5}-5 x^{4}+\frac{81}{32} x^{3}-\frac{9}{256} x^{2}-\frac{135}{4096} x -\frac{1215}{32768}\right) \sqrt{-4 x^{2}+3 x}-\frac{3645 \operatorname{RootOf}\left(\_Z^{2}+1\right) \ln \left(8 \operatorname{RootOf}\left(\_Z^{2}+1\right) \ldots\right.}{131072}$

input
```
int((-4*x^2+3*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/98304*(262144*x^5-491520*x^4+248832*x^3-3456*x^2-3240*x-3645)*x*(4*x-3)
/(-x*(4*x-3))^(1/2)+3645/131072*arcsin(-1+8/3*x)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int (3x-4x^2)^{5/2}\,dx = \frac{1}{98304}\left(262144\,x^5-491520\,x^4+248832\,x^3-3456\,x^2-3240\,x-3645\right)\sqrt{-4\,x^2+3\,x} - \frac{3645}{65536}\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

input
```
integrate((-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

output
```
1/98304*(262144*x^5 - 491520*x^4 + 248832*x^3 - 3456*x^2 - 3240*x - 3645)*
sqrt(-4*x^2 + 3*x) - 3645/65536*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int (3x-4x^2)^{5/2}\,dx = 9\sqrt{-4x^2+3x}\left(\frac{x^3}{4}-\frac{x^2}{32}-\frac{15x}{512}-\frac{135}{4096}\right) - 24\sqrt{-4x^2+3x}\left(\frac{x^4}{5}-\frac{3x^3}{160}-\frac{21x^2}{1280}-\frac{63x}{4096}-\frac{567}{32768}\right) + 16\sqrt{-4x^2+3x}\left(\frac{x^5}{6}-\frac{x^4}{80}-\frac{27x^3}{2560}-\frac{189x^2}{20480}-\frac{567x}{65536}-\frac{5103}{524288}\right) + \frac{3645\,\operatorname{asin}\left(\frac{8x}{3}-1\right)}{131072}$$

input
```
integrate((-4*x**2+3*x)**(5/2),x)
```

output
```
9*sqrt(-4*x**2 + 3*x)*(x**3/4 - x**2/32 - 15*x/512 - 135/4096) - 24*sqrt(-
4*x**2 + 3*x)*(x**4/5 - 3*x**3/160 - 21*x**2/1280 - 63*x/4096 - 567/32768)
 + 16*sqrt(-4*x**2 + 3*x)*(x**5/6 - x**4/80 - 27*x**3/2560 - 189*x**2/2048
0 - 567*x/65536 - 5103/524288) + 3645*asin(8*x/3 - 1)/131072
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \left(3x-4x^2\right)^{5/2}\,dx = \frac{1}{6}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}x-\frac{1}{16}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}+\frac{15}{128}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x-\frac{45}{1024}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{-4\,x^2+3\,x}x-\frac{1215}{32768}\sqrt{-4\,x^2+3\,x}-\frac{3645}{131072}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

input

```
integrate((-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

output

```
1/6*(-4*x^2 + 3*x)^(5/2)*x - 1/16*(-4*x^2 + 3*x)^(5/2) + 15/128*(-4*x^2 +
3*x)^(3/2)*x - 45/1024*(-4*x^2 + 3*x)^(3/2) + 405/4096*sqrt(-4*x^2 + 3*x)*
x - 1215/32768*sqrt(-4*x^2 + 3*x) - 3645/131072*arcsin(-8/3*x + 1)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \left(3x-4x^2\right)^{5/2}\,dx = \frac{1}{98304}\left(8\left(16\left(8\left(32\left(8\,x-15\right)x+243\right)x-27\right)x-405\right)x-3645\right)\sqrt{-4\,x^2+3\,x}+\frac{3645}{131072}\arcsin\left(\frac{8}{3}\,x-1\right)$$

input

```
integrate((-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

output

```
1/98304*(8*(16*(8*(32*(8*x - 15)*x + 243)*x - 27)*x - 405)*x - 3645)*sqrt(
-4*x^2 + 3*x) + 3645/131072*arcsin(8/3*x - 1)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \left(3x-4x^2\right)^{5/2}\,dx = \frac{3645\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{131072}+\frac{15\,\left(4\,x-\frac{3}{2}\right)\,\left(3\,x-4\,x^2\right)^{3/2}}{512}+\frac{\left(4\,x-\frac{3}{2}\right)\,\left(3\,x-4\,x^2\right)^{5/2}}{24}+\frac{405\,\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}}{2048}$$

input
```
int((3*x - 4*x^2)^(5/2),x)
```

output
```
(3645*asin((8*x)/3 - 1))/131072 + (15*(4*x - 3/2)*(3*x - 4*x^2)^(3/2))/512
 + ((4*x - 3/2)*(3*x - 4*x^2)^(5/2))/24 + (405*(x/2 - 3/16)*(3*x - 4*x^2)^
(1/2))/2048
```

3.8 $\int (3x-4x^2)^{3/2}\,dx$

3.8.1 Optimal result

Integrand size = 13, antiderivative size = 57

$$\int (3x-4x^2)^{3/2}\,dx = -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} - \frac{243\arcsin\left(1-\frac{8x}{3}\right)}{4096}$$

output
```
-1/32*(3-8*x)*(-4*x^2+3*x)^(3/2)+243/4096*arcsin(-1+8/3*x)-27/1024*(3-8*x)
*(-4*x^2+3*x)^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int (3x-4x^2)^{3/2}\,dx = \frac{\sqrt{-x(-3+4x)}\left(-2\sqrt{x}\sqrt{-3+4x}(81+72x-1152x^2+1024x^3)+243\log\left(-2\sqrt{x}+\sqrt{-3+\right.\right.}{2048\sqrt{x}\sqrt{-3+4x}}$$

input
```
Integrate[(3*x - 4*x^2)^(3/2),x]
```

output
```
(Sqrt[-(x*(-3 + 4*x))]*(-2*Sqrt[x]*Sqrt[-3 + 4*x]*(81 + 72*x - 1152*x^2 +
1024*x^3) + 243*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]]))/(2048*Sqrt[x]*Sqrt[-3 +
 4*x])
```

3.8.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1087, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x-4x^2\right)^{3/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{27}{64}\int \sqrt{3x-4x^2}dx-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}dx-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{27}{64}\left(-\frac{3}{64}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{27}{64}\left(-\frac{9}{64}\arcsin\left(\frac{1}{3}(3-8x)\right)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}$$

input
```
Int[(3*x - 4*x^2)^(3/2),x]
```

output
```
-1/32*((3 - 8*x)*(3*x - 4*x^2)^(3/2)) + (27*(-1/16*((3 - 8*x)*Sqrt[3*x - 4
*x^2]) - (9*ArcSin[(3 - 8*x)/3])/64))/64
```

3.8.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.8.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result
risch	$\frac{(1024x^3-1152x^2+72x+81)x(4x-3)}{1024\sqrt{-x(4x-3)}}+\frac{243\arcsin\left(-1+\frac{8x}{3}\right)}{4096}$
default	$-\frac{(3-8x)\left(-4x^2+3x\right)^{\frac{3}{2}}}{32}+\frac{243\arcsin\left(-1+\frac{8x}{3}\right)}{4096}-\frac{27(3-8x)\sqrt{-4x^2+3x}}{1024}$
pseudoelliptic	$-\frac{243\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)}{2048}+\frac{\left(-1024x^3+1152x^2-72x-81\right)\sqrt{-4x^2+3x}}{1024}$
meijerg	$-\frac{243i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{3}\,\left(\frac{5120}{27}x^3-\frac{640}{3}x^2+\frac{40}{3}x+15\right)\sqrt{-\frac{4x}{3}+1}}{360}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2\sqrt{3}\,\sqrt{x}}{3}\right)}{16}\right)}{128\sqrt{\pi}}$
trager	$\left(-x^3+\frac{9}{8}x^2-\frac{9}{128}x-\frac{81}{1024}\right)\sqrt{-4x^2+3x}+\frac{243\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-8\,\mathrm{RootOf}\left(\_Z^2+1\right)x+4\sqrt{-4x^2+3\right.}{4096}$

input
```
int((-4*x^2+3*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1024*(1024*x^3-1152*x^2+72*x+81)*x*(4*x-3)/(-x*(4*x-3))^(1/2)+243/4096*a
rcsin(-1+8/3*x)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \left(3x-4x^2\right)^{3/2} dx = -\frac{1}{1024}\left(1024\,x^3-1152\,x^2+72\,x+81\right)\sqrt{-4\,x^2+3\,x} - \frac{243}{2048}\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

input
```
integrate((-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/1024*(1024*x^3 - 1152*x^2 + 72*x + 81)*sqrt(-4*x^2 + 3*x) - 243/2048*ar
ctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \left(3x-4x^2\right)^{3/2} dx = 3\sqrt{-4x^2+3x}\left(\frac{x^2}{3}-\frac{x}{16}-\frac{9}{128}\right) - 4\sqrt{-4x^2+3x}\left(\frac{x^3}{4}-\frac{x^2}{32}-\frac{15x}{512}-\frac{135}{4096}\right) + \frac{243\operatorname{asin}\left(\frac{8x}{3}-1\right)}{4096}$$

input
```
integrate((-4*x**2+3*x)**(3/2),x)
```

output
```
3*sqrt(-4*x**2 + 3*x)*(x**2/3 - x/16 - 9/128) - 4*sqrt(-4*x**2 + 3*x)*(x**
3/4 - x**2/32 - 15*x/512 - 135/4096) + 243*asin(8*x/3 - 1)/4096
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \left(3x-4x^2\right)^{3/2} dx = \frac{1}{4}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x - \frac{3}{32}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}} + \frac{27}{128}\sqrt{-4\,x^2+3\,x}\,x - \frac{81}{1024}\sqrt{-4\,x^2+3\,x} - \frac{243}{4096}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

input
```
integrate((-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-4*x^2 + 3*x)^(3/2)*x - 3/32*(-4*x^2 + 3*x)^(3/2) + 27/128*sqrt(-4*x^
2 + 3*x)*x - 81/1024*sqrt(-4*x^2 + 3*x) - 243/4096*arcsin(-8/3*x + 1)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \left(3x-4x^2\right)^{3/2} dx = -\frac{1}{1024}\left(8\left(16\left(8x-9\right)x+9\right)x+81\right)\sqrt{-4x^2+3x} + \frac{243}{4096}\arcsin\left(\frac{8}{3}x-1\right)$$

input
```
integrate((-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

output
```
-1/1024*(8*(16*(8*x - 9)*x + 9)*x + 81)*sqrt(-4*x^2 + 3*x) + 243/4096*arcs
in(8/3*x - 1)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \left(3x-4x^2\right)^{3/2} dx = \frac{243\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{4096} + \frac{\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{3/2}}{16} + \frac{27\left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{3x-4x^2}}{64}$$

input
```
int((3*x - 4*x^2)^(3/2),x)
```

output
```
(243*asin((8*x)/3 - 1))/4096 + ((4*x - 3/2)*(3*x - 4*x^2)^(3/2))/16 + (27*
(x/2 - 3/16)*(3*x - 4*x^2)^(1/2))/64
```

3.9 $\int \sqrt{3x-4x^2}\,dx$

3.9.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \sqrt{3x-4x^2}\,dx = -\frac{1}{16}(3-8x)\sqrt{3x-4x^2} - \frac{9}{64}\arcsin\left(1-\frac{8x}{3}\right)$$

output
```
9/64*arcsin(-1+8/3*x)-1/16*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \sqrt{3x-4x^2}\,dx = \frac{1}{32}\sqrt{-x(-3+4x)}\left(-6+16x+\frac{9\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)}{\sqrt{x}\sqrt{-3+4x}}\right)$$

input
```
Integrate[Sqrt[3*x - 4*x^2],x]
```

output
```
(Sqrt[-(x*(-3 + 4*x))]*(-6 + 16*x + (9*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]])/(
Sqrt[x]*Sqrt[-3 + 4*x])))/32
```

3.9.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x-4x^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}dx-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{3}{64}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)$$

$$\Big\downarrow \text{223}$$

$$-\frac{9}{64}\arcsin\left(\frac{1}{3}(3-8x)\right)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)$$

input
```
Int[Sqrt[3*x - 4*x^2],x]
```

output
```
-1/16*((3 - 8*x)*Sqrt[3*x - 4*x^2]) - (9*ArcSin[(3 - 8*x)/3])/64
```

3.9.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.9.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result
default	$\frac{9 \arcsin\left(-1+\frac{8x}{3}\right)}{64}-\frac{(3-8x)\sqrt{-4x^2+3x}}{16}$
risch	$-\frac{(-3+8x)x(4x-3)}{16\sqrt{-x(4x-3)}}+\frac{9 \arcsin\left(-1+\frac{8x}{3}\right)}{64}$
pseudoelliptic	$-\frac{9 \arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)}{32}+\frac{(-3+8x)\sqrt{-4x^2+3x}}{16}$
meijerg	$-\frac{9i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{3}\,(3-8x)\sqrt{-\frac{4x}{3}+1}}{9}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2\sqrt{3}\,\sqrt{x}}{3}\right)}{2}\right)}{16\sqrt{\pi}}$
trager	$\left(-\frac{3}{16}+\frac{x}{2}\right)\sqrt{-4x^2+3x}-\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(8\,\mathrm{RootOf}\left(\_Z^2+1\right)x+4\sqrt{-4x^2+3x}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{64}$

input
```
int((-4*x^2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
9/64*arcsin(-1+8/3*x)-1/16*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \sqrt{3x-4x^2}\,dx = \frac{1}{16}\,\sqrt{-4\,x^2+3\,x}(8\,x-3)-\frac{9}{32}\,\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

input
```
integrate((-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) - 9/32*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \sqrt{3x-4x^2}\,dx = \left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{-4x^2+3x}+\frac{9\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{64}$$

input
```
integrate((-4*x**2+3*x)**(1/2),x)
```

output
```
(x/2 - 3/16)*sqrt(-4*x**2 + 3*x) + 9*asin(8*x/3 - 1)/64
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \sqrt{3x-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2+3\,x}x-\frac{3}{16}\sqrt{-4\,x^2+3\,x}-\frac{9}{64}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

input
```
integrate((-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*x^2 + 3*x)*x - 3/16*sqrt(-4*x^2 + 3*x) - 9/64*arcsin(-8/3*x +
1)
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \sqrt{3x-4x^2}\,dx = \frac{1}{16}\sqrt{-4\,x^2+3\,x}(8\,x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}\,x-1\right)$$

input
```
integrate((-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sqrt{3x-4x^2}\,dx=\frac{9\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{64}+\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}$$

input
```
int((3*x - 4*x^2)^(1/2),x)
```

output
```
(9*asin((8*x)/3 - 1))/64 + (x/2 - 3/16)*(3*x - 4*x^2)^(1/2)
```

3.10 $\int \sqrt{6x-x^2}\,dx$

3.10.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \sqrt{6x-x^2}\,dx=-\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{9}{2}\arcsin\left(1-\frac{x}{3}\right)$$

output
```
9/2*arcsin(-1+1/3*x)-1/2*(3-x)*(-x^2+6*x)^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \sqrt{6x-x^2}\,dx=\frac{1}{2}\sqrt{-((-6+x)x)}\left(-3+x+\frac{18\log\left(\sqrt{-6+x}-\sqrt{x}\right)}{\sqrt{-6+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[6*x - x^2],x]
```

output
```
(Sqrt[-((-6 + x)*x)]*(-3 + x + (18*Log[Sqrt[-6 + x] - Sqrt[x]])/(Sqrt[-6 +
 x]*Sqrt[x])))/2
```

3.10.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{6x-x^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{6x-x^2}}dx-\frac{1}{2}(3-x)\sqrt{6x-x^2}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{3}{4}\int \frac{1}{\sqrt{1-\frac{1}{36}(6-2x)^2}}d(6-2x)-\frac{1}{2}\sqrt{6x-x^2}(3-x)$$

$$\Big\downarrow \text{223}$$

$$-\frac{9}{2}\arcsin\left(\frac{1}{6}(6-2x)\right)-\frac{1}{2}\sqrt{6x-x^2}(3-x)$$

input
```
Int[Sqrt[6*x - x^2],x]
```

output
```
-1/2*((3 - x)*Sqrt[6*x - x^2]) - (9*ArcSin[(6 - 2*x)/6])/2
```

3.10.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.10.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

method	result	si
risch	$-\frac{(-3+x)x(-6+x)}{2\sqrt{-x(-6+x)}}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$	2
default	$-\frac{(6-2x)\sqrt{-x^2+6x}}{4}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$	2
pseudoelliptic	$-9\arctan\left(\frac{\sqrt{-x(-6+x)}}{x}\right)+\frac{(-3+x)\sqrt{-x(-6+x)}}{2}$	3
meijerg	$-\frac{18i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{6}\,(3-x)\sqrt{-\frac{x}{6}+1}}{36}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{6}\,\sqrt{x}}{6}\right)}{2}\right)}{\sqrt{\pi}}$	4
trager	$\left(-\frac{3}{2}+\frac{x}{2}\right)\sqrt{-x^2+6x}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+6x}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	5

input
```
int((-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-3+x)*x*(-6+x)/(-x*(-6+x))^(1/2)+9/2*arcsin(-1+1/3*x)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sqrt{6x-x^2}\,dx=\frac{1}{2}\sqrt{-x^2+6\,x}(x-3)-9\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/x)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sqrt{6x - x^2}\, dx = \left(\frac{x}{2} - \frac{3}{2}\right)\sqrt{-x^2 + 6x} + \frac{9\,\mathrm{asin}\left(\frac{x}{3} - 1\right)}{2}$$

input
```
integrate((-x**2+6*x)**(1/2),x)
```

output
```
(x/2 - 3/2)*sqrt(-x**2 + 6*x) + 9*asin(x/3 - 1)/2
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \sqrt{6x - x^2}\, dx = \frac{1}{2}\sqrt{-x^2 + 6\,x}x - \frac{3}{2}\sqrt{-x^2 + 6\,x} - \frac{9}{2}\arcsin\left(-\frac{1}{3}\,x + 1\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x + 1)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sqrt{6x - x^2}\, dx = \frac{1}{2}\sqrt{-x^2 + 6\,x}(x - 3) + \frac{9}{2}\arcsin\left(\frac{1}{3}\,x - 1\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sqrt{6x - x^2}\, dx = \frac{9\,\mathrm{asin}\left(\frac{x}{3} - 1\right)}{2} + \left(\frac{x}{2} - \frac{3}{2}\right)\sqrt{6\,x - x^2}$$

input
```
int((6*x - x^2)^(1/2),x)
```

output
```
(9*asin(x/3 - 1))/2 + (x/2 - 3/2)*(6*x - x^2)^(1/2)
```

3.11 $\int \sqrt{5x-9x^2}\,dx$

3.11.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \sqrt{5x-9x^2}\,dx=-\frac{1}{36}(5-18x)\sqrt{5x-9x^2}-\frac{25}{216}\arcsin\left(1-\frac{18x}{5}\right)$$

output
```
25/216*arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \sqrt{5x-9x^2}\,dx=\frac{1}{108}\sqrt{-x(-5+9x)}\left(-15+54x+\frac{25\log\left(-3\sqrt{x}+\sqrt{-5+9x}\right)}{\sqrt{x}\sqrt{-5+9x}}\right)$$

input
```
Integrate[Sqrt[5*x - 9*x^2],x]
```

output
```
(Sqrt[-(x*(-5 + 9*x))]*(-15 + 54*x + (25*Log[-3*Sqrt[x] + Sqrt[-5 + 9*x]])
/(Sqrt[x]*Sqrt[-5 + 9*x])))/108
```

3.11.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{5x-9x^2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{25}{72}\int \frac{1}{\sqrt{5x-9x^2}}dx-\frac{1}{36}(5-18x)\sqrt{5x-9x^2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{5}{216}\int \frac{1}{\sqrt{1-\frac{1}{25}(5-18x)^2}}d(5-18x)-\frac{1}{36}\sqrt{5x-9x^2}(5-18x)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{25}{216}\arcsin\left(\frac{1}{5}(5-18x)\right)-\frac{1}{36}\sqrt{5x-9x^2}(5-18x)$$

input
```
Int[Sqrt[5*x - 9*x^2],x]
```

output
```
-1/36*((5 - 18*x)*Sqrt[5*x - 9*x^2]) - (25*ArcSin[(5 - 18*x)/5])/216
```

3.11.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.11.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result
default	$\frac{25 \arcsin\left(-1+\frac{18x}{5}\right)}{216}-\frac{(5-18x)\sqrt{-9x^2+5x}}{36}$
risch	$-\frac{(-5+18x)x(9x-5)}{36\sqrt{-x(9x-5)}}+\frac{25 \arcsin\left(-1+\frac{18x}{5}\right)}{216}$
pseudoelliptic	$-\frac{25 \arctan\left(\frac{\sqrt{-9x^2+5x}}{3x}\right)}{108}+\frac{(-5+18x)\sqrt{-9x^2+5x}}{36}$
meijerg	$-\frac{25i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{5}\,\left(-\frac{54x}{5}+3\right)\sqrt{-\frac{9x}{5}+1}}{10}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{3\sqrt{5}\,\sqrt{x}}{5}\right)}{2}\right)}{54\sqrt{\pi}}$
trager	$\left(-\frac{5}{36}+\frac{x}{2}\right)\sqrt{-9x^2+5x}-\frac{25\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(18\,\mathrm{RootOf}\left(\_Z^2+1\right)x-5\,\mathrm{RootOf}\left(\_Z^2+1\right)+6\sqrt{-9x^2+5x}\right.}{216}$

input
```
int((-9*x^2+5*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
25/216*arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^(1/2)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \sqrt{5x-9x^2}\,dx = \frac{1}{36}\sqrt{-9\,x^2+5\,x}(18\,x-5)-\frac{25}{108}\arctan\left(\frac{\sqrt{-9\,x^2+5\,x}}{3\,x}\right)$$

input
```
integrate((-9*x^2+5*x)^(1/2),x, algorithm="fricas")
```

output
```
1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) - 25/108*arctan(1/3*sqrt(-9*x^2 + 5*x)/
x)
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \sqrt{5x-9x^2}\,dx = \left(\frac{x}{2}-\frac{5}{36}\right)\sqrt{-9x^2+5x}+\frac{25\,\mathrm{asin}\left(\frac{18x}{5}-1\right)}{216}$$

input
```
integrate((-9*x**2+5*x)**(1/2),x)
```

output
```
(x/2 - 5/36)*sqrt(-9*x**2 + 5*x) + 25*asin(18*x/5 - 1)/216
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \sqrt{5x-9x^2}\,dx = \frac{1}{2}\sqrt{-9\,x^2+5\,x}x-\frac{5}{36}\sqrt{-9\,x^2+5\,x}-\frac{25}{216}\arcsin\left(-\frac{18}{5}\,x+1\right)$$

input
```
integrate((-9*x^2+5*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-9*x^2 + 5*x)*x - 5/36*sqrt(-9*x^2 + 5*x) - 25/216*arcsin(-18/5*x
 + 1)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \sqrt{5x-9x^2}\,dx = \frac{1}{36}\sqrt{-9\,x^2+5\,x}(18\,x-5)+\frac{25}{216}\arcsin\left(\frac{18}{5}\,x-1\right)$$

input
```
integrate((-9*x^2+5*x)^(1/2),x, algorithm="giac")
```

output
```
1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) + 25/216*arcsin(18/5*x - 1)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sqrt{5x-9x^2}\,dx=\frac{25\,\mathrm{asin}\left(\frac{18\,x}{5}-1\right)}{216}+\left(\frac{x}{2}-\frac{5}{36}\right)\,\sqrt{5\,x-9\,x^2}$$

input
```
int((5*x - 9*x^2)^(1/2),x)
```

output
```
(25*asin((18*x)/5 - 1))/216 + (x/2 - 5/36)*(5*x - 9*x^2)^(1/2)
```

3.12 $\int (x-x^2)^{3/2}\,dx$

3.12.1 Optimal result

Integrand size = 11, antiderivative size = 51

$$\int (x-x^2)^{3/2}\,dx=-\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{128}\arcsin(1-2x)$$

output
```
-1/8*(1-2*x)*(-x^2+x)^(3/2)+3/128*arcsin(-1+2*x)-3/64*(1-2*x)*(-x^2+x)^(1/
2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int (x-x^2)^{3/2}\,dx=\frac{x(-3+x+26x^2-40x^3+16x^4)+6\sqrt{-1+x}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)}{64\sqrt{-((-1+x)x)}}$$

input
```
Integrate[(x - x^2)^(3/2),x]
```

output
```
(x*(-3 + x + 26*x^2 - 40*x^3 + 16*x^4) + 6*Sqrt[-1 + x]*Sqrt[x]*ArcTanh[Sq
rt[-1 + x]/(-1 + Sqrt[x])])/(64*Sqrt[-((-1 + x)*x)])
```

3.12.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1087, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x-x^2\right)^{3/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{3}{16} \int \sqrt{x-x^2}dx-\frac{1}{8}(1-2x) \left(x-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{3}{16} \left(\frac{1}{8} \int \frac{1}{\sqrt{x-x^2}}dx-\frac{1}{4}(1-2x)\sqrt{x-x^2}\right)-\frac{1}{8}(1-2x) \left(x-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{3}{16} \left(-\frac{1}{8} \int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)-\frac{1}{4}\sqrt{x-x^2}(1-2x)\right)-\frac{1}{8}(1-2x) \left(x-x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{16} \left(-\frac{1}{8} \arcsin(1-2x)-\frac{1}{4}\sqrt{x-x^2}(1-2x)\right)-\frac{1}{8}(1-2x) \left(x-x^2\right)^{3/2}$$

input
```
Int[(x - x^2)^(3/2),x]
```

output
```
-1/8*((1 - 2*x)*(x - x^2)^(3/2)) + (3*(-1/4*((1 - 2*x)*Sqrt[x - x^2]) - Ar
cSin[1 - 2*x]/8))/16
```

3.12.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.12.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
risch	$\frac{(16x^3-24x^2+2x+3)(-1+x)x}{64\sqrt{-(-1+x)x}}+\frac{3\arcsin(-1+2x)}{128}$
default	$-\frac{(1-2x)(-x^2+x)^{\frac{3}{2}}}{8}+\frac{3\arcsin(-1+2x)}{128}-\frac{3(1-2x)\sqrt{-x^2+x}}{64}$
pseudoelliptic	$-\frac{3\arctan\left(\frac{\sqrt{-(-1+x)x}}{x}\right)}{64}+\frac{(-16x^3+24x^2-2x-3)\sqrt{-(-1+x)x}}{64}$
meijerg	$-\frac{3i\left(-\frac{i\sqrt{\pi}\sqrt{x}\left(80x^3-120x^2+10x+15\right)\sqrt{1-x}}{240}+\frac{i\sqrt{\pi}\arcsin(\sqrt{x})}{16}\right)}{4\sqrt{\pi}}$
trager	$\left(-\frac{1}{4}x^3+\frac{3}{8}x^2-\frac{1}{32}x-\frac{3}{64}\right)\sqrt{-x^2+x}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}-\mathrm{RootOf}\right.}{128}$

input
```
int((-x^2+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*x^3-24*x^2+2*x+3)*(-1+x)*x/(-(-1+x)*x)^(1/2)+3/128*arcsin(-1+2*x)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(x-x^2\right)^{3/2}\,dx = -\frac{1}{64}\,\left(16\,x^{3}-24\,x^{2}+2\,x+3\right)\sqrt{-x^{2}+x}-\frac{3}{64}\,\arctan\left(\frac{\sqrt{-x^{2}+x}}{x}\right)$$

input
```
integrate((-x^2+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(16*x^3 - 24*x^2 + 2*x + 3)*sqrt(-x^2 + x) - 3/64*arctan(sqrt(-x^2 +
 x)/x)
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \left(x-x^2\right)^{3/2}\,dx = \sqrt{-x^{2}+x}\left(\frac{x^{2}}{3}-\frac{x}{12}-\frac{1}{8}\right) - \sqrt{-x^{2}+x}\left(\frac{x^{3}}{4}-\frac{x^{2}}{24}-\frac{5x}{96}-\frac{5}{64}\right)+\frac{3\operatorname{asin}\left(2x-1\right)}{128}$$

input
```
integrate((-x**2+x)**(3/2),x)
```

output
```
sqrt(-x**2 + x)*(x**2/3 - x/12 - 1/8) - sqrt(-x**2 + x)*(x**3/4 - x**2/24
- 5*x/96 - 5/64) + 3*asin(2*x - 1)/128
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \left(x-x^2\right)^{3/2}\,dx = \frac{1}{4}\,\left(-x^{2}+x\right)^{\frac{3}{2}}x-\frac{1}{8}\,\left(-x^{2}+x\right)^{\frac{3}{2}} + \frac{3}{32}\,\sqrt{-x^{2}+x}x-\frac{3}{64}\,\sqrt{-x^{2}+x}+\frac{3}{128}\,\arcsin\left(2\,x-1\right)$$

input
```
integrate((-x^2+x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-x^2 + x)^(3/2)*x - 1/8*(-x^2 + x)^(3/2) + 3/32*sqrt(-x^2 + x)*x - 3/
64*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int (x-x^2)^{3/2}\,dx = -\frac{1}{64}\,(2\,(4\,(2\,x-3)x+1)x+3)\sqrt{-x^2+x}+\frac{3}{128}\,\arcsin(2\,x-1)$$

input
```
integrate((-x^2+x)^(3/2),x, algorithm="giac")
```

output
```
-1/64*(2*(4*(2*x - 3)*x + 1)*x + 3)*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int (x-x^2)^{3/2}\,dx = \frac{3\,\mathrm{asin}(2\,x-1)}{128}+\frac{3\,\sqrt{x-x^2}\,\left(\frac{x}{2}-\frac{1}{4}\right)}{16}+\frac{(x-x^2)^{3/2}\,\left(x-\frac{1}{2}\right)}{4}$$

input
```
int((x - x^2)^(3/2),x)
```

output
```
(3*asin(2*x - 1))/128 + (3*(x - x^2)^(1/2)*(x/2 - 1/4))/16 + ((x - x^2)^(3
/2)*(x - 1/2))/4
```

3.13 $\int \sqrt{4x+x^2}\,dx$

3.13.1 Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \sqrt{4x+x^2}\,dx=\frac{1}{2}(2+x)\sqrt{4x+x^2}-4\text{arctanh}\left(\frac{x}{\sqrt{4x+x^2}}\right)$$

output
```
-4*arctanh(x/(x^2+4*x)^(1/2))+1/2*(2+x)*(x^2+4*x)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \sqrt{4x+x^2}\,dx=\frac{1}{2}\sqrt{x(4+x)}\left(2+x+\frac{8\log\left(-\sqrt{x}+\sqrt{4+x}\right)}{\sqrt{x}\sqrt{4+x}}\right)$$

input
```
Integrate[Sqrt[4*x + x^2],x]
```

output
```
(Sqrt[x*(4 + x)]*(2 + x + (8*Log[-Sqrt[x] + Sqrt[4 + x]])/(Sqrt[x]*Sqrt[4
+ x])))/2
```

3.13.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+4x}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-2\int \frac{1}{\sqrt{x^2+4x}}dx$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\int \frac{1}{1-\frac{x^2}{x^2+4x}}d\frac{x}{\sqrt{x^2+4x}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\text{arctanh}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

input
```
Int[Sqrt[4*x + x^2],x]
```

output
```
((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[x/Sqrt[4*x + x^2]]
```

3.13.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.13.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
trager	$\left(\frac{x}{2}+1\right)\sqrt{x^{2}+4x}-2\ln\left(2+x+\sqrt{x^{2}+4x}\right)$	32
default	$\frac{(2x+4)\sqrt{x^{2}+4x}}{4}-2\ln\left(2+x+\sqrt{x^{2}+4x}\right)$	33
risch	$\frac{(2+x)x(4+x)}{2\sqrt{x(4+x)}}-2\ln\left(2+x+\sqrt{x^{2}+4x}\right)$	33
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\left(3+\frac{3x}{2}\right)\sqrt{\frac{x}{4}+1}}{12}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{x}}{2}\right)}{2}\right)}{\sqrt{\pi}}$	38
pseudoelliptic	$\frac{8x^{2}\left((2+x)\sqrt{x(4+x)}+4\ln\left(\frac{\sqrt{x(4+x)}-x}{x}\right)-4\ln\left(\frac{x+\sqrt{x(4+x)}}{x}\right)\right)}{\left(-\sqrt{x(4+x)}+x\right)^{2}\left(x+\sqrt{x(4+x)}\right)^{2}}$	76

input
```
int((x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2*x+1)*(x^2+4*x)^(1/2)-2*ln(2+x+(x^2+4*x)^(1/2))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int\sqrt{4x+x^{2}}\,dx=\frac{1}{2}\,\sqrt{x^{2}+4\,x}(x+2)+2\,\log\left(-x+\sqrt{x^{2}+4\,x}-2\right)$$

input
```
integrate((x^2+4*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(-x + sqrt(x^2 + 4*x) - 2)
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \sqrt{4x+x^2}\,dx = \left(\frac{x}{2}+1\right)\sqrt{x^2+4x} - 2\log\left(2x+2\sqrt{x^2+4x}+4\right)$$

input
```
integrate((x**2+4*x)**(1/2),x)
```

output
```
(x/2 + 1)*sqrt(x**2 + 4*x) - 2*log(2*x + 2*sqrt(x**2 + 4*x) + 4)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \sqrt{4x+x^2}\,dx = \frac{1}{2}\sqrt{x^2+4\,x}x + \sqrt{x^2+4\,x} - 2\,\log\left(2\,x+2\,\sqrt{x^2+4\,x}+4\right)$$

input
```
integrate((x^2+4*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 + 4*x)*x + sqrt(x^2 + 4*x) - 2*log(2*x + 2*sqrt(x^2 + 4*x) +
4)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \sqrt{4x+x^2}\,dx = \frac{1}{2}\sqrt{x^2+4\,x}(x+2) + 2\,\log\left(\left|-x+\sqrt{x^2+4\,x}-2\right|\right)$$

input
```
integrate((x^2+4*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \sqrt{4x+x^2}\,dx = \sqrt{x^2+4\,x}\,\left(\frac{x}{2}+1\right)-2\,\ln\left(x+\sqrt{x\,\left(x+4\right)}+2\right)$$

input
```
int((4*x + x^2)^(1/2),x)
```

output
```
(4*x + x^2)^(1/2)*(x/2 + 1) - 2*log(x + (x*(x + 4))^(1/2) + 2)
```

3.14 $\int \sqrt{-8x+x^2}\,dx$

3.14.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \sqrt{-8x+x^2}\,dx=-\frac{1}{2}(4-x)\sqrt{-8x+x^2}-16\operatorname{arctanh}\left(\frac{x}{\sqrt{-8x+x^2}}\right)$$

output
```
-16*arctanh(x/(x^2-8*x)^(1/2))-1/2*(4-x)*(x^2-8*x)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \sqrt{-8x+x^2}\,dx=\frac{1}{2}\sqrt{(-8+x)x}\left(-4+x+\frac{32\log\left(\sqrt{-8+x}-\sqrt{x}\right)}{\sqrt{-8+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[-8*x + x^2],x]
```

output
```
(Sqrt[(-8 + x)*x]*(-4 + x + (32*Log[Sqrt[-8 + x] - Sqrt[x]])/(Sqrt[-8 + x]
*Sqrt[x])))/2
```

3.14.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2-8x}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-8\int \frac{1}{\sqrt{x^2-8x}}dx-\frac{1}{2}\sqrt{x^2-8x}(4-x)$$

$$\Big\downarrow \text{1091}$$

$$-16\int \frac{1}{1-\frac{x^2}{x^2-8x}}d\frac{x}{\sqrt{x^2-8x}}-\frac{1}{2}\sqrt{x^2-8x}(4-x)$$

$$\Big\downarrow \text{219}$$

$$-16\text{arctanh}\left(\frac{x}{\sqrt{x^2-8x}}\right)-\frac{1}{2}\sqrt{x^2-8x}(4-x)$$

input
```
Int[Sqrt[-8*x + x^2],x]
```

output
```
-1/2*((4 - x)*Sqrt[-8*x + x^2]) - 16*ArcTanh[x/Sqrt[-8*x + x^2]]
```

3.14.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.14.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
trager	$\left(\frac{x}{2}-2\right)\sqrt{x^{2}-8x}-8\ln\left(x-4+\sqrt{x^{2}-8x}\right)$	32
default	$\frac{(2x-8)\sqrt{x^{2}-8x}}{4}-8\ln\left(x-4+\sqrt{x^{2}-8x}\right)$	33
risch	$\frac{(x-4)x(x-8)}{2\sqrt{x(x-8)}}-8\ln\left(x-4+\sqrt{x^{2}-8x}\right)$	33
meijerg	$-\frac{32i\sqrt{\operatorname{signum}(x-8)}\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\left(-\frac{3x}{4}+3\right)\sqrt{-\frac{x}{8}+1}}{24}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{4}\right)}{2}\right)}{\sqrt{\pi}\,\sqrt{-\operatorname{signum}(x-8)}}$	61
pseudoelliptic	$\frac{32\left((x-4)\sqrt{x(x-8)}+16\ln\left(\frac{\sqrt{x(x-8)}-x}{x}\right)-16\ln\left(\frac{x+\sqrt{x(x-8)}}{x}\right)\right)x^{2}}{\left(-\sqrt{x(x-8)}+x\right)^{2}\left(x+\sqrt{x(x-8)}\right)^{2}}$	76

input
```
int((x^2-8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2*x-2)*(x^2-8*x)^(1/2)-8*ln(x-4+(x^2-8*x)^(1/2))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int\sqrt{-8x+x^{2}}\,dx=\frac{1}{2}\sqrt{x^{2}-8\,x}(x-4)+8\,\log\left(-x+\sqrt{x^{2}-8\,x}+4\right)$$

input
```
integrate((x^2-8*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(-x + sqrt(x^2 - 8*x) + 4)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \sqrt{-8x+x^2}\,dx = \left(\frac{x}{2}-2\right)\sqrt{x^2-8x} - 8\log\left(2x+2\sqrt{x^2-8x}-8\right)$$

input
```
integrate((x**2-8*x)**(1/2),x)
```

output
```
(x/2 - 2)*sqrt(x**2 - 8*x) - 8*log(2*x + 2*sqrt(x**2 - 8*x) - 8)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \sqrt{-8x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-8\,x}x - 2\sqrt{x^2-8\,x} - 8\,\log\left(2\,x+2\sqrt{x^2-8\,x}-8\right)$$

input
```
integrate((x^2-8*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 - 8*x)*x - 2*sqrt(x^2 - 8*x) - 8*log(2*x + 2*sqrt(x^2 - 8*x)
- 8)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \sqrt{-8x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-8\,x}(x-4) + 8\,\log\left(\left|-x+\sqrt{x^2-8\,x}+4\right|\right)$$

input
```
integrate((x^2-8*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(abs(-x + sqrt(x^2 - 8*x) + 4))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \sqrt{-8x+x^2}\,dx=\left(\frac{x}{2}-2\right)\,\sqrt{x^2-8\,x}-8\,\ln\left(x+\sqrt{x\,\left(x-8\right)}-4\right)$$

input
```
int((x^2 - 8*x)^(1/2),x)
```

output
```
(x/2 - 2)*(x^2 - 8*x)^(1/2) - 8*log(x + (x*(x - 8))^(1/2) - 4)
```

3.15 $\int \sqrt{-x+x^2}\,dx$

3.15.1 Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \sqrt{-x+x^2}\,dx=-\frac{1}{4}(1-2x)\sqrt{-x+x^2}-\frac{1}{4}\text{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)$$

output
```
-1/4*arctanh(x/(x^2-x)^(1/2))-1/4*(1-2*x)*(x^2-x)^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \sqrt{-x+x^2}\,dx=\frac{1}{4}\sqrt{(-1+x)x}\left(-1+2x-\frac{2\text{arctanh}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)}{\sqrt{-1+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[-x + x^2],x]
```

output
```
(Sqrt[(-1 + x)*x]*(-1 + 2*x - (2*ArcTanh[Sqrt[-1 + x]/(-1 + Sqrt[x])])/(Sq
rt[-1 + x]*Sqrt[x])))/4
```

3.15.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2-x}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{1}{8}\int \frac{1}{\sqrt{x^2-x}}dx-\frac{1}{4}\sqrt{x^2-x}(1-2x)$$

$$\Big\downarrow \text{1091}$$

$$-\frac{1}{4}\int \frac{1}{1-\frac{x^2}{x^2-x}}d\frac{x}{\sqrt{x^2-x}}-\frac{1}{4}\sqrt{x^2-x}(1-2x)$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{4}\text{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{1}{4}\sqrt{x^2-x}(1-2x)$$

input
```
Int[Sqrt[-x + x^2],x]
```

output
```
-1/4*((1 - 2*x)*Sqrt[-x + x^2]) - ArcTanh[x/Sqrt[-x + x^2]]/4
```

3.15.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.15.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{(-1+2x)\sqrt{x^2-x}}{4}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)}{8}$	33
risch	$\frac{(-1+2x)(-1+x)x}{4\sqrt{(-1+x)x}}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)}{8}$	35
trager	$\left(-\frac{1}{4}+\frac{x}{2}\right)\sqrt{x^2-x}-\frac{\ln\left(2x-1+2\sqrt{x^2-x}\right)}{8}$	36
meijerg	$-\frac{i\sqrt{\operatorname{signum}(-1+x)}\left(-\frac{i\sqrt{\pi}\sqrt{x}(3-6x)\sqrt{1-x}}{6}+\frac{i\sqrt{\pi}\arcsin(\sqrt{x})}{2}\right)}{2\sqrt{\pi}\sqrt{-\operatorname{signum}(-1+x)}}$	53
pseudoelliptic	$\frac{x^2\left(4\sqrt{(-1+x)x}\,x+\ln\left(\frac{\sqrt{(-1+x)x}-x}{x}\right)-\ln\left(\frac{x+\sqrt{(-1+x)x}}{x}\right)-2\sqrt{(-1+x)x}\right)}{8\left(\sqrt{(-1+x)x}-x\right)^2\left(x+\sqrt{(-1+x)x}\right)^2}$	82

input
```
int((x^2-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-1+2*x)*(x^2-x)^(1/2)-1/8*ln(x-1/2+(x^2-x)^(1/2))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int\sqrt{-x+x^2}\,dx=\frac{1}{4}\sqrt{x^2-x}(2\,x-1)+\frac{1}{8}\,\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)$$

input
```
integrate((x^2-x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(-2*x + 2*sqrt(x^2 - x) + 1)
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \sqrt{-x+x^2}\,dx = \left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x}-\frac{\log\left(2x+2\sqrt{x^2-x}-1\right)}{8}$$

input
```
integrate((x**2-x)**(1/2),x)
```

output
```
(x/2 - 1/4)*sqrt(x**2 - x) - log(2*x + 2*sqrt(x**2 - x) - 1)/8
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \sqrt{-x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-x}x-\frac{1}{4}\sqrt{x^2-x}-\frac{1}{8}\log\left(2\,x+2\sqrt{x^2-x}-1\right)$$

input
```
integrate((x^2-x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 - x)*x - 1/4*sqrt(x^2 - x) - 1/8*log(2*x + 2*sqrt(x^2 - x) -
1)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sqrt{-x+x^2}\,dx = \frac{1}{4}\sqrt{x^2-x}(2\,x-1)+\frac{1}{8}\log\left(\left|-2\,x+2\sqrt{x^2-x}+1\right|\right)$$

input
```
integrate((x^2-x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{-x+x^2}\,dx=\sqrt{x^2-x}\left(\frac{x}{2}-\frac{1}{4}\right)-\frac{\ln\left(x+\sqrt{x\,(x-1)}-\frac{1}{2}\right)}{8}$$

input
```
int((x^2 - x)^(1/2),x)
```

output
```
(x^2 - x)^(1/2)*(x/2 - 1/4) - log(x + (x*(x - 1))^(1/2) - 1/2)/8
```

3.16 $\int \frac{1}{(bx+cx^2)^{7/2}}\,dx$

3.16.1 Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}} + \frac{32c(b+2cx)}{15b^4(bx+cx^2)^{3/2}} - \frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}$$

output
```
-2/5*(2*c*x+b)/b^2/(c*x^2+b*x)^(5/2)+32/15*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(3/
2)-256/15*c^2*(2*c*x+b)/b^6/(c*x^2+b*x)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2(3b^5-10b^4cx+80b^3c^2x^2+480b^2c^3x^3+640bc^4x^4+256c^5x^5)}{15b^6(x(b+cx))^{5/2}}$$

input
```
Integrate[(b*x + c*x^2)^(-7/2),x]
```

output
```
(-2*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^3 + 640*b*c^4*x^4
 + 256*c^5*x^5))/(15*b^6*(x*(b + c*x))^(5/2))
```

3.16.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{16c\int \frac{1}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{16c\left(-\frac{8c\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}-\frac{16c\left(\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}$$

input
```
Int[(b*x + c*x^2)^(-7/2),x]
```

output
```
(-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) - (16*c*((-2*(b + 2*c*x))/(3*
b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])))/
(5*b^2)
```

3.16.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.16.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{2x(cx+b)\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+480b^{2}c^{3}x^{3}+80x^{2}b^{3}c^{2}-10cx\,b^{4}+3b^{5}\right)}{15b^{6}\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}$	75
default	$-\frac{2(2cx+b)}{5b^{2}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^{2}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^{4}\sqrt{c\,x^{2}+bx}}\right)}{5b^{2}}$	76
pseudoelliptic	$\frac{-\frac{512}{15}c^{5}x^{5}-\frac{256}{3}b\,x^{4}c^{4}-64b^{2}c^{3}x^{3}-\frac{32}{3}x^{2}b^{3}c^{2}+\frac{4}{3}cx\,b^{4}-\frac{2}{5}b^{5}}{x^{2}(cx+b)^{2}\sqrt{x(cx+b)}\,b^{6}}$	77
trager	$-\frac{2\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+480b^{2}c^{3}x^{3}+80x^{2}b^{3}c^{2}-10cx\,b^{4}+3b^{5}\right)\sqrt{c\,x^{2}+bx}}{15b^{6}(cx+b)^{3}x^{3}}$	79
risch	$-\frac{2(cx+b)\left(128c^{2}x^{2}-19bcx+3b^{2}\right)}{15b^{6}x^{2}\sqrt{x(cx+b)}}-\frac{2c^{3}\left(128c^{2}x^{2}+275bcx+150b^{2}\right)x}{15\sqrt{x(cx+b)}\,(c^{2}x^{2}+2bcx+b^{2})b^{6}}$	98

input
```
int(1/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(c*x+b)*(256*c^5*x^5+640*b*c^4*x^4+480*b^2*c^3*x^3+80*b^3*c^2*x^2-
10*b^4*c*x+3*b^5)/b^6/(c*x^2+b*x)^(7/2)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\,(256\,c^5x^5+640\,bc^4x^4+480\,b^2c^3x^3+80\,b^3c^2x^2-10\,b^4cx+3\,b^5)\sqrt{cx^2+bx}}{15\,(b^6c^3x^6+3\,b^7c^2x^5+3\,b^8cx^4+b^9x^3)}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(256*c^5*x^5 + 640*b*c^4*x^4 + 480*b^2*c^3*x^3 + 80*b^3*c^2*x^2 - 10
*b^4*c*x + 3*b^5)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c
*x^4 + b^9*x^3)
```

3.16.6 Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral((b*x + c*x**2)**(-7/2), x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{4\,cx}{5\,(cx^2+bx)^{\frac{5}{2}}b^2}+\frac{64\,c^2x}{15\,(cx^2+bx)^{\frac{3}{2}}b^4} - \frac{512\,c^3x}{15\,\sqrt{cx^2+bx}b^6}-\frac{2}{5\,(cx^2+bx)^{\frac{5}{2}}b}+\frac{32\,c}{15\,(cx^2+bx)^{\frac{3}{2}}b^3}-\frac{256\,c^2}{15\,\sqrt{cx^2+bx}b^5}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
-4/5*c*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*x/((c*x^2 + b*x)^(3/2)*b^4)
 - 512/15*c^3*x/(sqrt(c*x^2 + b*x)*b^6) - 2/5/((c*x^2 + b*x)^(5/2)*b) + 32
/15*c/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c^2/(sqrt(c*x^2 + b*x)*b^5)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\left(2\left(8\left(2\left(4\,x\left(\frac{2\,c^5x}{b^6}+\frac{5\,c^4}{b^5}\right)+\frac{15\,c^3}{b^4}\right)x+\frac{5\,c^2}{b^3}\right)x-\frac{5\,c}{b^2}\right)x+\frac{3}{b}\right)}{15\,(cx^2+bx)^{\frac{5}{2}}}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(2*(8*(2*(4*x*(2*c^5*x/b^6 + 5*c^4/b^5) + 15*c^3/b^4)*x + 5*c^2/b^3)
*x - 5*c/b^2)*x + 3/b)/(c*x^2 + b*x)^(5/2)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{6\,b^5+256\,b\,c^2\,(c\,x^2+b\,x)^2+512\,c^3\,x\,(c\,x^2+b\,x)^2-32\,b^3\,c\,(c\,x^2+b\,x)+12\,b^4\,c\,x-64\,b^2\,c^2\,x\,(c\,x^2+b\,x)}{15\,b^6\,(c\,x^2+b\,x)^{5/2}}$$

input
```
int(1/(b*x + c*x^2)^(7/2),x)
```

output
```
-(6*b^5 + 256*b*c^2*(b*x + c*x^2)^2 + 512*c^3*x*(b*x + c*x^2)^2 - 32*b^3*c
*(b*x + c*x^2) + 12*b^4*c*x - 64*b^2*c^2*x*(b*x + c*x^2))/(15*b^6*(b*x + c
*x^2)^(5/2))
```

3.17 $\int \frac{1}{\sqrt{3ix+4x^2}}\,dx$

3.17.1 Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx=\frac{1}{2}i\arcsin\left(1-\frac{8ix}{3}\right)$$

output
```
-1/2*I*arcsin(-1+8/3*I*x)
```

3.17.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 51 vs. 2(16) = 32.

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx=-\frac{\sqrt{x}\sqrt{3i+4x}\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{\sqrt{x(3i+4x)}}$$

input
```
Integrate[1/Sqrt[(3*I)*x + 4*x^2],x]
```

output
```
-((Sqrt[x]*Sqrt[3*I + 4*x]*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/Sqrt[x*(3*I
+ 4*x)])
```

3.17.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2+3ix}}\,dx$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)$$

input
```
Int[1/Sqrt[(3*I)*x + 4*x^2],x]
```

output
```
ArcSinh[(3*I + 8*x)/3]/2
```

3.17.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.17.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{\operatorname{arcsinh}\left(i+\frac{8x}{3}\right)}{2}$	10
trager	$\frac{\ln\left(440x+144+165i-192i\sqrt{4x^2+3ix}-384ix+220\sqrt{4x^2+3ix}\right)}{2}$	44
pseudoelliptic	$\frac{\ln\left(\frac{\sqrt{x(3i+4x)}+2x}{x}\right)}{2}-\frac{\ln\left(\frac{-2x+\sqrt{x(3i+4x)}}{x}\right)}{2}$	44

input
```
int(1/(3*I*x+4*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(I+8/3*x)
```

3.17.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = -\frac{1}{2}\,\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)$$

input
```
integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I)
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{2}$$

input
```
integrate(1/(3*I*x+4*x**2)**(1/2),x)
```

output
```
asinh(8*x/3 + I)/2
```

3.17.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = \frac{1}{2}\,\log\left(8\,x + 4\,\sqrt{4\,x^2 + 3i\,x} + 3i\right)$$

input
```
integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

3.17.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(8) = 16.

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.88

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = \frac{1}{32}\,\sqrt{8\,x^2 + 2\,\sqrt{16\,x^2 + 9}x}(8\,x + 3i)\left(\frac{3i\,x}{4\,x^2 + \sqrt{16\,x^4 + 9\,x^2}} + 1\right) - \frac{9}{64}\,\log\left(2\,\sqrt{8\,x^2 + 2\,\sqrt{16\,x^2 + 9}x}\left(\frac{3i\,x}{4\,x^2 + \sqrt{16\,x^4 + 9\,x^2}} + 1\right) - 8\,x - 3i\right)$$

input
```
integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/32*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(1
6*x^4 + 9*x^2)) + 1) - 9/64*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*
x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx=\frac{\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{2}$$

input
```
int(1/(x*3i + 4*x^2)^(1/2),x)
```

output
```
log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8)/2
```

3.18 $\int \frac{1}{(3ix+4x^2)^{3/2}} \, dx$

3.18.1 Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \frac{1}{(3ix+4x^2)^{3/2}} \, dx = \frac{2(3i+8x)}{9\sqrt{3ix+4x^2}}$$

output
```
2/9*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(3ix+4x^2)^{3/2}} \, dx = \frac{2(3i+8x)}{9\sqrt{x(3i+4x)}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(-3/2),x]
```

output
```
(2*(3*I + 8*x))/(9*Sqrt[x*(3*I + 4*x)])
```

3.18.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^2+3ix)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

input
```
Int[((3*I)*x + 4*x^2)^(-3/2),x]
```

output
```
(2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])
```

3.18.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

3.18.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{\frac{2i}{3}+\frac{16x}{9}}{\sqrt{x(3i+4x)}}$	19
pseudoelliptic	$\frac{\frac{2i}{3}+\frac{16x}{9}}{\sqrt{x(3i+4x)}}$	19
default	$\frac{\frac{2i}{3}+\frac{16x}{9}}{\sqrt{4x^2+3ix}}$	21
gosper	$\frac{2x(3i+4x)(3i+8x)}{9(4x^2+3ix)^{\frac{3}{2}}}$	28
trager	$\frac{\left(-\frac{14}{225}+\frac{16i}{75}\right)(24ix+32x+12i-9)\sqrt{4x^2+3ix}}{x(12ix-16x-12i-9)}$	44

input
```
int(1/(3*I*x+4*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(3*I+8*x)/(x*(3*I+4*x))^(1/2)
```

3.18.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \frac{2\left(16\,x^2+\sqrt{4\,x^2+3i\,x}(8\,x+3i)+12i\,x\right)}{9\,(4\,x^2+3i\,x)}$$

input
```
integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(16*x^2 + sqrt(4*x^2 + 3*I*x)*(8*x + 3*I) + 12*I*x)/(4*x^2 + 3*I*x)
```

3.18.6 Sympy [F]

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \int \frac{1}{(4x^2+3ix)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*I*x+4*x**2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*I*x)**(-3/2), x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \frac{16\,x}{9\,\sqrt{4\,x^2+3i\,x}} + \frac{2i}{3\,\sqrt{4\,x^2+3i\,x}}$$

input
```
integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

output
```
16/9*x/sqrt(4*x^2 + 3*I*x) + 2/3*I/sqrt(4*x^2 + 3*I*x)
```

3.18.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(18) = 36.

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \frac{\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}(8\,x+3i)\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)}{9\,(4\,x^2+3i\,x)}$$

input
```
integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/9*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(16
*x^4 + 9*x^2)) + 1)/(4*x^2 + 3*I*x)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \frac{16\,x+6\mathrm{i}}{9\,\sqrt{4\,x^2+x\,3\mathrm{i}}}$$

input
```
int(1/(x*3i + 4*x^2)^(3/2),x)
```

output
```
(16*x + 6i)/(9*(x*3i + 4*x^2)^(1/2))
```

3.19 $\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx$

3.19.1 Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{2(3i+8x)}{27\,(3ix+4x^2)^{3/2}} + \frac{64(3i+8x)}{243\sqrt{3ix+4x^2}}$$

output
```
2/27*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+64/243*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{54i - 432x + 2304ix^2 + 2048x^3}{243(x(3i+4x))^{3/2}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(-5/2),x]
```

output
```
(54*I - 432*x + (2304*I)*x^2 + 2048*x^3)/(243*(x*(3*I + 4*x))^(3/2))
```

3.19.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^2+3ix)^{5/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{32}{27}\int \frac{1}{(4x^2+3ix)^{3/2}}dx+\frac{2(8x+3i)}{27\,(4x^2+3ix)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\,(4x^2+3ix)^{3/2}}$$

input
```
Int[((3*I)*x + 4*x^2)^(-5/2),x]
```

output
```
(2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[
(3*I)*x + 4*x^2])
```

3.19.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.19.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{2x(3i+4x)(1024x^3+1152ix^2-216x+27i)}{243(4x^2+3ix)^{\frac{5}{2}}}$	39
risch	$\frac{\frac{2048}{243}x^3+\frac{256}{27}ix^2-\frac{16}{9}x+\frac{2}{9}i}{x(3i+4x)\sqrt{x(3i+4x)}}$	41
pseudoelliptic	$\frac{2048x^3+2304ix^2-432x+54i}{243x(3i+4x)\sqrt{x(3i+4x)}}$	41
default	$\frac{\frac{2i}{9}+\frac{16x}{27}}{(4x^2+3ix)^{\frac{3}{2}}}+\frac{\frac{64i}{81}+\frac{512x}{243}}{\sqrt{4x^2+3ix}}$	42
trager	$\frac{\left(\frac{88}{151875}+\frac{26i}{16875}\right)(-76800ix^3-102400x^3-115200ix^2+86400x^2+16200ix+21600x-2700i+2025)\sqrt{4x^2+3ix}}{(12ix-16x-12i-9)^2x^2}$	66

input
```
int(1/(3*I*x+4*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/243*x*(3*I+4*x)*(1152*I*x^2+1024*x^3+27*I-216*x)/(3*I*x+4*x^2)^(5/2)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{2\left(2048\,x^4+3072i\,x^3-1152\,x^2+(1024\,x^3+1152i\,x^2-216\,x+27i)\sqrt{4\,x^2+3i\,x}\right)}{243\,(16\,x^4+24i\,x^3-9\,x^2)}$$

input
```
integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/243*(2048*x^4 + 3072*I*x^3 - 1152*x^2 + (1024*x^3 + 1152*I*x^2 - 216*x +
 27*I)*sqrt(4*x^2 + 3*I*x))/(16*x^4 + 24*I*x^3 - 9*x^2)
```

3.19.6 Sympy [F]

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \int \frac{1}{(4x^2+3ix)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(3*I*x+4*x**2)**(5/2),x)
```

output
```
Integral((4*x**2 + 3*I*x)**(-5/2), x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{512\,x}{243\,\sqrt{4\,x^2+3i\,x}} + \frac{64i}{81\,\sqrt{4\,x^2+3i\,x}} + \frac{16\,x}{27\,(4\,x^2+3i\,x)^{\frac{3}{2}}} + \frac{2i}{9\,(4\,x^2+3i\,x)^{\frac{3}{2}}}$$

input
```
integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

output
```
512/243*x/sqrt(4*x^2 + 3*I*x) + 64/81*I/sqrt(4*x^2 + 3*I*x) + 16/27*x/(4*x
^2 + 3*I*x)^(3/2) + 2/9*I/(4*x^2 + 3*I*x)^(3/2)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{(8\,(16\,(8\,x+9i)x-27)x+27i)\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)}{243\,(4\,x^2+3i\,x)^2}$$

input
```
integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/243*(8*(16*(8*x + 9*I)*x - 27)*x + 27*I)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)
*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1)/(4*x^2 + 3*I*x)^2
```

3.19.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{(16\,x+6\mathrm{i})\,\left(128\,x^2+x\,96\mathrm{i}+9\right)}{243\,\left(4\,x^2+x\,3\mathrm{i}\right)^{3/2}}$$

input
```
int(1/(x*3i + 4*x^2)^(5/2),x)
```

output
```
((16*x + 6i)*(x*96i + 128*x^2 + 9))/(243*(x*3i + 4*x^2)^(3/2))
```

3.20 $\int \frac{1}{\left(3ix+4x^2\right)^{7/2}}\,dx$

3.20.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{1}{\left(3ix+4x^2\right)^{7/2}}\,dx=\frac{2(3i+8x)}{45\left(3ix+4x^2\right)^{5/2}}+\frac{128(3i+8x)}{1215\left(3ix+4x^2\right)^{3/2}}+\frac{4096(3i+8x)}{10935\sqrt{3ix+4x^2}}$$

output
```
2/45*(3*I+8*x)/(3*I*x+4*x^2)^(5/2)+128/1215*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+
4096/10935*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(3ix+4x^2\right)^{7/2}}\,dx=\frac{1458i-6480x-69120ix^2-552960x^3+983040ix^4+524288x^5}{10935(x(3i+4x))^{5/2}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(-7/2),x]
```

output
```
(1458*I - 6480*x - (69120*I)*x^2 - 552960*x^3 + (983040*I)*x^4 + 524288*x^
5)/(10935*(x*(3*I + 4*x))^(5/2))
```

3.20.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^2+3ix)^{7/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{64}{45}\int \frac{1}{(4x^2+3ix)^{5/2}}dx+\frac{2(8x+3i)}{45(4x^2+3ix)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{64}{45}\left(\frac{32}{27}\int \frac{1}{(4x^2+3ix)^{3/2}}dx+\frac{2(8x+3i)}{27(4x^2+3ix)^{3/2}}\right)+\frac{2(8x+3i)}{45(4x^2+3ix)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(8x+3i)}{45(4x^2+3ix)^{5/2}}+\frac{64}{45}\left(\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27(4x^2+3ix)^{3/2}}\right)$$

input
```
Int[((3*I)*x + 4*x^2)^(-7/2),x]
```

output
```
(2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^(5/2)) + (64*((2*(3*I + 8*x))/(27*((
3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[(3*I)*x + 4*x^2])))/45
```

3.20.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.20.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

method	result
gosper	$\frac{2x(3i+4x)(262144x^5+491520ix^4-276480x^3-34560ix^2-3240x+729i)}{10935(4x^2+3ix)^{\frac{7}{2}}}$
risch	$\frac{\frac{524288}{10935}x^5+\frac{65536}{729}ix^4-\frac{4096}{81}x^3-\frac{512}{81}ix^2-\frac{16}{27}x+\frac{2}{15}i}{x^2(3i+4x)^2\sqrt{x(3i+4x)}}$
pseudoelliptic	$\frac{\frac{524288}{10935}x^5+\frac{65536}{729}ix^4-\frac{4096}{81}x^3-\frac{512}{81}ix^2-\frac{16}{27}x+\frac{2}{15}i}{x^2(3i+4x)^2\sqrt{x(3i+4x)}}$
default	$\frac{\frac{2i}{15}+\frac{16x}{45}}{(4x^2+3ix)^{\frac{5}{2}}}+\frac{\frac{128i}{405}+\frac{1024x}{1215}}{(4x^2+3ix)^{\frac{3}{2}}}+\frac{\frac{4096i}{3645}+\frac{32768x}{10935}}{\sqrt{4x^2+3ix}}$
trager	$\frac{\left(\frac{1054}{4271484375}+\frac{224i}{1423828125}\right)(12288000000ix^5+16384000000x^5+30720000000ix^4-23040000000x^4-12960000000ix^3-1728000}{(12ix-16x-12i-9)^3}$

input
```
int(1/(3*I*x+4*x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/10935*x*(3*I+4*x)*(491520*I*x^4+262144*x^5-34560*I*x^2-276480*x^3+729*I-
3240*x)/(3*I*x+4*x^2)^(7/2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \frac{2\left(524288\,x^6+1179648i\,x^5-884736\,x^4-221184i\,x^3+(262144\,x^5+491520i\,x^4-2\right.}{10935\,(64\,x^6+144i\,x^5-108\,x^4-27}$$

input
```
integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

output
```
2/10935*(524288*x^6 + 1179648*I*x^5 - 884736*x^4 - 221184*I*x^3 + (262144*
x^5 + 491520*I*x^4 - 276480*x^3 - 34560*I*x^2 - 3240*x + 729*I)*sqrt(4*x^2
 + 3*I*x))/(64*x^6 + 144*I*x^5 - 108*x^4 - 27*I*x^3)
```

3.20.6 Sympy [F]

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \int \frac{1}{(4x^2+3ix)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(3*I*x+4*x**2)**(7/2),x)
```

output
```
Integral((4*x**2 + 3*I*x)**(-7/2), x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \frac{32768\,x}{10935\,\sqrt{4\,x^2+3i\,x}} + \frac{4096i}{3645\,\sqrt{4\,x^2+3i\,x}} + \frac{1024\,x}{1215\,(4\,x^2+3i\,x)^{\frac{3}{2}}} + \frac{128i}{405\,(4\,x^2+3i\,x)^{\frac{3}{2}}} + \frac{16\,x}{45\,(4\,x^2+3i\,x)^{\frac{5}{2}}} + \frac{2i}{15\,(4\,x^2+3i\,x)^{\frac{5}{2}}}$$

input
```
integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

output
```
32768/10935*x/sqrt(4*x^2 + 3*I*x) + 4096/3645*I/sqrt(4*x^2 + 3*I*x) + 1024
/1215*x/(4*x^2 + 3*I*x)^(3/2) + 128/405*I/(4*x^2 + 3*I*x)^(3/2) + 16/45*x/
(4*x^2 + 3*I*x)^(5/2) + 2/15*I/(4*x^2 + 3*I*x)^(5/2)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \frac{(8\,(32\,(8\,(16\,(8\,x+15i)x-135)x-135i)x-405)x+729i)\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}\,(}{10935\,(4\,x^2+3i\,x)^3}$$

input
```
integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

output
```
1/10935*(8*(32*(8*(16*(8*x + 15*I)*x - 135)*x - 135*I)*x - 405)*x + 729*I)
*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2))
+ 1)/(4*x^2 + 3*I*x)^3
```

3.20.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = -\frac{-524288\,x^5 - x^4\,983040\mathrm{i} + 552960\,x^3 + x^2\,69120\mathrm{i} + 6480\,x - 1458\mathrm{i}}{10935\,(x\ (4\,x+3\mathrm{i}))^{5/2}}$$

input
```
int(1/(x*3i + 4*x^2)^(7/2),x)
```

output
```
-(6480*x + x^2*69120i + 552960*x^3 - x^4*983040i - 524288*x^5 - 1458i)/(10
935*(x*(4*x + 3i))^(5/2))
```

3.21 $\int \frac{1}{\sqrt{3x-4x^2}} \, dx$

3.21.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{1}{\sqrt{3x-4x^2}} \, dx = -\frac{1}{2} \arcsin\left(1-\frac{8x}{3}\right)$$

output
```
1/2*arcsin(-1+8/3*x)
```

3.21.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(12) = 24.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.83

$$\int \frac{1}{\sqrt{3x-4x^2}} \, dx = -\frac{\sqrt{x}\sqrt{-3+4x}\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)}{\sqrt{-x(-3+4x)}}$$

input
```
Integrate[1/Sqrt[3*x - 4*x^2],x]
```

output
```
-((Sqrt[x]*Sqrt[-3 + 4*x]*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]])/Sqrt[-(x*(-3 +
 4*x))])
```

3.21.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x-4x^2}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{6}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{1}{2}\arcsin\left(\frac{1}{3}(3-8x)\right)$$

input
```
Int[1/Sqrt[3*x - 4*x^2],x]
```

output
```
-1/2*ArcSin[(3 - 8*x)/3]
```

3.21.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.21.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\arcsin\left(-1+\frac{8x}{3}\right)}{2}$	9
meijerg	$\arcsin\left(\frac{2\sqrt{3}\,\sqrt{x}}{3}\right)$	10
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$	20
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-8\operatorname{RootOf}\left(\_Z^2+1\right)x+4\sqrt{-4x^2+3x}+3\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{2}$	41

input
```
int(1/(-4*x^2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(-1+8/3*x)
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{8x}{3}-1\right)}{2}$$

input
```
integrate(1/(-4*x**2+3*x)**(1/2),x)
```

output
```
asin(8*x/3 - 1)/2
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\frac{1}{2}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arcsin(-8/3*x + 1)
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(8) = 16.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = \frac{1}{16}\sqrt{-4\,x^2+3\,x}(8\,x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}\,x-1\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = \frac{\mathrm{asin}\left(\frac{8x}{3}-1\right)}{2}$$

input
```
int(1/(3*x - 4*x^2)^(1/2),x)
```

output
```
asin((8*x)/3 - 1)/2
```

3.22 $\int \frac{1}{(3x-4x^2)^{3/2}}\,dx$

3.22.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = -\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

output
```
-2/9*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = \frac{2(-3+8x)}{9\sqrt{-x(-3+4x)}}$$

input
```
Integrate[(3*x - 4*x^2)^(-3/2),x]
```

output
```
(2*(-3 + 8*x))/(9*Sqrt[-(x*(-3 + 4*x))])
```

3.22.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx$$
$$\Big\downarrow \ 1088$$
$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

input
```
Int[(3*x - 4*x^2)^(-3/2),x]
```

output
```
(-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])
```

3.22.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

3.22.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2(3-8x)}{9\sqrt{-4x^2+3x}}$	19
pseudoelliptic	$\frac{-\frac{2}{3}+\frac{16x}{9}}{\sqrt{-4x^2+3x}}$	19
meijerg	$-\frac{2\sqrt{3}\,(1-\frac{8x}{3})}{9\sqrt{x}\,\sqrt{-\frac{4x}{3}+1}}$	21
gosper	$-\frac{2x(4x-3)(-3+8x)}{9(-4x^2+3x)^{\frac{3}{2}}}$	25
trager	$-\frac{2(-3+8x)\sqrt{-4x^2+3x}}{9x(4x-3)}$	29

input
```
int(1/(-4*x^2+3*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = -\frac{2\,\sqrt{-4\,x^2+3\,x}(8\,x-3)}{9\,(4\,x^2-3\,x)}$$

input
```
integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

3.22.6 Sympy [F]

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = \int \frac{1}{(-4x^2+3x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-4*x**2+3*x)**(3/2),x)
```

output
```
Integral((-4*x**2 + 3*x)**(-3/2), x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = \frac{16\,x}{9\,\sqrt{-4\,x^2+3\,x}} - \frac{2}{3\,\sqrt{-4\,x^2+3\,x}}$$

input
```
integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

output
```
16/9*x/sqrt(-4*x^2 + 3*x) - 2/3/sqrt(-4*x^2 + 3*x)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = -\frac{2\,\sqrt{-4\,x^2+3\,x}(8\,x-3)}{9\,(4\,x^2-3\,x)}$$

input
```
integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

output
```
-2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = \frac{16\,x-6}{9\,\sqrt{3\,x-4\,x^2}}$$

input
```
int(1/(3*x - 4*x^2)^(3/2),x)
```

output
```
(16*x - 6)/(9*(3*x - 4*x^2)^(1/2))
```

3.23 $\int \frac{1}{(3x-4x^2)^{5/2}}\,dx$

3.23.1 Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = -\frac{2(3-8x)}{27(3x-4x^2)^{3/2}} - \frac{64(3-8x)}{243\sqrt{3x-4x^2}}$$

output
```
-2/27*(3-8*x)/(-4*x^2+3*x)^(3/2)-64/243*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = -\frac{54+432x-2304x^2+2048x^3}{243(-x(-3+4x))^{3/2}}$$

input
```
Integrate[(3*x - 4*x^2)^(-5/2),x]
```

output
```
-1/243*(54 + 432*x - 2304*x^2 + 2048*x^3)/(-(x*(-3 + 4*x)))^(3/2)
```

3.23.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-4x^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{32}{27} \int \frac{1}{(3x-4x^2)^{3/2}} dx - \frac{2(3-8x)}{27\,(3x-4x^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}} - \frac{2(3-8x)}{27\,(3x-4x^2)^{3/2}}$$

input
```
Int[(3*x - 4*x^2)^(-5/2),x]
```

output
```
(-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqrt[3*x - 4
*x^2])
```

3.23.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.23.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

method	result	size
meijerg	$-\frac{2\sqrt{3}\,\left(\frac{1024}{27}x^{3}-\frac{128}{3}x^{2}+8x+1\right)}{81x^{\frac{3}{2}}\left(-\frac{4x}{3}+1\right)^{\frac{3}{2}}}$	31
gosper	$\frac{2x(4x-3)\left(1024x^{3}-1152x^{2}+216x+27\right)}{243\left(-4x^{2}+3x\right)^{\frac{5}{2}}}$	35
default	$-\frac{2(3-8x)}{27\left(-4x^{2}+3x\right)^{\frac{3}{2}}}-\frac{64(3-8x)}{243\sqrt{-4x^{2}+3x}}$	38
trager	$-\frac{2\left(1024x^{3}-1152x^{2}+216x+27\right)\sqrt{-4x^{2}+3x}}{243\left(4x-3\right)^{2}x^{2}}$	39
pseudoelliptic	$\frac{2048x^{3}-2304x^{2}+432x+54}{\sqrt{-4x^{2}+3x}\,\left(972x^{2}-729x\right)}$	39

input
```
int(1/(-4*x^2+3*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/81/x^(3/2)*3^(1/2)*(1024/27*x^3-128/3*x^2+8*x+1)/(-4/3*x+1)^(3/2)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = -\frac{2\left(1024\,x^{3}-1152\,x^{2}+216\,x+27\right)\sqrt{-4\,x^{2}+3\,x}}{243\left(16\,x^{4}-24\,x^{3}+9\,x^{2}\right)}$$

input
```
integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/243*(1024*x^3 - 1152*x^2 + 216*x + 27)*sqrt(-4*x^2 + 3*x)/(16*x^4 - 24*
x^3 + 9*x^2)
```

3.23.6 Sympy [F]

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = \int \frac{1}{(-4x^2+3x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-4*x**2+3*x)**(5/2),x)
```

output
```
Integral((-4*x**2 + 3*x)**(-5/2), x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = \frac{512\,x}{243\,\sqrt{-4\,x^2+3\,x}} - \frac{64}{81\,\sqrt{-4\,x^2+3\,x}} + \frac{16\,x}{27\,(-4\,x^2+3\,x)^{\frac{3}{2}}} - \frac{2}{9\,(-4\,x^2+3\,x)^{\frac{3}{2}}}$$

input
```
integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

output
```
512/243*x/sqrt(-4*x^2 + 3*x) - 64/81/sqrt(-4*x^2 + 3*x) + 16/27*x/(-4*x^2
+ 3*x)^(3/2) - 2/9/(-4*x^2 + 3*x)^(3/2)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = -\frac{2\,(8\,(16\,(8\,x-9)x+27)x+27)\sqrt{-4\,x^2+3\,x}}{243\,(4\,x^2-3\,x)^2}$$

input
```
integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

output
```
-2/243*(8*(16*(8*x - 9)*x + 27)*x + 27)*sqrt(-4*x^2 + 3*x)/(4*x^2 - 3*x)^2
```

3.23.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = \frac{(16\,x-6)\,\left(-128\,x^2+96\,x+9\right)}{243\,{\left(3\,x-4\,x^2\right)}^{3/2}}$$

input
```
int(1/(3*x - 4*x^2)^(5/2),x)
```

output
```
((16*x - 6)*(96*x - 128*x^2 + 9))/(243*(3*x - 4*x^2)^(3/2))
```

3.24 $\int \frac{1}{(3x-4x^2)^{7/2}} \, dx$

3.24.1 Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{(3x-4x^2)^{7/2}} \, dx = -\frac{2(3-8x)}{45 (3x-4x^2)^{5/2}} - \frac{128(3-8x)}{1215 (3x-4x^2)^{3/2}} - \frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}$$

output
```
-2/45*(3-8*x)/(-4*x^2+3*x)^(5/2)-128/1215*(3-8*x)/(-4*x^2+3*x)^(3/2)-4096/
10935*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \frac{1}{(3x-4x^2)^{7/2}} \, dx = \frac{2(-729-3240x-34560x^2+276480x^3-491520x^4+262144x^5)}{10935(-x(-3+4x))^{5/2}}$$

input
```
Integrate[(3*x - 4*x^2)^(-7/2),x]
```

output
```
(2*(-729 - 3240*x - 34560*x^2 + 276480*x^3 - 491520*x^4 + 262144*x^5))/(10
935*(-(x*(-3 + 4*x)))^(5/2))
```

3.24.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-4x^2)^{7/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{64}{45}\int \frac{1}{(3x-4x^2)^{5/2}}dx-\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{64}{45}\left(\frac{32}{27}\int \frac{1}{(3x-4x^2)^{3/2}}dx-\frac{2(3-8x)}{27\,(3x-4x^2)^{3/2}}\right)-\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{64}{45}\left(-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\,(3x-4x^2)^{3/2}}\right)-\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}}$$

input
```
Int[(3*x - 4*x^2)^(-7/2),x]
```

output
```
(-2*(3 - 8*x))/(45*(3*x - 4*x^2)^(5/2)) + (64*((-2*(3 - 8*x))/(27*(3*x - 4
*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqrt[3*x - 4*x^2])))/45
```

3.24.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.24.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
meijerg	$-\frac{2\sqrt{3}\left(-\frac{262144}{243}x^{5}+\frac{163840}{81}x^{4}-\frac{10240}{9}x^{3}+\frac{1280}{9}x^{2}+\frac{40}{3}x+3\right)}{1215x^{\frac{5}{2}}\left(-\frac{4x}{3}+1\right)^{\frac{5}{2}}}$	41
gosper	$-\frac{2x(4x-3)\left(262144x^{5}-491520x^{4}+276480x^{3}-34560x^{2}-3240x-729\right)}{10935\left(-4x^{2}+3x\right)^{\frac{7}{2}}}$	45
trager	$-\frac{2\left(262144x^{5}-491520x^{4}+276480x^{3}-34560x^{2}-3240x-729\right)\sqrt{-4x^{2}+3x}}{10935(4x-3)^{3}x^{3}}$	49
pseudoelliptic	$\frac{\frac{524288}{10935}x^{5}-\frac{65536}{729}x^{4}+\frac{4096}{81}x^{3}-\frac{512}{81}x^{2}-\frac{16}{27}x-\frac{2}{15}}{(4x-3)^{2}x^{2}\sqrt{-4x^{2}+3x}}$	49
default	$-\frac{2(3-8x)}{45\left(-4x^{2}+3x\right)^{\frac{5}{2}}}-\frac{128(3-8x)}{1215\left(-4x^{2}+3x\right)^{\frac{3}{2}}}-\frac{4096(3-8x)}{10935\sqrt{-4x^{2}+3x}}$	56

input
```
int(1/(-4*x^2+3*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1215/x^(5/2)*3^(1/2)*(-262144/243*x^5+163840/81*x^4-10240/9*x^3+1280/9*
x^2+40/3*x+3)/(-4/3*x+1)^(5/2)
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = -\frac{2\,(262144\,x^5-491520\,x^4+276480\,x^3-34560\,x^2-3240\,x-729)\sqrt{-4\,x^2+3\,x}}{10935\,(64\,x^6-144\,x^5+108\,x^4-27\,x^3)}$$

input
```
integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/10935*(262144*x^5 - 491520*x^4 + 276480*x^3 - 34560*x^2 - 3240*x - 729)
*sqrt(-4*x^2 + 3*x)/(64*x^6 - 144*x^5 + 108*x^4 - 27*x^3)
```

3.24.6 Sympy [F]

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = \int \frac{1}{(-4x^2+3x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-4*x**2+3*x)**(7/2),x)
```

output
```
Integral((-4*x**2 + 3*x)**(-7/2), x)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = \frac{32768\,x}{10935\,\sqrt{-4\,x^2+3\,x}} - \frac{4096}{3645\,\sqrt{-4\,x^2+3\,x}} + \frac{1024\,x}{1215\,(-4\,x^2+3\,x)^{\frac{3}{2}}} - \frac{128}{405\,(-4\,x^2+3\,x)^{\frac{3}{2}}} + \frac{16\,x}{45\,(-4\,x^2+3\,x)^{\frac{5}{2}}} - \frac{2}{15\,(-4\,x^2+3\,x)^{\frac{5}{2}}}$$

input
```
integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

output
```
32768/10935*x/sqrt(-4*x^2 + 3*x) - 4096/3645/sqrt(-4*x^2 + 3*x) + 1024/121
5*x/(-4*x^2 + 3*x)^(3/2) - 128/405/(-4*x^2 + 3*x)^(3/2) + 16/45*x/(-4*x^2
+ 3*x)^(5/2) - 2/15/(-4*x^2 + 3*x)^(5/2)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = -\frac{2\,(8\,(32\,(8\,(16\,(8\,x-15)x+135)x-135)x-405)x-729)\sqrt{-4\,x^2+3\,x}}{10935\,(4\,x^2-3\,x)^3}$$

input
```
integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

output
```
-2/10935*(8*(32*(8*(16*(8*x - 15)*x + 135)*x - 135)*x - 405)*x - 729)*sqrt
(-4*x^2 + 3*x)/(4*x^2 - 3*x)^3
```

3.24.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx =$$
$$-\frac{6480\,x-9216\,x\,(3\,x-4\,x^2)-32768\,x\,(3\,x-4\,x^2)^2+12288\,(3\,x-4\,x^2)^2-13824\,x^2+1458}{(3\,x-4\,x^2)^{3/2}\,(32805\,x-43740\,x^2)}$$

input
```
int(1/(3*x - 4*x^2)^(7/2),x)
```

output
```
-(6480*x - 9216*x*(3*x - 4*x^2) - 32768*x*(3*x - 4*x^2)^2 + 12288*(3*x - 4
*x^2)^2 - 13824*x^2 + 1458)/((3*x - 4*x^2)^(3/2)*(32805*x - 43740*x^2))
```

3.25 $\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx$

3.25.1 Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = -\frac{\arcsin(1-2bx)}{b}$$

output
```
arcsin(2*b*x-1)/b
```

3.25.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.75

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{-1+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{-1+bx}\right)}{\sqrt{b}\sqrt{-bx(-1+bx)}}$$

input
```
Integrate[1/Sqrt[b*x - b^2*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[-1 + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[-1 + b*x]])/(Sqrt
[b]*Sqrt[-(b*x*(-1 + b*x))])
```

3.25.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx$$

$$\Big\downarrow \; 1090$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{\left(b-2b^2x\right)^2}{b^2}}}d\left(b-2b^2x\right)}{b^2}$$

$$\Big\downarrow \; 223$$

$$-\frac{\arcsin\left(\frac{b-2b^2x}{b}\right)}{b}$$

input
```
Int[1/Sqrt[b*x - b^2*x^2],x]
```

output
```
-(ArcSin[(b - 2*b^2*x)/b]/b)
```

3.25.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 2.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

method	result	size
pseudoelliptic	$-\frac{2\arctan\left(\frac{\sqrt{-bx(bx-1)}}{xb}\right)}{b}$	25
default	$\frac{\arctan\left(\frac{\sqrt{b^2}\left(x-\frac{1}{2b}\right)}{\sqrt{-b^2x^2+bx}}\right)}{\sqrt{b^2}}$	35

input
```
int(1/(-b^2*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctan((-b*x*(b*x-1))^(1/2)/x/b)/b
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-b^2x^2+bx}}{bx}\right)}{b}$$

input
```
integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-b^2*x^2 + b*x)/(b*x))/b
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = \begin{cases} \frac{\log\left(-2b^2x+b+2\sqrt{-b^2}\sqrt{-b^2x^2+bx}\right)}{\sqrt{-b^2}} & \text{for } b^2 \neq 0 \\ \frac{2\sqrt{bx}}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-b**2*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((log(-2*b**2*x + b + 2*sqrt(-b**2)*sqrt(-b**2*x**2 + b*x))/sqrt(
-b**2), Ne(b**2, 0)), (2*sqrt(b*x)/b, Ne(b, 0)), (zoo*x, True))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = -\frac{\arcsin\left(-\frac{2\,b^2x-b}{b}\right)}{b}$$

input
```
integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-(2*b^2*x - b)/b)/b
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = \frac{1}{4}\sqrt{-b^2x^2+bx}\left(2\,x-\frac{1}{b}\right)-\frac{\arcsin\left(-2\,bx+1\right)\operatorname{sgn}(b)}{8\,|b|}$$

input
```
integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-b^2*x^2 + b*x)*(2*x - 1/b) - 1/8*arcsin(-2*b*x + 1)*sgn(b)/abs(b
)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{1}{\sqrt{bx - b^2x^2}}\,dx = \frac{\ln\left(\frac{\frac{b}{2} - b^2\,x}{\sqrt{-b^2}} + \sqrt{b\,x - b^2\,x^2}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(b*x - b^2*x^2)^(1/2),x)
```

output
```
log((b/2 - b^2*x)/(-b^2)^(1/2) + (b*x - b^2*x^2)^(1/2))/(-b^2)^(1/2)
```

3.26 $\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx$

3.26.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{2\text{arctanh}\left(\frac{bx}{\sqrt{bx+b^2x^2}}\right)}{b}$$

output
```
2*arctanh(b*x/(b^2*x^2+b*x)^(1/2))/b
```

3.26.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 58 vs. 2(24) = 48.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{4\sqrt{x}\sqrt{1+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-1+\sqrt{1+bx}}\right)}{\sqrt{b}\sqrt{bx(1+bx)}}$$

input
```
Integrate[1/Sqrt[b*x + b^2*x^2],x]
```

output
```
(4*Sqrt[x]*Sqrt[1 + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-1 + Sqrt[1 + b*x])])/
(Sqrt[b]*Sqrt[b*x*(1 + b*x)])
```

3.26.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b^2x^2+bx}}dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{b^2x^2}{b^2x^2+bx}}d\frac{x}{\sqrt{b^2x^2+bx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

input
```
Int[1/Sqrt[b*x + b^2*x^2],x]
```

output
```
(2*ArcTanh[(b*x)/Sqrt[b*x + b^2*x^2]])/b
```

3.26.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.26.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2+bx}\right)}{\sqrt{b^2}}$	37
pseudoelliptic	$\frac{-\ln\left(\frac{-bx+\sqrt{bx(bx+1)}}{x}\right)+\ln\left(\frac{bx+\sqrt{bx(bx+1)}}{x}\right)}{b}$	47

input
```
int(1/(b^2*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+b*x)^(1/2))/(b^2)^(1/2)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = -\frac{\log\left(-2\,bx+2\,\sqrt{b^2x^2+bx}-1\right)}{b}$$

input
```
integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-log(-2*b*x + 2*sqrt(b^2*x^2 + b*x) - 1)/b
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(20) = 40.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \begin{cases} \frac{\log\left(2b^2x+b+2\sqrt{b^2x^2+bx}\sqrt{b^2}\right)}{\sqrt{b^2}} & \text{for } b^2 \neq 0 \\ \frac{2\sqrt{bx}}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b**2*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((log(2*b**2*x + b + 2*sqrt(b**2*x**2 + b*x)*sqrt(b**2))/sqrt(b**
2), Ne(b**2, 0)), (2*sqrt(b*x)/b, Ne(b, 0)), (zoo*x, True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx}b+b\right)}{b}$$

input
```
integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x)*b + b)/b
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(22) = 44.

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{1}{4}\,\sqrt{b^2x^2+bx}\left(2\,x+\frac{1}{b}\right)+\frac{\log\left(\left|-2\,\left(x|b|-\sqrt{b^2x^2+bx}\right)|b|-b\right|\right)}{8\,|b|}$$

input
```
integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b^2*x^2 + b*x)*(2*x + 1/b) + 1/8*log(abs(-2*(x*abs(b) - sqrt(b^2*
x^2 + b*x))*abs(b) - b))/abs(b)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{bx + b^2x^2}}\,dx = \frac{\ln\left(\frac{x\,b^2+\frac{b}{2}}{\sqrt{b^2}} + \sqrt{b^2\,x^2+b\,x}\right)}{\sqrt{b^2}}$$

input
```
int(1/(b*x + b^2*x^2)^(1/2),x)
```

output
```
log((b/2 + b^2*x)/(b^2)^(1/2) + (b*x + b^2*x^2)^(1/2))/(b^2)^(1/2)
```

3.27 $\int \frac{1}{\sqrt{6x-x^2}}\,dx$

3.27.1 Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = -\arcsin\left(1-\frac{x}{3}\right)$$

output
```
arcsin(-1+1/3*x)
```

3.27.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(10) = 20.

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 4.00

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = -\frac{2\sqrt{-6+x}\sqrt{x}\log\left(\sqrt{-6+x}-\sqrt{x}\right)}{\sqrt{-((-6+x)x)}}$$

input
```
Integrate[1/Sqrt[6*x - x^2],x]
```

output
```
(-2*Sqrt[-6 + x]*Sqrt[x]*Log[Sqrt[-6 + x] - Sqrt[x]])/Sqrt[-((-6 + x)*x)]
```

3.27.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{6x-x^2}}dx$$

$$\Big\downarrow \; 1090$$

$$-\frac{1}{6}\int \frac{1}{\sqrt{1-\frac{1}{36}(6-2x)^2}}d(6-2x)$$

$$\Big\downarrow \; 223$$

$$-\arcsin\left(\frac{1}{6}(6-2x)\right)$$

input
```
Int[1/Sqrt[6*x - x^2],x]
```

output
```
-ArcSin[(6 - 2*x)/6]
```

3.27.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.27.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\arcsin\left(-1+\frac{x}{3}\right)$	7
meijerg	$2\arcsin\left(\frac{\sqrt{6}\,\sqrt{x}}{6}\right)$	12
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-x(-6+x)}}{x}\right)$	16
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+6x}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)$	38

input
```
int(1/(-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(-1+1/3*x)
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = -2\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

input
```
integrate(1/(-x^2+6*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-x^2 + 6*x)/x)
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{3}-1\right)$$

input
```
integrate(1/(-x**2+6*x)**(1/2),x)
```

output
```
asin(x/3 - 1)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{6x - x^2}}\, dx = -\arcsin\left(-\frac{1}{3}\,x + 1\right)$$

input
```
integrate(1/(-x^2+6*x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/3*x + 1)
```

3.27.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{6x - x^2}}\, dx = \frac{1}{2}\,\sqrt{-x^2 + 6\,x}(x - 3) + \frac{9}{2}\,\arcsin\left(\frac{1}{3}\,x - 1\right)$$

input
```
integrate(1/(-x^2+6*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{6x - x^2}}\, dx = \mathrm{asin}\left(\frac{x}{3} - 1\right)$$

input
```
int(1/(6*x - x^2)^(1/2),x)
```

output
```
asin(x/3 - 1)
```

3.28 $\int \frac{1}{\sqrt{4x+x^2}}\,dx$

3.28.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = 2\text{arctanh}\left(\frac{x}{\sqrt{4x+x^2}}\right)$$

output
```
2*arctanh(x/(x^2+4*x)^(1/2))
```

3.28.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{4+x}\log\left(-\sqrt{x}+\sqrt{4+x}\right)}{\sqrt{x(4+x)}}$$

input
```
Integrate[1/Sqrt[4*x + x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[4 + x]*Log[-Sqrt[x] + Sqrt[4 + x]])/Sqrt[x*(4 + x)]
```

3.28.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+4x}}dx$$

$$\Big\downarrow \ 1091$$

$$2\int \frac{1}{1-\frac{x^2}{x^2+4x}}d\frac{x}{\sqrt{x^2+4x}}$$

$$\Big\downarrow \ 219$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

input
```
Int[1/Sqrt[4*x + x^2],x]
```

output
```
2*ArcTanh[x/Sqrt[4*x + x^2]]
```

3.28.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.28.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

method	result	size
meijerg	$2 \operatorname{arcsinh}\left(\frac{\sqrt{x}}{2}\right)$	9
default	$\ln\left(2+x+\sqrt{x^2+4x}\right)$	14
trager	$\ln\left(2+x+\sqrt{x^2+4x}\right)$	14
pseudoelliptic	$2 \operatorname{arctanh}\left(\frac{\sqrt{x(4+x)}}{x}\right)$	15

input
```
int(1/(x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(1/2*x^(1/2))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+4\,x}-2\right)$$

input
```
integrate(1/(x^2+4*x)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + 4*x) - 2)
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = \log\left(2x+2\sqrt{x^2+4x}+4\right)$$

input
```
integrate(1/(x**2+4*x)**(1/2),x)
```

output
```
log(2*x + 2*sqrt(x**2 + 4*x) + 4)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = \log\left(2\,x + 2\,\sqrt{x^2+4\,x} + 4\right)$$

input
```
integrate(1/(x^2+4*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 + 4*x) + 4)
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2+4\,x}(x+2) + 2\,\log\left(\left|-x+\sqrt{x^2+4\,x}-2\right|\right)$$

input
```
integrate(1/(x^2+4*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

3.28.9 Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = \ln\left(x + \sqrt{x\,(x+4)} + 2\right)$$

input
```
int(1/(4*x + x^2)^(1/2),x)
```

output
```
log(x + (x*(x + 4))^(1/2) + 2)
```

3.29 $\int \frac{1}{\sqrt{-2x+x^2}} dx$

3.29.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\sqrt{-2x+x^2}} dx = 2\text{arctanh}\left(\frac{x}{\sqrt{-2x+x^2}}\right)$$

output
```
2*arctanh(x/(x^2-2*x)^(1/2))
```

3.29.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{-2x+x^2}} dx = -\frac{2\sqrt{-2+x}\sqrt{x}\log\left(\sqrt{-2+x}-\sqrt{x}\right)}{\sqrt{(-2+x)x}}$$

input
```
Integrate[1/Sqrt[-2*x + x^2],x]
```

output
```
(-2*Sqrt[-2 + x]*Sqrt[x]*Log[Sqrt[-2 + x] - Sqrt[x]])/Sqrt[(-2 + x)*x]
```

3.29.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-2x}}dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{x^2}{x^2-2x}}d\frac{x}{\sqrt{x^2-2x}}$$

$$\Big\downarrow \text{219}$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

input
```
Int[1/Sqrt[-2*x + x^2],x]
```

output
```
2*ArcTanh[x/Sqrt[-2*x + x^2]]
```

3.29.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.29.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\ln\left(-1+x+\sqrt{x^{2}-2x}\right)$	14
trager	$\ln\left(-1+x+\sqrt{x^{2}-2x}\right)$	14
pseudoelliptic	$2\,\operatorname{arctanh}\left(\frac{\sqrt{x(-2+x)}}{x}\right)$	15
meijerg	$\frac{2\sqrt{-\operatorname{signum}(-2+x)}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{\sqrt{\operatorname{signum}(-2+x)}}$	26

input
```
int(1/(x^2-2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(-1+x+(x^2-2*x)^(1/2))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = -\log\left(-x+\sqrt{x^2-2\,x}+1\right)$$

input
```
integrate(1/(x^2-2*x)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 - 2*x) + 1)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = \log\left(2x+2\sqrt{x^2-2x}-2\right)$$

input
```
integrate(1/(x**2-2*x)**(1/2),x)
```

output
```
log(2*x + 2*sqrt(x**2 - 2*x) - 2)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = \log\left(2\,x + 2\,\sqrt{x^2-2\,x} - 2\right)$$

input
```
integrate(1/(x^2-2*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 - 2*x) - 2)
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2-2\,x}(x-1) + \frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-2\,x}+1\right|\right)$$

input
```
integrate(1/(x^2-2*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 2*x)*(x - 1) + 1/2*log(abs(-x + sqrt(x^2 - 2*x) + 1))
```

3.29.9 Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = \ln\left(x+\sqrt{x\,(x-2)}-1\right)$$

input
```
int(1/(x^2 - 2*x)^(1/2),x)
```

output
```
log(x + (x*(x - 2))^(1/2) - 1)
```

3.30 $\int (bx + cx^2)^{4/3} \, dx$

3.30.1 Optimal result

Integrand size = 13, antiderivative size = 448

$$\int (bx+cx^2)^{4/3}\,dx = \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)(bx+cx^2)^{4/3}}{55c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}$$

$$+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)(bx+cx^2)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}$$

$$+\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}\,b^2(bx+cx^2)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\ \text{Elli}}{55c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+c}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+}{b^2}}\right.}}}$$

output
```
3/55*(-c*x*(c*x+b)/b^2)^(1/3)*(2*c*x+b)*(c*x^2+b*x)^(4/3)/c/(-c*(c*x^2+b*x
)/b^2)^(4/3)+3/22*(-c*x*(c*x+b)/b^2)^(4/3)*(2*c*x+b)*(c*x^2+b*x)^(4/3)/c/(
-c*(c*x^2+b*x)/b^2)^(4/3)+1/55*2^(1/3)*3^(3/4)*b^2*(c*x^2+b*x)^(4/3)*(1-2^
(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1
/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-
c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1
/2)/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(4/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)
^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.30.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11

$$\int \left(bx+cx^2\right)^{4/3}\,dx=\frac{3bx^2\sqrt[3]{x(b+cx)}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{7}{3},\frac{10}{3},-\frac{cx}{b}\right)}{7\sqrt[3]{1+\frac{cx}{b}}}$$

input
```
Integrate[(b*x + c*x^2)^(4/3),x]
```

output
```
(3*b*x^2*(x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3, 7/3, 10/3, -((c*x)/b)
])/(7*(1 + (c*x)/b)^(1/3))
```

3.30.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1087, 1087, 1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{4/3}\,dx$$

$$\Big\downarrow \; 1087$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c}-\frac{2b^2\int \sqrt[3]{cx^2+bx}dx}{11c}$$

$\downarrow$ 1087

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c}-\frac{2b^2\left(\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}-\frac{b^2\int \frac{1}{\left(cx^2+bx\right)^{2/3}}dx}{10c}\right)}{11c}$$

$\downarrow$ 1093

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c}-\frac{2b^2\left(\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}-\frac{b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{2/3}}dx}{10c(bx+cx^2)^{2/3}}\right)}{11c}$$

$\downarrow$ 1090

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c}-\frac{2b^2\left(\frac{b^4\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{2/3}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{5\ 2^{2/3}c^3(bx+cx^2)^{2/3}}+\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}\right)}{11c}$$

$\downarrow$ 234

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c}-\frac{2b^2\left(\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\int \frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{10\ 2^{2/3}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)(bx+cx^2)^{2/3}}\right)}{11c}$$

$\downarrow$ 760

3.30. $\int \left(bx+cx^2\right)^{4/3}\,dx$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c}-$$

$$\frac{2b^2\left(\frac{3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\frac{2xc^2}{b^2}+\frac{c}{b}-\sqrt{3}+1}\right),-\right.}{5\ 2^{2/3}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\left(bx+cx^2\right)^{2/3}}\right.}{11c}$$

input
```
Int[(b*x + c*x^2)^(4/3),x]
```

output
```
(3*(b + 2*c*x)*(b*x + c*x^2)^(4/3))/(22*c) - (2*b^2*((3*(b + 2*c*x)*(b*x +
 c*x^2)^(1/3))/(10*c) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(1 + c/b + (2*c^2*x
)/b^2)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)
^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b
+ (2*c^2*x)/b^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(
1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(5*2^(2/3)*c*(-(c/b)
 - (2*c^2*x)/b^2)*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2
*c^2*x)/b^2)^2)]*(b*x + c*x^2)^(2/3))))/(11*c)
```

3.30.3.1 Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.30.4 Maple [F]

$$\int \left(c\,x^{2}+b x \right)^{\frac{4}{3}}d x$$

input
```
int((c*x^2+b*x)^(4/3),x)
```

output
```
int((c*x^2+b*x)^(4/3),x)
```

3.30.5 Fricas [F]

$$\int \left(bx+cx^2\right)^{4/3}\,dx=\int \left(cx^{2}+bx\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((c*x^2+b*x)^(4/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(4/3), x)
```

3.30.6 Sympy [F]

$$\int \left(bx + cx^2\right)^{4/3} dx = \int \left(bx + cx^2\right)^{\frac{4}{3}} dx$$

input
```
integrate((c*x**2+b*x)**(4/3),x)
```

output
```
Integral((b*x + c*x**2)**(4/3), x)
```

3.30.7 Maxima [F]

$$\int \left(bx + cx^2\right)^{4/3} dx = \int \left(cx^2 + bx\right)^{\frac{4}{3}} dx$$

input
```
integrate((c*x^2+b*x)^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(4/3), x)
```

3.30.8 Giac [F]

$$\int \left(bx + cx^2\right)^{4/3} dx = \int \left(cx^2 + bx\right)^{\frac{4}{3}} dx$$

input
```
integrate((c*x^2+b*x)^(4/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(4/3), x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.08

$$\int \left(bx+cx^2\right)^{4/3}\,dx=\frac{3\,x\,{\left(c\,x^2+b\,x\right)}^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{c\,x}{b}\right)}{7\,{\left(\frac{c\,x}{b}+1\right)}^{4/3}}$$

input
```
int((b*x + c*x^2)^(4/3),x)
```

output
```
(3*x*(b*x + c*x^2)^(4/3)*hypergeom([-4/3, 7/3], 10/3, -(c*x)/b))/(7*((c*x)
/b + 1)^(4/3))
```

3.31 $\int \sqrt[3]{bx+cx^2}\,dx$

3.31.1 Optimal result

Integrand size = 13, antiderivative size = 387

$$\int \sqrt[3]{bx+cx^2}\,dx = \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\ \text{Ellipti}}{5\ 2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b}{}}\right.}}}$$

output
```
3/10*(-c*x*(c*x+b)/b^2)^(1/3)*(2*c*x+b)*(c*x^2+b*x)^(1/3)/c/(-c*(c*x^2+b*x
)/b^2)^(1/3)+1/10*3^(3/4)*b^2*(c*x^2+b*x)^(1/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b
^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/
3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/
2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3
))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)*2^(1/3)/c/(2*c*x+
b)/(-c*(c*x^2+b*x)/b^2)^(1/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^
(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.31.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.12

$$\int \sqrt[3]{bx+cx^2}\,dx=\frac{3x\sqrt[3]{x(b+cx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},-\frac{cx}{b}\right)}{4\sqrt[3]{1+\frac{cx}{b}}}$$

input
```
Integrate[(b*x + c*x^2)^(1/3),x]
```

output
```
(3*x*(x*(b + c*x))^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -((c*x)/b)])/(4
*(1 + (c*x)/b)^(1/3))
```

3.31.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1087, 1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}-\frac{b^2\int \frac{1}{(cx^2+bx)^{2/3}}dx}{10c}$$

$$\Big\downarrow \text{1093}$$

$$\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}-\frac{b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{2/3}}dx}{10c\,(bx+cx^2)^{2/3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{b^4\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{2/3}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{5\ 2^{2/3}c^3\left(bx+cx^2\right)^{2/3}}+\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}$$

$$\Big\downarrow \ 234$$

$$\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{10\ 2^{2/3}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\left(bx+cx^2\right)^{2/3}}$$

$$\Big\downarrow \ 760$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\frac{2xc^2}{b^2}\cdots}{\frac{2xc^2}{b^2}\cdots}\right.\right.}{5\ 2^{2/3}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\left(bx+cx^2\right)^{2/3}}$$

$$\frac{3(b+2cx)\sqrt[3]{bx+cx^2}}{10c}$$

input
```
Int[(b*x + c*x^2)^(1/3),x]
```

output
```
(3*(b + 2*c*x)*(b*x + c*x^2)^(1/3))/(10*c) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*b^
2*(1 + c/b + (2*c^2*x)/b^2)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[(1 + (-(
c/b) - (2*c^2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3)
)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + c
/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])
/(5*2^(2/3)*c*(-(c/b) - (2*c^2*x)/b^2)*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1
 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)]*(b*x + c*x^2)^(2/3))
```

3.31. $\int \sqrt[3]{bx+cx^2}\,dx$

3.31.3.1 Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.31.4 Maple [F]

$$\int \left(c\,x^{2}+b x \right)^{\frac{1}{3}}d x$$

input
```
int((c*x^2+b*x)^(1/3),x)
```

output
```
int((c*x^2+b*x)^(1/3),x)
```

3.31.5 Fricas [F]

$$\int \sqrt[3]{bx+cx^2}\,dx=\int \left(cx^2+bx\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(1/3), x)
```

3.31.6 Sympy [F]

$$\int \sqrt[3]{bx+cx^2}\,dx=\int \sqrt[3]{bx+cx^2}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/3),x)
```

output
```
Integral((b*x + c*x**2)**(1/3), x)
```

3.31.7 Maxima [F]

$$\int \sqrt[3]{bx+cx^2}\,dx=\int \left(cx^2+bx\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(1/3), x)
```

3.31.8 Giac [F]

$$\int \sqrt[3]{bx+cx^2}\,dx = \int (cx^2+bx)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(1/3), x)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.09

$$\int \sqrt[3]{bx+cx^2}\,dx = \frac{3\,x\,(c\,x^2+b\,x)^{1/3}\;{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{4}{3};\ \frac{7}{3};\ -\frac{c\,x}{b}\right)}{4\,\left(\frac{c\,x}{b}+1\right)^{1/3}}$$

input
```
int((b*x + c*x^2)^(1/3),x)
```

output
```
(3*x*(b*x + c*x^2)^(1/3)*hypergeom([-1/3, 4/3], 7/3, -(c*x)/b))/(4*((c*x)/
b + 1)^(1/3))
```

3.32 $\int \frac{1}{(bx+cx^2)^{2/3}}\,dx$

3.32.1 Optimal result

Integrand size = 13, antiderivative size = 322

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx = \frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\cdots}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\cdots}\right.\cdots}}\cdots}{c(b+2cx)\left(bx+cx^2\right)^{2/3}\sqrt{-\frac{\cdots}{\left(1\cdots\right.}}\cdots}$$

output
```
2^(1/3)*3^(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(2/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^
2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3
)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2
))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3)
)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c/(2*c*x+b)/(c*x^2
+b*x)^(2/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b
)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.32.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.13

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx = \frac{3x\left(1+\frac{cx}{b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{cx}{b}\right)}{(x(b+cx))^{2/3}}$$

input
```
Integrate[(b*x + c*x^2)^(-2/3),x]
```

output
```
(3*x*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((c*x)/b)])/(x*
(b + c*x))^(2/3)
```

3.32.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{2/3}}dx}{(bx+cx^2)^{2/3}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\sqrt[3]{2}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{2/3}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c^2\,(bx+cx^2)^{2/3}}$$

$$\Big\downarrow \text{234}$$

$$\frac{3\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{2^{2/3}\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)(bx+cx^2)^{2/3}}$$

$$\Big\downarrow \text{760}$$

$$-\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{2}{2}\right.\right.}{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}(bx+cx^2)^{2/3}}$$

input
```
Int[(b*x + c*x^2)^(-2/3),x]
```

output
```
-((2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*(1 + c/b + (2*c^2*x)/b^2)*(-((c*(b*x
+ c*x^2))/b^2))^(2/3)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 - (b^2*(-(
c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2
]*EllipticF[ArcSin[(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b
+ (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/((-(c/b) - (2*c^2*x)/b^2)*Sqrt[-((1 +
c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)]*(b*x + c*x^2)
^(2/3)))
```

3.32.3.1 Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.32.4 Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{2}{3}}}d x$$

input
```
int(1/(c*x^2+b*x)^(2/3),x)
```

output
```
int(1/(c*x^2+b*x)^(2/3),x)
```

3.32.5 Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(-2/3), x)
```

3.32.6 Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(2/3),x)
```

output
```
Integral((b*x + c*x**2)**(-2/3), x)
```

3.32.7 Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-2/3), x)
```

3.32.8 Giac [F]

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-2/3), x)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.11

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx = \frac{3\,x\left(\frac{c\,x}{b}+1\right)^{2/3}{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{4}{3};\ -\frac{c\,x}{b}\right)}{(c\,x^2+b\,x)^{2/3}}$$

input
```
int(1/(b*x + c*x^2)^(2/3),x)
```

output
```
(3*x*((c*x)/b + 1)^(2/3)*hypergeom([1/3, 2/3], 4/3, -(c*x)/b))/(b*x + c*x^
2)^(2/3)
```

3.33 $\int \frac{1}{(bx+cx^2)^{5/3}}\,dx$

3.33.1 Optimal result

Integrand size = 13, antiderivative size = 384

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx = \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}}$$

$$+\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\,\mathrm{E}}{c(b+2cx)(bx+cx^2)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+c}{b^2}}\right.}}}$$

output
```
3/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(5/3)/c/(-c*x*(c*x+b)/b^2)^(2/3)/(c*x^2
+b*x)^(5/3)+2^(1/3)*3^(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(5/3)*(1-2^(2/3)*(-c*
x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2
))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2
)-1/2*2^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b
)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c/(2*c
*x+b)/(c*x^2+b*x)^(5/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*
(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.33.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.12

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx = -\frac{3\left(1+\frac{cx}{b}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{1}{3},-\frac{cx}{b}\right)}{2b(x(b+cx))^{2/3}}$$

input
```
Integrate[(b*x + c*x^2)^(-5/3),x]
```

output
```
(-3*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 1/3, -((c*x)/b)])/(2*
b*(x*(b + c*x))^(2/3))
```

3.33.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1089, 1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{c\int \frac{1}{(cx^2+bx)^{2/3}}dx}{b^2}-\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}$$

$$\Big\downarrow \text{1093}$$

$$-\frac{c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{2/3}}dx}{b^2(bx+cx^2)^{2/3}}-\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\sqrt[3]{2}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{2/3}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c(bx+cx^2)^{2/3}}-\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}$$

$$\Big\downarrow \; 234$$

$$-\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{2^{2/3}b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)(bx+cx^2)^{2/3}}-\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}$$

$$\Big\downarrow \; 760$$

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}c\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\left(bx+cx^2\right)^{2/3}}$$

$$\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}$$

input
```
Int[(b*x + c*x^2)^(-5/3),x]
```

output
```
(-3*(b + 2*c*x))/(2*b^2*(b*x + c*x^2)^(2/3)) + (2^(1/3)*3^(3/4)*Sqrt[2 - S
qrt[3]]*c*(1 + c/b + (2*c^2*x)/b^2)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[
(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^
2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)], -7 + 4*S
qrt[3]])/(b^2*(-(c/b) - (2*c^2*x)/b^2)*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1
 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)]*(b*x + c*x^2)^(2/3))
```

3.33.3.1 Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.33.4 Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{5}{3}}}d x$$

input
```
int(1/(c*x^2+b*x)^(5/3),x)
```

output
```
int(1/(c*x^2+b*x)^(5/3),x)
```

3.33.5 Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(1/3)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

3.33.6 Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(5/3),x)
```

output
```
Integral((b*x + c*x**2)**(-5/3), x)
```

3.33.7 Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-5/3), x)
```

3.33.8 Giac [F]

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-5/3), x)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.09

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx = -\frac{3\,x\,\left(\frac{c\,x}{b}+1\right)^{5/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{5}{3};\ \frac{1}{3};\ -\frac{c\,x}{b}\right)}{2\,(c\,x^2+b\,x)^{5/3}}$$

input
```
int(1/(b*x + c*x^2)^(5/3),x)
```

output
```
-(3*x*((c*x)/b + 1)^(5/3)*hypergeom([-2/3, 5/3], 1/3, -(c*x)/b))/(2*(b*x +
 c*x^2)^(5/3))
```

3.34 $\int \frac{1}{(bx+cx^2)^{8/3}}\,dx$

3.34.1 Optimal result

Integrand size = 13, antiderivative size = 448

$$\int \frac{1}{(bx+cx^2)^{8/3}}\,dx = \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}} + \frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}}$$
$$+\frac{14\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\cdots}{5c(b+2cx)(bx+cx^2)^{8/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+\cdots}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+\cdots}{b^2}}\right)\cdots}}}$$

output
```
3/5*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(8/3)/c/(-c*x*(c*x+b)/b^2)^(5/3)/(c*x^2
+b*x)^(8/3)+21/5*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(8/3)/c/(-c*x*(c*x+b)/b^2)
^(2/3)/(c*x^2+b*x)^(8/3)+14/5*2^(1/3)*3^(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(8/
3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)
/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^
(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^
(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2
))^2)^(1/2)/c/(2*c*x+b)/(c*x^2+b*x)^(8/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^
(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.34.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.11

$$\int \frac{1}{(bx+cx^2)^{8/3}}\,dx = -\frac{3\left(1+\frac{cx}{b}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{8}{3},-\frac{2}{3},-\frac{cx}{b}\right)}{5b^2x(x(b+cx))^{2/3}}$$

input
```
Integrate[(b*x + c*x^2)^(-8/3),x]
```

output
```
(-3*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[-5/3, 8/3, -2/3, -((c*x)/b)])/(5
*b^2*x*(x*(b + c*x))^(2/3))
```

3.34.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1089, 1089, 1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{8/3}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{14c\int \frac{1}{(cx^2+bx)^{5/3}}dx}{5b^2}-\frac{3(b+2cx)}{5b^2(bx+cx^2)^{5/3}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{14c\left(-\frac{c\int \frac{1}{(cx^2+bx)^{2/3}}dx}{b^2}-\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}\right)}{5b^2}-\frac{3(b+2cx)}{5b^2(bx+cx^2)^{5/3}}$$

$$\Big\downarrow \text{1093}$$

$$-\frac{14c\left(-\frac{c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{2/3}}dx}{b^2(bx+cx^2)^{2/3}}-\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}\right)}{5b^2}-\frac{3(b+2cx)}{5b^2\,(bx+cx^2)^{5/3}}$$

$\Big\downarrow$ 1090

$$-\frac{14c\left(\frac{\sqrt[3]{2}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{2/3}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c(bx+cx^2)^{2/3}}-\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}\right)}{5b^2}-\frac{3(b+2cx)}{5b^2\,(bx+cx^2)^{5/3}}$$

$\Big\downarrow$ 234

$$-\frac{14c\left(-\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{2^{2/3}b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)(bx+cx^2)^{2/3}}-\frac{3(b+2cx)}{2b^2(bx+cx^2)^{2/3}}\right)}{5b^2}-\frac{3(b+2cx)}{5b^2\,(bx+cx^2)^{5/3}}$$

$\Big\downarrow$ 760

$$-\frac{14c\left(\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}c\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}\cdots}{\frac{2xc^2}{b^2}+\frac{c}{b}-\sqrt{3}\cdots}\right.\right.}{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}(bx+cx^2)^{2/3}}\cdots\right)}{5b^2}$$

$$-\frac{3(b+2cx)}{5b^2\,(bx+cx^2)^{5/3}}$$

3.34. $\int \frac{1}{(bx+cx^2)^{8/3}}\,dx$

input
```
Int[(b*x + c*x^2)^(-8/3),x]
```

output
```
(-3*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/3)) - (14*c*((-3*(b + 2*c*x))/(2*
b^2*(b*x + c*x^2)^(2/3)) + (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*c*(1 + c/b +
 (2*c^2*x)/b^2)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[(1 + (-(c/b) - (2*c^
2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt[
3] + c/b + (2*c^2*x)/b^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + c/b + (2*c^2*
x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(b^2*(-(c/b
) - (2*c^2*x)/b^2)*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (
2*c^2*x)/b^2)^2)]*(b*x + c*x^2)^(2/3))))/(5*b^2)
```

3.34.3.1 Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.34.4 Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{8}{3}}}d x$$

input
```
int(1/(c*x^2+b*x)^(8/3),x)
```

output
```
int(1/(c*x^2+b*x)^(8/3),x)
```

3.34.5 Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{8/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(1/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^
3), x)
```

3.34.6 Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{8/3}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(8/3),x)
```

output
```
Integral((b*x + c*x**2)**(-8/3), x)
```

3.34.7 Maxima [F]

$$\int \frac{1}{(bx + cx^2)^{8/3}} \, dx = \int \frac{1}{(cx^2 + bx)^{\frac{8}{3}}} \, dx$$

input
```
integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-8/3), x)
```

3.34.8 Giac [F]

$$\int \frac{1}{(bx + cx^2)^{8/3}} \, dx = \int \frac{1}{(cx^2 + bx)^{\frac{8}{3}}} \, dx$$

input
```
integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-8/3), x)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.08

$$\int \frac{1}{(bx + cx^2)^{8/3}} \, dx = -\frac{3\,x\,\left(\frac{c\,x}{b}+1\right)^{8/3}\,{}_2\mathrm{F}_1\left(-\frac{5}{3},\frac{8}{3};\ -\frac{2}{3};\ -\frac{c\,x}{b}\right)}{5\,\left(c\,x^2+b\,x\right)^{8/3}}$$

input
```
int(1/(b*x + c*x^2)^(8/3),x)
```

output
```
-(3*x*((c*x)/b + 1)^(8/3)*hypergeom([-5/3, 8/3], -2/3, -(c*x)/b))/(5*(b*x
+ c*x^2)^(8/3))
```

3.35 $\int (bx+cx^2)^{5/3}\,dx$

3.35.1 Optimal result

Integrand size = 13, antiderivative size = 842

$$\int \left(bx+cx^2\right)^{5/3}\,dx=\frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}$$

$$+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}$$

$$-\frac{15(b+2cx)\left(bx+cx^2\right)^{5/3}}{182\sqrt[3]{2}c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}$$

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(bx+cx^2\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}E\Bigg(\text{a}}{364\sqrt[3]{2}c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+c}{b^2}}}}$$

$$+\frac{5\,3^{3/4}b^2\left(bx+cx^2\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\,\text{EllipticF}\Bigg(\text{arcs}}{91\,2^{5/6}c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx}{b^2}}}}$$

output
```
15/364*(-c*x*(c*x+b)/b^2)^(2/3)*(2*c*x+b)*(c*x^2+b*x)^(5/3)/c/(-c*(c*x^2+b
*x)/b^2)^(5/3)+3/26*(-c*x*(c*x+b)/b^2)^(5/3)*(2*c*x+b)*(c*x^2+b*x)^(5/3)/c
/(-c*(c*x^2+b*x)/b^2)^(5/3)-15/364*(2*c*x+b)*(c*x^2+b*x)^(5/3)*2^(2/3)/c/(
-c*(c*x^2+b*x)/b^2)^(5/3)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))+5/1
82*3^(3/4)*b^2*(c*x^2+b*x)^(5/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*Elli
pticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b
)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+
2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^
(1/2))^2)^(1/2)*2^(1/6)/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(5/3)/((-1+2^(2/3
)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2
)^(1/2)-15/728*3^(1/4)*b^2*(c*x^2+b*x)^(5/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)
^(1/3))*EllipticE((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*
(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)
/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b
^2)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/
(-c*(c*x^2+b*x)/b^2)^(5/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/
3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.35.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.06

$$\int \left(bx+cx^2\right)^{5/3} dx = \frac{3bx^2(x(b+cx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{8}{3},\frac{11}{3},-\frac{cx}{b}\right)}{8\left(1+\frac{cx}{b}\right)^{2/3}}$$

input
```
Integrate[(b*x + c*x^2)^(5/3),x]
```

output
```
(3*b*x^2*(x*(b + c*x))^(2/3)*Hypergeometric2F1[-5/3, 8/3, 11/3, -((c*x)/b)
])/(8*(1 + (c*x)/b)^(2/3))
```

3.35.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 727, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {1087, 1087, 1093, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/3}\,dx$$

$$\Big\downarrow \ 1087$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c}-\frac{5b^2\int \left(cx^2+bx\right)^{2/3}dx}{26c}$$

$$\Big\downarrow \ 1087$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c}-\frac{5b^2\left(\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{b^2\int \frac{1}{\sqrt[3]{cx^2+bx}}dx}{7c}\right)}{26c}$$

$$\Big\downarrow \ 1093$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c}-\frac{5b^2\left(\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{1}{\sqrt[3]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{7c\sqrt[3]{bx+cx^2}}\right)}{26c}$$

$$\Big\downarrow \ 1090$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c}-\frac{5b^2\left(\frac{b^4\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{1}{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{7\sqrt[3]{2}c^3\sqrt[3]{bx+cx^2}}+\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}\right)}{26c}$$

$$\Big\downarrow \ 233$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c}-\frac{5b^2\left(\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int\frac{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{14\sqrt[3]{2}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}\right)}{26c}$$

$\downarrow$ 833

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c}-\frac{5b^2\left(\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\right)}{14\sqrt[3]{2}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}\right)}{26c}$$

$\downarrow$ 760

3.35. $\int \left(bx+cx^2\right)^{5/3}\,dx$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c}-\frac{5b^2\left(\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(-\int\frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\frac{2\sqrt{2-\cdots}}{\cdots}\right)}{14\sqrt[3]{2}c\left(-\frac{2c}{b}\cdots\right.}\right)}{26c}$$

$\big\downarrow$ 2418

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c}-\frac{5b^2\left(\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\cdots\right.}{\cdots}}\cdots}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\cdots\right.}}}{\sqrt[4]{3}\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Int[(b*x + c*x^2)^(5/3),x]
```

3.35. $\int \left(bx+cx^2\right)^{5/3}\,dx$

output
```
(3*(b + 2*c*x)*(b*x + c*x^2)^(5/3))/(26*c) - (5*b^2*((3*(b + 2*c*x)*(b*x +
 c*x^2)^(2/3))/(14*c) - (3*b^2*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2
)]*(-((c*(b*x + c*x^2))/b^2))^(1/3)*((-2*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b
^2)^2)/c^2)])/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2) + (3^(1/4)*Sqrt[2 + Sqrt
[3]]*(1 + c/b + (2*c^2*x)/b^2)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 -
 (b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*
x)/b^2)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[
3] + c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(Sqrt[-((b^2*(-(c/b) - (2*c^2
*x)/b^2)^2)/c^2)]*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2
*c^2*x)/b^2)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 + c/b + (2*c^2*x
)/b^2)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x
)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2]*EllipticF[Arc
Sin[(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2
)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)
]*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)]
)))/(14*2^(1/3)*c*(-(c/b) - (2*c^2*x)/b^2)*(b*x + c*x^2)^(1/3))))/(26*c)
```

3.35.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.35.4 Maple [F]

$$\int \left(c\,x^{2}+b x \right)^{\frac{5}{3}}d x$$

input
```
int((c*x^2+b*x)^(5/3),x)
```

output
```
int((c*x^2+b*x)^(5/3),x)
```

3.35.5 Fricas [F]

$$\int \left(bx+cx^2\right)^{5/3} \, dx = \int \left(cx^2+bx\right)^{\frac{5}{3}} \, dx$$

input
```
integrate((c*x^2+b*x)^(5/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(5/3), x)
```

3.35.6 Sympy [F]

$$\int \left(bx+cx^2\right)^{5/3} \, dx = \int \left(bx+cx^2\right)^{\frac{5}{3}} \, dx$$

input
```
integrate((c*x**2+b*x)**(5/3),x)
```

output
```
Integral((b*x + c*x**2)**(5/3), x)
```

3.35.7 Maxima [F]

$$\int \left(bx+cx^2\right)^{5/3} \, dx = \int \left(cx^2+bx\right)^{\frac{5}{3}} \, dx$$

input
```
integrate((c*x^2+b*x)^(5/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/3), x)
```

3.35.8 Giac [F]

$$\int \left(bx+cx^2\right)^{5/3}\,dx=\int \left(cx^2+bx\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/3), x)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.04

$$\int \left(bx+cx^2\right)^{5/3}\,dx=\frac{3\,x\,\left(c\,x^2+b\,x\right)^{5/3}\,{}_2\mathrm{F}_1\left(-\frac{5}{3},\frac{8}{3};\ \frac{11}{3};\ -\frac{c\,x}{b}\right)}{8\,\left(\frac{c\,x}{b}+1\right)^{5/3}}$$

input
```
int((b*x + c*x^2)^(5/3),x)
```

output
```
(3*x*(b*x + c*x^2)^(5/3)*hypergeom([-5/3, 8/3], 11/3, -(c*x)/b))/(8*((c*x)
/b + 1)^(5/3))
```

3.36 $\int (bx + cx^2)^{2/3} \, dx$

3.36.1 Optimal result

Integrand size = 13, antiderivative size = 781

$$\int \left(bx+cx^2\right)^{2/3}\,dx = \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}}$$

$$-\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{7\sqrt[3]{2}c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\,E\left(\mathrm{arc}\ldots\right.}{14\sqrt[3]{2}c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\ldots}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\ldots\right.}}}$$

$$+\frac{\sqrt[6]{2}\,3^{3/4}b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\,\mathrm{EllipticF}\left(\mathrm{ar}\ldots\right.}{7c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\ldots}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\ldots}}}$$

output
```
3/14*(-c*x*(c*x+b)/b^2)^(2/3)*(2*c*x+b)*(c*x^2+b*x)^(2/3)/c/(-c*(c*x^2+b*x
)/b^2)^(2/3)-3/14*(2*c*x+b)*(c*x^2+b*x)^(2/3)*2^(2/3)/c/(-c*(c*x^2+b*x)/b^
2)^(2/3)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))+1/7*2^(1/6)*3^(3/4)*
b^2*(c*x^2+b*x)^(2/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^
(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3
)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(
-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(
1/2)/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(2/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2
)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)-3/28*3^(1/4
)*b^2*(c*x^2+b*x)^(2/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticE((1-
2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1
/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)
*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^
(2/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)
^(1/3)-3^(1/2))^2)^(1/2)
```

3.36.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.06

$$\int \left(bx+cx^2\right)^{2/3} dx = \frac{3x(x(b+cx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{8}{3},-\frac{cx}{b}\right)}{5\left(1+\frac{cx}{b}\right)^{2/3}}$$

input
```
Integrate[(b*x + c*x^2)^(2/3),x]
```

output
```
(3*x*(x*(b + c*x))^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((c*x)/b)])/(5
*(1 + (c*x)/b)^(2/3))
```

3.36.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {1087, 1093, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{2/3}\,dx$$

$$\Big\downarrow \; 1087$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{b^2\int \frac{1}{\sqrt[3]{cx^2+bx}}dx}{7c}$$

$$\Big\downarrow \; 1093$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{1}{\sqrt[3]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{7c\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \; 1090$$

$$\frac{b^4\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{1}{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{7\sqrt[3]{2}c^3\sqrt[3]{bx+cx^2}}+\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}$$

$$\Big\downarrow \; 233$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{14\sqrt[3]{2}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \; 833$$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\int \frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}\right.}{14\sqrt[3]{2}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \; 760$$

3.36. $\int \left(bx+cx^2\right)^{2/3}\,dx$

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(-\int\frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)}{}\right.}{14\sqrt[3]{2}c\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}$$

$\downarrow$ 2418

$$\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c}-\frac{3b^2\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\text{EllipticI}}{\sqrt[4]{3}\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}}\right.}{}$$

input
```
Int[(b*x + c*x^2)^(2/3),x]
```

3.36. $\int \left(bx+cx^2\right)^{2/3}\,dx$

output
```
(3*(b + 2*c*x)*(b*x + c*x^2)^(2/3))/(14*c) - (3*b^2*Sqrt[-((b^2*(-(c/b) -
(2*c^2*x)/b^2)^2)/c^2)]*(-((c*(b*x + c*x^2))/b^2))^(1/3)*((-2*Sqrt[-((b^2*
(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)])/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2) + (
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + c/b + (2*c^2*x)/b^2)*Sqrt[(1 + (-(c/b) - (2
*c^2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sq
rt[3] + c/b + (2*c^2*x)/b^2)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + c/b + (2*c
^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(Sqrt[-(
(b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)]*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1
 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3]
)*(1 + c/b + (2*c^2*x)/b^2)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 - (b
^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*x)/
b^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3]
+ c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((b^2*(-(c/b) - (
2*c^2*x)/b^2)^2)/c^2)]*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b
 + (2*c^2*x)/b^2)^2)])))/(14*2^(1/3)*c*(-(c/b) - (2*c^2*x)/b^2)*(b*x + c*x
^2)^(1/3))
```

3.36.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.36.4 Maple [F]

$$\int \left(c\,x^{2}+b x \right)^{\frac{2}{3}}d x$$

input
```
int((c*x^2+b*x)^(2/3),x)
```

output
```
int((c*x^2+b*x)^(2/3),x)
```

3.36.5 Fricas [F]

$$\int \left(bx+cx^2\right)^{2/3} dx = \int \left(cx^2+bx\right)^{\frac{2}{3}} dx$$

input
```
integrate((c*x^2+b*x)^(2/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(2/3), x)
```

3.36.6 Sympy [F]

$$\int \left(bx+cx^2\right)^{2/3} dx = \int \left(bx+cx^2\right)^{\frac{2}{3}} dx$$

input
```
integrate((c*x**2+b*x)**(2/3),x)
```

output
```
Integral((b*x + c*x**2)**(2/3), x)
```

3.36.7 Maxima [F]

$$\int \left(bx+cx^2\right)^{2/3} dx = \int \left(cx^2+bx\right)^{\frac{2}{3}} dx$$

input
```
integrate((c*x^2+b*x)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(2/3), x)
```

3.36.8 Giac [F]

$$\int \left(bx+cx^2\right)^{2/3}\,dx=\int \left(cx^2+bx\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*x^2+b*x)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(2/3), x)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.05

$$\int \left(bx+cx^2\right)^{2/3}\,dx=\frac{3\,x\,\left(c\,x^2+b\,x\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{5}{3};\ \frac{8}{3};\ -\frac{c\,x}{b}\right)}{5\,\left(\frac{c\,x}{b}+1\right)^{2/3}}$$

input
```
int((b*x + c*x^2)^(2/3),x)
```

output
```
(3*x*(b*x + c*x^2)^(2/3)*hypergeom([-2/3, 5/3], 8/3, -(c*x)/b))/(5*((c*x)/
b + 1)^(2/3))
```

3.37 $\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx$

3.37.1 Optimal result

Integrand size = 13, antiderivative size = 715

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx = -\frac{3(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}{\sqrt[3]{2}c\sqrt[3]{bx+cx^2}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\,E\ldots}{2\sqrt[3]{2}c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\ldots}}}$$

$$+\frac{\sqrt[6]{2}3^{3/4}b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\,\text{Elliptic}\ldots}{c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\ldots}}}$$

output
```
-3/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(1/3)*2^(2/3)/c/(c*x^2+b*x)^(1/3)/(1-2
^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))+2^(1/6)*3^(3/4)*b^2*(-c*(c*x^2+b*
x)/b^2)^(1/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-
c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2
)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*
x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c/(
2*c*x+b)/(c*x^2+b*x)^(1/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/
3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)-3/4*3^(1/4)*b^2*(-c*(c*x^2+b
*x)/b^2)^(1/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticE((1-2^(2/3)*(
-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/
2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c
*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)*(1
/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(c*x^2+b*x)^(1/3)/((-1+2^(2/3)
*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)
^(1/2)
```

3.37.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx=\frac{3x\sqrt[3]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{cx}{b}\right)}{2\sqrt[3]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-1/3),x]
```

output
```
(3*x*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((c*x)/b)])/(2*
(x*(b + c*x))^(1/3))
```

3.37.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 657, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1093, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}dx$$

$$\Big\downarrow \ 1093$$

$$\frac{\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[3]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \ 1090$$

$$-\frac{b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{\sqrt[3]{2}c^2\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \ 233$$

$$\frac{3\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{2\sqrt[3]{2}\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \ 833$$

$$\frac{3\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\left((1+\sqrt{3})\int \frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\int \frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\right.}{2\sqrt[3]{2}\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \ 760$$

3.37. $\int \frac{1}{\sqrt[3]{bx+cx^2}}dx$

$$\frac{3\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(-\int\frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\sqrt{\cdots}}{\cdots}\right.}{2\sqrt[3]{2}\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{3\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\cdots\right.}{\sqrt[4]{3}\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}}\right.}{\cdots}$$

input
```
Int[(b*x + c*x^2)^(-1/3),x]
```

output
```
(3*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)]*(-((c*(b*x + c*x^2))/b^2)
)^(1/3)*((-2*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)])/(1 - Sqrt[3] +
 c/b + (2*c^2*x)/b^2) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + c/b + (2*c^2*x)/b^
2)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/b^
2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2]*EllipticE[ArcSin[
(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)],
-7 + 4*Sqrt[3]])/(Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)]*Sqrt[-((1
+ c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)]) - (2*Sqrt[
2 - Sqrt[3]]*(1 + Sqrt[3])*(1 + c/b + (2*c^2*x)/b^2)*Sqrt[(1 + (-(c/b) - (
2*c^2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - S
qrt[3] + c/b + (2*c^2*x)/b^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + c/b + (2*
c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(3^(1/4
)*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)]*Sqrt[-((1 + c/b + (2*c^2*x
)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)])))/(2*2^(1/3)*(-(c/b) - (2*
c^2*x)/b^2)*(b*x + c*x^2)^(1/3))
```

3.37. $\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx$

3.37.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.37.4 Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+b x\right)^{\frac{1}{3}}}d x$$

input
```
int(1/(c*x^2+b*x)^(1/3),x)
```

output
```
int(1/(c*x^2+b*x)^(1/3),x)
```

3.37.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(-1/3), x)
```

3.37.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx = \int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(1/3),x)
```

output
```
Integral((b*x + c*x**2)**(-1/3), x)
```

3.37.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-1/3), x)
```

3.37.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-1/3), x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx = \frac{3\,x\,\left(\frac{c\,x}{b}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{c\,x}{b}\right)}{2\,(c\,x^2+b\,x)^{1/3}}$$

input
```
int(1/(b*x + c*x^2)^(1/3),x)
```

output
```
(3*x*((c*x)/b + 1)^(1/3)*hypergeom([1/3, 2/3], 5/3, -(c*x)/b))/(2*(b*x + c
*x^2)^(1/3))
```

3.38 $\int \frac{1}{(bx+cx^2)^{4/3}}\,dx$

3.38.1 Optimal result

Integrand size = 13, antiderivative size = 773

$$\int \frac{1}{(bx+cx^2)^{4/3}}\,dx = \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{4/3}}$$

$$+\frac{3\,2^{2/3}(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\,(bx+cx^2)^{4/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}E\Bigg(}{\sqrt[3]{2}c(b+2cx)\,(bx+cx^2)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}}}$$

$$-\frac{2\sqrt[6]{2}3^{3/4}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\,\text{EllipticF}}{c(b+2cx)\,(bx+cx^2)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}}}$$

output
```
3*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(4/3)/c/(-c*x*(c*x+b)/b^2)^(1/3)/(c*x^2+b
*x)^(4/3)+3*2^(2/3)*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(4/3)/c/(c*x^2+b*x)^(4/
3)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))-2*2^(1/6)*3^(3/4)*b^2*(-c*
(c*x^2+b*x)/b^2)^(4/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2
^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/
3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*
(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^
(1/2)/c/(2*c*x+b)/(c*x^2+b*x)^(4/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))
/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)+3/2*3^(1/4)*b^2*(-c
*(c*x^2+b*x)/b^2)^(4/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticE((1-
2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1
/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)
*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(c*x^2+b*x)^(4/3)/((-
1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^
(1/2))^2)^(1/2)
```

3.38.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.06

$$\int \frac{1}{(bx+cx^2)^{4/3}}\,dx = -\frac{3\sqrt[3]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},-\frac{cx}{b}\right)}{b\sqrt[3]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-4/3),x]
```

output
```
(-3*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, -((c*x)/b)])/(b*
(x*(b + c*x))^(1/3))
```

3.38.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 684, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {1089, 1093, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{4/3}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2c\int \frac{1}{\sqrt[3]{cx^2+bx}}dx}{b^2}-\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \text{1093}$$

$$\frac{2c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[3]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{b^2\sqrt[3]{bx+cx^2}}-\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2^{2/3}\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c\sqrt[3]{bx+cx^2}}-\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \text{233}$$

$$\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{\sqrt[3]{2}b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}-\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}$$

$$\Big\downarrow \text{833}$$

$$\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\,(bx+cx^2)}{b^2}}\left((1+\sqrt{3})\int\frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\int\frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\right.}{\sqrt[3]{2}b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}$$

$$\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}$$

$\downarrow$ 760

$$\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\,(bx+cx^2)}{b^2}}\left(-\int\frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\sqrt{}}{}\right.}{\sqrt[3]{2}b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}$$

$$\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}$$

$\downarrow$ 2418

$$\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\,(bx+cx^2)}{b^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\text{EllipticF}}{\sqrt[4]{3}\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}}\right.}{}$$

$$\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}$$

input
```
Int[(b*x + c*x^2)^(-4/3),x]
```

3.38. $\int\frac{1}{(bx+cx^2)^{4/3}}\,dx$

output
```
(-3*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/3)) + (3*c*Sqrt[-((b^2*(-(c/b) - (2
*c^2*x)/b^2)^2)/c^2)]*(-((c*(b*x + c*x^2))/b^2))^(1/3)*((-2*Sqrt[-((b^2*(-
(c/b) - (2*c^2*x)/b^2)^2)/c^2)])/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*(1 + c/b + (2*c^2*x)/b^2)*Sqrt[(1 + (-(c/b) - (2*c
^2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt
[3] + c/b + (2*c^2*x)/b^2)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + c/b + (2*c^2
*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(Sqrt[-((b
^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)]*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 -
 Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*
(1 + c/b + (2*c^2*x)/b^2)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 - (b^2
*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^
2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] +
c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((b^2*(-(c/b) - (2*
c^2*x)/b^2)^2)/c^2)]*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b +
 (2*c^2*x)/b^2)^2)])))/(2^(1/3)*b^2*(-(c/b) - (2*c^2*x)/b^2)*(b*x + c*x^2)
^(1/3))
```

3.38.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.38.4 Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{4}{3}}}d x$$

input
```
int(1/(c*x^2+b*x)^(4/3),x)
```

output
```
int(1/(c*x^2+b*x)^(4/3),x)
```

3.38.5 Fricas [F]

$$\int \frac{1}{(bx + cx^2)^{4/3}} \, dx = \int \frac{1}{(cx^2 + bx)^{\frac{4}{3}}} \, dx$$

input
```
integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(2/3)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

3.38.6 Sympy [F]

$$\int \frac{1}{(bx + cx^2)^{4/3}} \, dx = \int \frac{1}{(bx + cx^2)^{\frac{4}{3}}} \, dx$$

input
```
integrate(1/(c*x**2+b*x)**(4/3),x)
```

output
```
Integral((b*x + c*x**2)**(-4/3), x)
```

3.38.7 Maxima [F]

$$\int \frac{1}{(bx + cx^2)^{4/3}} \, dx = \int \frac{1}{(cx^2 + bx)^{\frac{4}{3}}} \, dx$$

input
```
integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-4/3), x)
```

3.38.8 Giac [F]

$$\int \frac{1}{(bx+cx^2)^{4/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-4/3), x)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.05

$$\int \frac{1}{(bx+cx^2)^{4/3}}\,dx = -\frac{3\,x\,\left(\frac{c\,x}{b}+1\right)^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{4}{3};\ \frac{2}{3};\ -\frac{c\,x}{b}\right)}{\left(c\,x^2+b\,x\right)^{4/3}}$$

input
```
int(1/(b*x + c*x^2)^(4/3),x)
```

output
```
-(3*x*((c*x)/b + 1)^(4/3)*hypergeom([-1/3, 4/3], 2/3, -(c*x)/b))/(b*x + c*
x^2)^(4/3)
```

3.39 $\int \frac{1}{(bx+cx^2)^{7/3}}\,dx$

3.39.1 Optimal result

Integrand size = 13, antiderivative size = 838

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx = \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(bx+cx^2)^{7/3}}$$

$$+\frac{15(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{7/3}}+\frac{15(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{\sqrt[3]{2}c\,(bx+cx^2)^{7/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}$$

$$+\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}E\Bigg(}{2\sqrt[3]{2}c(b+2cx)(bx+cx^2)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right.}}}$$

$$-\frac{5\sqrt[6]{2}3^{3/4}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}\,\mathrm{EllipticF}}{c(b+2cx)(bx+cx^2)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right.}}}$$

output
```
3/4*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(7/3)/c/(-c*x*(c*x+b)/b^2)^(4/3)/(c*x^2
+b*x)^(7/3)+15/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(7/3)/c/(-c*x*(c*x+b)/b^2)
^(1/3)/(c*x^2+b*x)^(7/3)+15/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(7/3)*2^(2/3)
/c/(c*x^2+b*x)^(7/3)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))-5*2^(1/6
)*3^(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(7/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/
3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*
x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2
)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^
(1/3)-3^(1/2))^2)^(1/2)/c/(2*c*x+b)/(c*x^2+b*x)^(7/3)/((-1+2^(2/3)*(-c*x*(
c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)+1
5/4*3^(1/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(7/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(
1/3))*EllipticE((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-
c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b
^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2
)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(c
*x^2+b*x)^(7/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c
*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.06

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx = -\frac{3\sqrt[3]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{7}{3},-\frac{1}{3},-\frac{cx}{b}\right)}{4b^2x\sqrt[3]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-7/3),x]
```

output
```
(-3*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[-4/3, 7/3, -1/3, -((c*x)/b)])/(4
*b^2*x*(x*(b + c*x))^(1/3))
```

3.39.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 719, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {1089, 1089, 1093, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{5c\int \frac{1}{(cx^2+bx)^{4/3}}dx}{2b^2}-\frac{3(b+2cx)}{4b^2(bx+cx^2)^{4/3}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{5c\left(\frac{2c\int \frac{1}{\sqrt[3]{cx^2+bx}}dx}{b^2}-\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}\right)}{2b^2}-\frac{3(b+2cx)}{4b^2(bx+cx^2)^{4/3}}$$

$$\Big\downarrow \text{1093}$$

$$-\frac{5c\left(\frac{2c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[3]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{b^2\sqrt[3]{bx+cx^2}}-\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}\right)}{2b^2}-\frac{3(b+2cx)}{4b^2(bx+cx^2)^{4/3}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{5c\left(-\frac{2^{2/3}\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c\sqrt[3]{bx+cx^2}}-\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}\right)}{2b^2}-\frac{3(b+2cx)}{4b^2(bx+cx^2)^{4/3}}$$

$$\Big\downarrow \; 233$$

$$-\frac{5c\left(\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}{\sqrt[3]{2}b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}-\frac{3(b+2cx)}{b^2\sqrt[3]{bx+cx^2}}\right)}{2b^2}-\frac{3(b+2cx)}{4b^2\left(bx+cx^2\right)^{4/3}}$$

$$\Big\downarrow \; 833$$

$$-\frac{5c\left(\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\int \frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{\cdots}\right.}{\sqrt[3]{2}b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}\cdots\right.}{2b^2}-\frac{3(b+2cx)}{4b^2\left(bx+cx^2\right)^{4/3}}$$

$$\Big\downarrow \; 760$$

3.39. $\int \frac{1}{\left(bx+cx^2\right)^{7/3}}\,dx$

$$-\frac{5c\left(\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(-\int\frac{\frac{2xc^2}{b^2}+\frac{c}{b}+\sqrt{3}+1}{\sqrt{-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\sqrt[3]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\sqrt{\cdots}}{\cdots}\right)}{\sqrt[3]{2}b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)\sqrt[3]{bx+cx^2}}\cdots\right)}{2b^2\cdots}$$

$$\frac{3(b+2cx)}{4b^2\left(bx+cx^2\right)^{4/3}}$$

$\downarrow$ 2418

$$-\frac{5c\left(\frac{3c\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2c^2x}{b^2}+\frac{c}{b}+1\right)\sqrt{\frac{\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2+\sqrt[3]{1-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\cdots\right)}{\sqrt[4]{3}\sqrt{-\frac{b^2\left(-\frac{2c^2x}{b^2}-\frac{c}{b}\right)^2}{c^2}}\sqrt{-\frac{\frac{2c^2x}{b^2}+\frac{c}{b}+1}{\left(\frac{2c^2x}{b^2}+\frac{c}{b}-\sqrt{3}+1\right)^2}}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{3(b+2cx)}{4b^2\left(bx+cx^2\right)^{4/3}}$$

3.39. $\int\frac{1}{\left(bx+cx^2\right)^{7/3}}\,dx$

input
```
Int[(b*x + c*x^2)^(-7/3),x]
```

output
```
(-3*(b + 2*c*x))/(4*b^2*(b*x + c*x^2)^(4/3)) - (5*c*((-3*(b + 2*c*x))/(b^2
*(b*x + c*x^2)^(1/3)) + (3*c*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)]
*(-((c*(b*x + c*x^2))/b^2))^(1/3)*((-2*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2
)^2)/c^2)])/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2) + (3^(1/4)*Sqrt[2 + Sqrt[3
]]*(1 + c/b + (2*c^2*x)/b^2)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 - (
b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*x)
/b^2)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3]
 + c/b + (2*c^2*x)/b^2)], -7 + 4*Sqrt[3]])/(Sqrt[-((b^2*(-(c/b) - (2*c^2*x
)/b^2)^2)/c^2)]*Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c
^2*x)/b^2)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 + c/b + (2*c^2*x)/
b^2)*Sqrt[(1 + (-(c/b) - (2*c^2*x)/b^2)^2 + (1 - (b^2*(-(c/b) - (2*c^2*x)/
b^2)^2)/c^2)^(1/3))/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2]*EllipticF[ArcSi
n[(1 + Sqrt[3] + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)]
, -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((b^2*(-(c/b) - (2*c^2*x)/b^2)^2)/c^2)]*
Sqrt[-((1 + c/b + (2*c^2*x)/b^2)/(1 - Sqrt[3] + c/b + (2*c^2*x)/b^2)^2)]))
)/(2^(1/3)*b^2*(-(c/b) - (2*c^2*x)/b^2)*(b*x + c*x^2)^(1/3))))/(2*b^2)
```

3.39.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.39.4 Maple [F]

$$\int \frac{1}{(c\,x^{2}+bx)^{\frac{7}{3}}}dx$$

input
```
int(1/(c*x^2+b*x)^(7/3),x)
```

output
```
int(1/(c*x^2+b*x)^(7/3),x)
```

3.39.5 Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(2/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^
3), x)
```

3.39.6 Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(7/3),x)
```

output
```
Integral((b*x + c*x**2)**(-7/3), x)
```

3.39.7 Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-7/3), x)
```

3.39.8 Giac [F]

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-7/3), x)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.04

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx = -\frac{3\,x\,\left(\frac{c\,x}{b}+1\right)^{7/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{7}{3};\ -\frac{1}{3};\ -\frac{c\,x}{b}\right)}{4\,(c\,x^2+b\,x)^{7/3}}$$

input
```
int(1/(b*x + c*x^2)^(7/3),x)
```

output
```
-(3*x*((c*x)/b + 1)^(7/3)*hypergeom([-4/3, 7/3], -1/3, -(c*x)/b))/(4*(b*x
+ c*x^2)^(7/3))
```

3.40 $\int (bx+cx^2)^{5/4}\,dx$

3.40.1 Optimal result

Integrand size = 13, antiderivative size = 119

$$\int (bx+cx^2)^{5/4}\,dx = -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}+\frac{5b^5\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right),2\right)}{84\sqrt{2}c^3\left(bx+cx^2\right)^{3/4}}$$

output
```
-5/84*b^2*(2*c*x+b)*(c*x^2+b*x)^(1/4)/c^2+1/7*(2*c*x+b)*(c*x^2+b*x)^(5/4)/
c+5/168*b^5*(-c*(c*x^2+b*x)/b^2)^(3/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2
)/cos(1/2*arcsin(1+2*c*x/b))*EllipticF(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))
/c^3/(c*x^2+b*x)^(3/4)*2^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.40

$$\int (bx+cx^2)^{5/4}\,dx = \frac{4bx^2\sqrt[4]{x(b+cx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4},\frac{13}{4},-\frac{cx}{b}\right)}{9\sqrt[4]{1+\frac{cx}{b}}}$$

input
```
Integrate[(b*x + c*x^2)^(5/4),x]
```

output
```
(4*b*x^2*(x*(b + c*x))^(1/4)*Hypergeometric2F1[-5/4, 9/4, 13/4, -((c*x)/b)
])/(9*(1 + (c*x)/b)^(1/4))
```

3.40.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1087, 1087, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/4}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\int \sqrt[4]{cx^2+bx}dx}{28c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\left(\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\int \frac{1}{\left(cx^2+bx\right)^{3/4}}dx}{12c}\right)}{28c}$$

$$\Big\downarrow \text{1093}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\left(\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{3/4}}dx}{12c\left(bx+cx^2\right)^{3/4}}\right)}{28c}$$

$$\Big\downarrow \text{1090}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\left(\frac{b^4\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{3/4}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{6\sqrt{2}c^3\left(bx+cx^2\right)^{3/4}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}\right)}{28c}$$

$$\Big\downarrow \text{230}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\left(\frac{b^3\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right),2\right)}{3\sqrt{2}c^2\left(bx+cx^2\right)^{3/4}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}\right)}{28c}$$

input
```
Int[(b*x + c*x^2)^(5/4),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(5/4))/(7*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x^
2)^(1/4))/(3*c) + (b^3*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[(
b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(3*Sqrt[2]*c^2*(b*x + c*x^2)^(3/4)))
)/(28*c)
```

3.40.3.1 Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.40.4 Maple [F]

$$\int \left(c\,x^{2}+bx\right)^{\frac{5}{4}}dx$$

input
```
int((c*x^2+b*x)^(5/4),x)
```

output
```
int((c*x^2+b*x)^(5/4),x)
```

3.40.5 Fricas [F]

$$\int \left(bx+cx^2\right)^{5/4} dx = \int \left(cx^{2}+bx\right)^{\frac{5}{4}} dx$$

input
```
integrate((c*x^2+b*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(5/4), x)
```

3.40.6 Sympy [F]

$$\int \left(bx+cx^2\right)^{5/4} dx = \int \left(bx+cx^{2}\right)^{\frac{5}{4}} dx$$

input
```
integrate((c*x**2+b*x)**(5/4),x)
```

output
```
Integral((b*x + c*x**2)**(5/4), x)
```

3.40.7 Maxima [F]

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/4), x)
```

3.40.8 Giac [F]

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/4), x)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\frac{4\,x\,\left(c\,x^2+b\,x\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{9}{4};\ \frac{13}{4};\ -\frac{c\,x}{b}\right)}{9\,\left(\frac{c\,x}{b}+1\right)^{5/4}}$$

input
```
int((b*x + c*x^2)^(5/4),x)
```

output
```
(4*x*(b*x + c*x^2)^(5/4)*hypergeom([-5/4, 9/4], 13/4, -(c*x)/b))/(9*((c*x)
/b + 1)^(5/4))
```

3.41 $\int (bx+cx^2)^{3/4}\,dx$

3.41.1 Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

output
```
1/5*(2*c*x+b)*(c*x^2+b*x)^(3/4)/c-3/20*b^3*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos
(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(
1/2*arcsin(1+2*c*x/b)),2^(1/2))/c^2/(c*x^2+b*x)^(1/4)*2^(1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.50

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\frac{4x(x(b+cx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{11}{4},-\frac{cx}{b}\right)}{7\left(1+\frac{cx}{b}\right)^{3/4}}$$

input
```
Integrate[(b*x + c*x^2)^(3/4),x]
```

output
```
(4*x*(x*(b + c*x))^(3/4)*Hypergeometric2F1[-3/4, 7/4, 11/4, -((c*x)/b)])/(
7*(1 + (c*x)/b)^(3/4))
```

3.41.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1087, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/4}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^2\int \frac{1}{\sqrt[4]{cx^2+bx}}dx}{20c}$$

$$\Big\downarrow \text{ 1093}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{20c\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{3b^4\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{20\sqrt{2}c^3\sqrt[4]{bx+cx^2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}$$

$$\Big\downarrow \text{ 226}$$

$$\frac{3b^3\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}$$

input
```
Int[(b*x + c*x^2)^(3/4),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(3/4))/(5*c) + (3*b^3*(-((c*(b*x + c*x^2))/b^2)
)^(1/4)*EllipticE[ArcSin[(b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(10*Sqrt[2
]*c^2*(b*x + c*x^2)^(1/4))
```

3.41.3.1 Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.41.4 Maple [F]

$$\int \left(c\,x^{2}+b x \right)^{\frac{3}{4}}d x$$

input
```
int((c*x^2+b*x)^(3/4),x)
```

output
```
int((c*x^2+b*x)^(3/4),x)
```

3.41.5 Fricas [F]

$$\int (bx+cx^2)^{3/4}\,dx = \int (cx^2+bx)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/4), x)
```

3.41.6 Sympy [F]

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\int \left(bx+cx^2\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/4),x)
```

output
```
Integral((b*x + c*x**2)**(3/4), x)
```

3.41.7 Maxima [F]

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\int \left(cx^2+bx\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/4), x)
```

3.41.8 Giac [F]

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\int \left(cx^2+bx\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/4), x)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \left(bx + cx^2\right)^{3/4} \, dx = \frac{4\,x\,\left(c\,x^2 + b\,x\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{7}{4};\ \frac{11}{4};\ -\frac{c\,x}{b}\right)}{7\,\left(\frac{c\,x}{b} + 1\right)^{3/4}}$$

input
```
int((b*x + c*x^2)^(3/4),x)
```

output
```
(4*x*(b*x + c*x^2)^(3/4)*hypergeom([-3/4, 7/4], 11/4, -(c*x)/b))/(7*((c*x)
/b + 1)^(3/4))
```

3.42 $\int \sqrt[4]{bx+cx^2}\,dx$

3.42.1 Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \sqrt[4]{bx+cx^2}\,dx=\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right),2\right)}{3\sqrt{2}c^2\left(bx+cx^2\right)^{3/4}}$$

output
```
1/3*(2*c*x+b)*(c*x^2+b*x)^(1/4)/c-1/6*b^3*(-c*(c*x^2+b*x)/b^2)^(3/4)*(cos(
1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticF(sin(1
/2*arcsin(1+2*c*x/b)),2^(1/2))/c^2/(c*x^2+b*x)^(3/4)*2^(1/2)
```

3.42.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.50

$$\int \sqrt[4]{bx+cx^2}\,dx=\frac{4x\sqrt[4]{x(b+cx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{9}{4},-\frac{cx}{b}\right)}{5\sqrt[4]{1+\frac{cx}{b}}}$$

input
```
Integrate[(b*x + c*x^2)^(1/4),x]
```

output
```
(4*x*(x*(b + c*x))^(1/4)*Hypergeometric2F1[-1/4, 5/4, 9/4, -((c*x)/b)])/(5
*(1 + (c*x)/b)^(1/4))
```

3.42.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1087, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{bx+cx^2}\,dx$$

$$\Big\downarrow \ 1087$$

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\int \frac{1}{(cx^2+bx)^{3/4}}dx}{12c}$$

$$\Big\downarrow \ 1093$$

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{3/4}}dx}{12c\,(bx+cx^2)^{3/4}}$$

$$\Big\downarrow \ 1090$$

$$\frac{b^4\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{3/4}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{6\sqrt{2}c^3\,(bx+cx^2)^{3/4}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}$$

$$\Big\downarrow \ 230$$

$$\frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right),2\right)}{3\sqrt{2}c^2\,(bx+cx^2)^{3/4}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}$$

input
```
Int[(b*x + c*x^2)^(1/4),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(1/4))/(3*c) + (b^3*(-((c*(b*x + c*x^2))/b^2))^
(3/4)*EllipticF[ArcSin[(b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(3*Sqrt[2]*c
^2*(b*x + c*x^2)^(3/4))
```

3.42.3.1 Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.42.4 Maple [F]

$$\int \left(c\,x^{2}+b x \right)^{\frac{1}{4}}d x$$

input
```
int((c*x^2+b*x)^(1/4),x)
```

output
```
int((c*x^2+b*x)^(1/4),x)
```

3.42.5 Fricas [F]

$$\int \sqrt[4]{bx+cx^2}\,dx=\int (cx^2+bx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(1/4), x)
```

3.42.6 Sympy [F]

$$\int \sqrt[4]{bx+cx^2}\,dx = \int \sqrt[4]{bx+cx^2}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/4),x)
```

output
```
Integral((b*x + c*x**2)**(1/4), x)
```

3.42.7 Maxima [F]

$$\int \sqrt[4]{bx+cx^2}\,dx = \int (cx^2+bx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(1/4), x)
```

3.42.8 Giac [F]

$$\int \sqrt[4]{bx+cx^2}\,dx = \int (cx^2+bx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(1/4), x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \sqrt[4]{bx+cx^2}\,dx=\frac{4\,x\,{\left(c\,x^2+b\,x\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{c\,x}{b}\right)}{5\,{\left(\frac{c\,x}{b}+1\right)}^{1/4}}$$

input
```
int((b*x + c*x^2)^(1/4),x)
```

output
```
(4*x*(b*x + c*x^2)^(1/4)*hypergeom([-1/4, 5/4], 9/4, -(c*x)/b))/(5*((c*x)/
b + 1)^(1/4))
```

3.43 $\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx$

3.43.1 Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx=\frac{\sqrt{2}b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

output
```
b*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*
arcsin(1+2*c*x/b))*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/2)/c
/(c*x^2+b*x)^(1/4)
```

3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx=\frac{4x\sqrt[4]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{cx}{b}\right)}{3\sqrt[4]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-1/4),x]
```

output
```
(4*x*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((c*x)/b)])/(3*
(x*(b + c*x))^(1/4))
```

3.43.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{b^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt{2}b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

input
```
Int[(b*x + c*x^2)^(-1/4),x]
```

output
```
-((Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[(b*(-(c/b)
- (2*c^2*x)/b^2))/c]/2, 2])/(c*(b*x + c*x^2)^(1/4)))
```

3.43.3.1 Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.43.4 Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{1}{4}}}d x$$

input
```
int(1/(c*x^2+b*x)^(1/4),x)
```

output
```
int(1/(c*x^2+b*x)^(1/4),x)
```

3.43.5 Fricas [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx=\int \frac{1}{(cx^2+bx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(-1/4), x)
```

3.43.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(1/4),x)
```

output
```
Integral((b*x + c*x**2)**(-1/4), x)
```

3.43.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-1/4), x)
```

3.43.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-1/4), x)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx=\frac{4\,x\,{\left(\frac{c\,x}{b}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{c\,x}{b}\right)}{3\,{\left(c\,x^2+b\,x\right)}^{1/4}}$$

input
```
int(1/(b*x + c*x^2)^(1/4),x)
```

output
```
(4*x*((c*x)/b + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -(c*x)/b))/(3*(b*x + c
*x^2)^(1/4))
```

3.44 $\int \frac{1}{(bx+cx^2)^{3/4}}\,dx$

3.44.1 Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \frac{2\sqrt{2}b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right),2\right)}{c\,(bx+cx^2)^{3/4}}$$

output
```
2*b*(-c*(c*x^2+b*x)/b^2)^(3/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/
2*arcsin(1+2*c*x/b))*EllipticF(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/2)
/c/(c*x^2+b*x)^(3/4)
```

3.44.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \frac{4x\left(1+\frac{cx}{b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{cx}{b}\right)}{(x(b+cx))^{3/4}}$$

input
```
Integrate[(b*x + c*x^2)^(-3/4),x]
```

output
```
(4*x*(1 + (c*x)/b)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((c*x)/b)])/(x*
(b + c*x))^(3/4)
```

3.44.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{3/4}} \, dx$$

$$\Big\downarrow \; 1093$$

$$\frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} \int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{3/4}}dx}{(bx+cx^2)^{3/4}}$$

$$\Big\downarrow \; 1090$$

$$-\frac{\sqrt{2}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} \int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{3/4}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c^2\,(bx+cx^2)^{3/4}}$$

$$\Big\downarrow \; 230$$

$$-\frac{2\sqrt{2}b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right),2\right)}{c\,(bx+cx^2)^{3/4}}$$

input
```
Int[(b*x + c*x^2)^(-3/4),x]
```

output
```
(-2*Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[(b*(-(c/b)
 - (2*c^2*x)/b^2))/c]/2, 2])/(c*(b*x + c*x^2)^(3/4))
```

3.44.3.1 Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.44.4 Maple [F]

$$\int \frac{1}{(c\,x^2+bx)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x^2+b*x)^(3/4),x)
```

output
```
int(1/(c*x^2+b*x)^(3/4),x)
```

3.44.5 Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(-3/4), x)
```

3.44.6 Sympy [F]

$$\int \frac{1}{(bx + cx^2)^{3/4}}\,dx = \int \frac{1}{(bx + cx^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(3/4),x)
```

output
```
Integral((b*x + c*x**2)**(-3/4), x)
```

3.44.7 Maxima [F]

$$\int \frac{1}{(bx + cx^2)^{3/4}}\,dx = \int \frac{1}{(cx^2 + bx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-3/4), x)
```

3.44.8 Giac [F]

$$\int \frac{1}{(bx + cx^2)^{3/4}}\,dx = \int \frac{1}{(cx^2 + bx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-3/4), x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\;\frac{5}{4};\;-\frac{c\,x}{b}\right)}{\left(c\,x^2+b\,x\right)^{3/4}}$$

input
```
int(1/(b*x + c*x^2)^(3/4),x)
```

output
```
(4*x*((c*x)/b + 1)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(c*x)/b))/(b*x + c*x^
2)^(3/4)
```

3.45 $\int \frac{1}{(bx+cx^2)^{5/4}} dx$

3.45.1 Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{1}{(bx+cx^2)^{5/4}} dx = -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}} + \frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}$$

output
```
-4*(2*c*x+b)/b^2/(c*x^2+b*x)^(1/4)+4*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*a
rcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(1/2*ar
csin(1+2*c*x/b)),2^(1/2))*2^(1/2)/b/(c*x^2+b*x)^(1/4)
```

3.45.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{1}{(bx+cx^2)^{5/4}} dx = -\frac{4\sqrt[4]{1+\frac{cx}{b}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{cx}{b}\right)}{b\sqrt[4]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-5/4),x]
```

output
```
(-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((c*x)/b)])/(b*
(x*(b + c*x))^(1/4))
```

3.45.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx$$

$$\Big\downarrow \ 1089$$

$$\frac{4c\int \frac{1}{\sqrt[4]{cx^2+bx}}dx}{b^2}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \ 1093$$

$$\frac{4c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{b^2\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \ 1090$$

$$-\frac{2\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \ 226$$

$$-\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

input
```
Int[(b*x + c*x^2)^(-5/4),x]
```

output
```
(-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4)) - (4*Sqrt[2]*(-((c*(b*x + c*x^2
))/b^2))^(1/4)*EllipticE[ArcSin[(b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(b*
(b*x + c*x^2)^(1/4))
```

3.45.3.1 Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.45.4 Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{5}{4}}}d x$$

input
```
int(1/(c*x^2+b*x)^(5/4),x)
```

output
```
int(1/(c*x^2+b*x)^(5/4),x)
```

3.45.5 Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/4)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

3.45.6 Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(5/4),x)
```

output
```
Integral((b*x + c*x**2)**(-5/4), x)
```

3.45.7 Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-5/4), x)
```

3.45.8 Giac [F]

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-5/4), x)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.43

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = -\frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ -\frac{c\,x}{b}\right)}{\left(c\,x^2+b\,x\right)^{5/4}}$$

input
```
int(1/(b*x + c*x^2)^(5/4),x)
```

output
```
-(4*x*((c*x)/b + 1)^(5/4)*hypergeom([-1/4, 5/4], 3/4, -(c*x)/b))/(b*x + c*
x^2)^(5/4)
```

3.46 $\int \frac{1}{(bx+cx^2)^{9/4}}\,dx$

3.46.1 Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = -\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}+\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{48\sqrt{2}c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right)\middle|2\right)}{5b^3\sqrt[4]{bx+cx^2}}$$

output
```
-4/5*(2*c*x+b)/b^2/(c*x^2+b*x)^(5/4)+48/5*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(1/4
)-48/5*c*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/c
os(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^
(1/2)/b^3/(c*x^2+b*x)^(1/4)
```

3.46.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.43

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4},-\frac{1}{4},-\frac{cx}{b}\right)}{5b^2x\sqrt[4]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-9/4),x]
```

output
```
(-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, -((c*x)/b)])/(5
*b^2*x*(x*(b + c*x))^(1/4))
```

3.46.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1089, 1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{9/4}} \, dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{12c\int \frac{1}{(cx^2+bx)^{5/4}}dx}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{12c\left(\frac{4c\int \frac{1}{\sqrt[4]{cx^2+bx}}dx}{b^2}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}$$

$$\Big\downarrow \text{1093}$$

$$-\frac{12c\left(\frac{4c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{b^2\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{12c\left(-\frac{2\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2\,(bx+cx^2)^{5/4}}$$

$$\Big\downarrow \; 226$$

$$-\frac{12c\left(-\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2\,(bx+cx^2)^{5/4}}$$

input
```
Int[(b*x + c*x^2)^(-9/4),x]
```

output
```
(-4*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/4)) - (12*c*((-4*(b + 2*c*x))/(b^
2*(b*x + c*x^2)^(1/4)) - (4*Sqrt[2]*(-((c*(b*x + c*x^2))/b^2))^(1/4)*Ellip
ticE[ArcSin[(b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(b*(b*x + c*x^2)^(1/4))
))/(5*b^2)
```

3.46.3.1 Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.46.4 Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{9}{4}}}d x$$

input
```
int(1/(c*x^2+b*x)^(9/4),x)
```

output
```
int(1/(c*x^2+b*x)^(9/4),x)
```

3.46.5 Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/4)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^
3), x)
```

3.46.6 Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(9/4),x)
```

output
```
Integral((b*x + c*x**2)**(-9/4), x)
```

3.46.7 Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-9/4), x)
```

3.46.8 Giac [F]

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-9/4), x)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.31

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = -\frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{9/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{9}{4};\ -\frac{1}{4};\ -\frac{c\,x}{b}\right)}{5\,\left(c\,x^2+b\,x\right)^{9/4}}$$

input
```
int(1/(b*x + c*x^2)^(9/4),x)
```

output
```
-(4*x*((c*x)/b + 1)^(9/4)*hypergeom([-5/4, 9/4], -1/4, -(c*x)/b))/(5*(b*x
+ c*x^2)^(9/4))
```

3.47 $\int \frac{1}{(bx+cx^2)^{13/4}}\,dx$

3.47.1 Optimal result

Integrand size = 13, antiderivative size = 146

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = -\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}+\frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}} -\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}}+\frac{448\sqrt{2}c^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right)\middle|2\right)}{15b^5\sqrt[4]{bx+cx^2}}$$

output
```
-4/9*(2*c*x+b)/b^2/(c*x^2+b*x)^(9/4)+112/45*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(5
/4)-448/15*c^2*(2*c*x+b)/b^6/(c*x^2+b*x)^(1/4)+448/15*c^2*(-c*(c*x^2+b*x)/
b^2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))
*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/2)/b^5/(c*x^2+b*x)^(1/
4)
```

3.47.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.34

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{13}{4},-\frac{5}{4},-\frac{cx}{b}\right)}{9b^3x^2\sqrt[4]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-13/4),x]
```

output
```
(-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-9/4, 13/4, -5/4, -((c*x)/b)])/(
9*b^3*x^2*(x*(b + c*x))^(1/4))
```

3.47.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1089, 1089, 1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{13/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{28c\int \frac{1}{(cx^2+bx)^{9/4}}dx}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{28c\left(-\frac{12c\int \frac{1}{(cx^2+bx)^{5/4}}dx}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{28c\left(-\frac{12c\left(\frac{4c\int \frac{1}{\sqrt[4]{cx^2+bx}}dx}{b^2}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

$$\Big\downarrow \text{1093}$$

$$-\frac{28c\left(-\frac{12c\left(\frac{4c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int\frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{b^2\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

$\Big\downarrow$ 1090

$$-\frac{28c\left(-\frac{12c\left(-\frac{2\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int\frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

$\Big\downarrow$ 226

3.47. $\int\frac{1}{(bx+cx^2)^{13/4}}\,dx$

$$-\frac{28c\left(-\frac{12c\left(-\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}}$$

input
```
Int[(b*x + c*x^2)^(-13/4),x]
```

output
```
(-4*(b + 2*c*x))/(9*b^2*(b*x + c*x^2)^(9/4)) - (28*c*((-4*(b + 2*c*x))/(5*
b^2*(b*x + c*x^2)^(5/4)) - (12*c*((-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4
)) - (4*Sqrt[2]*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[(b*(-(c/
b) - (2*c^2*x)/b^2))/c]/2, 2])/(b*(b*x + c*x^2)^(1/4))))/(5*b^2)))/(9*b^2)
```

3.47.3.1 Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.47.4 Maple [F]

$$\int \frac{1}{(c\,x^2+bx)^{\frac{13}{4}}}dx$$

input
```
int(1/(c*x^2+b*x)^(13/4),x)
```

output
```
int(1/(c*x^2+b*x)^(13/4),x)
```

3.47.5 Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/4)/(c^4*x^8 + 4*b*c^3*x^7 + 6*b^2*c^2*x^6 + 4*b^
3*c*x^5 + b^4*x^4), x)
```

3.47.6 Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(13/4),x)
```

output
```
Integral((b*x + c*x**2)**(-13/4), x)
```

3.47.7 Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-13/4), x)
```

3.47.8 Giac [F]

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-13/4), x)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.25

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = -\frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{13/4}\,{}_2\mathrm{F}_1\left(-\frac{9}{4},\frac{13}{4};\ -\frac{5}{4};\ -\frac{c\,x}{b}\right)}{9\,\left(c\,x^2+b\,x\right)^{13/4}}$$

input
```
int(1/(b*x + c*x^2)^(13/4),x)
```

output
```
-(4*x*((c*x)/b + 1)^(13/4)*hypergeom([-9/4, 13/4], -5/4, -(c*x)/b))/(9*(b*
x + c*x^2)^(13/4))
```

3.48 $\int (bx + cx^2)^p \, dx$

3.48.1 Optimal result

Integrand size = 11, antiderivative size = 55

$$\int (bx + cx^2)^p \, dx = -\frac{\left(-\frac{cx}{b}\right)^{-1-p} (bx + cx^2)^{1+p} \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{b+cx}{b}\right)}{b(1+p)}$$

output
```
-(-c*x/b)^(-1-p)*(c*x^2+b*x)^(p+1)*hypergeom([-p, p+1],[2+p],(c*x+b)/b)/b/
(p+1)
```

3.48.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int (bx + cx^2)^p \, dx = \frac{x(x(b+cx))^p \left(1 + \frac{cx}{b}\right)^{-p} \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, -\frac{cx}{b}\right)}{1+p}$$

input
```
Integrate[(b*x + c*x^2)^p,x]
```

output
```
(x*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x)/b)])/((1 +
p)*(1 + (c*x)/b)^p)
```

3.48.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1096}$$

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1} \left(bx+cx^2\right)^{p+1} \text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+cx}{b}\right)}{b(p+1)}$$

input
```
Int[(b*x + c*x^2)^p,x]
```

output
```
-(((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p
, 2 + p, (b + c*x)/b])/(b*(1 + p)))
```

3.48.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

3.48.4 Maple [F]

$$\int \left(c\,x^2+bx\right)^p dx$$

input
```
int((c*x^2+b*x)^p,x)
```

output
```
int((c*x^2+b*x)^p,x)
```

3.48.5 Fricas [F]

$$\int (bx + cx^2)^p \, dx = \int (cx^2 + bx)^p \, dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p, x)
```

3.48.6 Sympy [F]

$$\int (bx + cx^2)^p \, dx = \int (bx + cx^2)^p \, dx$$

input
```
integrate((c*x**2+b*x)**p,x)
```

output
```
Integral((b*x + c*x**2)**p, x)
```

3.48.7 Maxima [F]

$$\int (bx + cx^2)^p \, dx = \int (cx^2 + bx)^p \, dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p, x)
```

3.48.8 Giac [F]

$$\int (bx + cx^2)^p \, dx = \int (cx^2 + bx)^p \, dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p, x)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int (bx + cx^2)^p \, dx = \frac{x\,(c\,x^2 + b\,x)^p \,{}_2\mathrm{F}_1\!\left(-p, p+1;\; p+2;\; -\frac{c\,x}{b}\right)}{\left(\frac{c\,x}{b} + 1\right)^p \,(p+1)}$$

input
```
int((b*x + c*x^2)^p,x)
```

output
```
(x*(b*x + c*x^2)^p*hypergeom([-p, p + 1], p + 2, -(c*x)/b))/(((c*x)/b + 1)
^p*(p + 1))
```

3.49 $\int (a+cx^2)^4 \, dx$

3.49.1 Optimal result

Integrand size = 9, antiderivative size = 51

$$\int (a+cx^2)^4 \, dx = a^4x + \frac{4}{3}a^3cx^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}ac^3x^7 + \frac{c^4x^9}{9}$$

output
```
a^4*x+4/3*a^3*c*x^3+6/5*a^2*c^2*x^5+4/7*a*c^3*x^7+1/9*c^4*x^9
```

3.49.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int (a+cx^2)^4 \, dx = a^4x + \frac{4}{3}a^3cx^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}ac^3x^7 + \frac{c^4x^9}{9}$$

input
```
Integrate[(a + c*x^2)^4,x]
```

output
```
a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/
9
```

3.49.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^4 dx$$

$$\Big\downarrow \; 210$$

$$\int \left(a^4+4a^3cx^2+6a^2c^2x^4+4ac^3x^6+c^4x^8\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

input
```
Int[(a + c*x^2)^4,x]
```

output
```
a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/
9
```

3.49.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.49.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
gosper	$a^4x+\frac{4}{3}c\,a^3x^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}a\,c^3x^7+\frac{1}{9}c^4x^9$	44
default	$a^4x+\frac{4}{3}c\,a^3x^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}a\,c^3x^7+\frac{1}{9}c^4x^9$	44
norman	$a^4x+\frac{4}{3}c\,a^3x^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}a\,c^3x^7+\frac{1}{9}c^4x^9$	44
risch	$a^4x+\frac{4}{3}c\,a^3x^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}a\,c^3x^7+\frac{1}{9}c^4x^9$	44
parallelrisch	$a^4x+\frac{4}{3}c\,a^3x^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}a\,c^3x^7+\frac{1}{9}c^4x^9$	44

input
```
int((c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+4/3*c*a^3*x^3+6/5*a^2*c^2*x^5+4/7*a*c^3*x^7+1/9*c^4*x^9
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a+cx^2\right)^4\,dx=\frac{1}{9}\,c^4x^9+\frac{4}{7}\,ac^3x^7+\frac{6}{5}\,a^2c^2x^5+\frac{4}{3}\,a^3cx^3+a^4x$$

input
```
integrate((c*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \left(a+cx^2\right)^4\,dx=a^4x+\frac{4a^3cx^3}{3}+\frac{6a^2c^2x^5}{5}+\frac{4ac^3x^7}{7}+\frac{c^4x^9}{9}$$

input
```
integrate((c*x**2+a)**4,x)
```

output
```
a**4*x + 4*a**3*c*x**3/3 + 6*a**2*c**2*x**5/5 + 4*a*c**3*x**7/7 + c**4*x**
9/9
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a+cx^2\right)^4 \, dx = \frac{1}{9}c^4x^9 + \frac{4}{7}ac^3x^7 + \frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3cx^3 + a^4x$$

input
```
integrate((c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a+cx^2\right)^4 \, dx = \frac{1}{9}c^4x^9 + \frac{4}{7}ac^3x^7 + \frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3cx^3 + a^4x$$

input
```
integrate((c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a+cx^2\right)^4 \, dx = a^4\,x + \frac{4\,a^3\,c\,x^3}{3} + \frac{6\,a^2\,c^2\,x^5}{5} + \frac{4\,a\,c^3\,x^7}{7} + \frac{c^4\,x^9}{9}$$

input
```
int((a + c*x^2)^4,x)
```

output
```
a^4*x + (c^4*x^9)/9 + (4*a^3*c*x^3)/3 + (4*a*c^3*x^7)/7 + (6*a^2*c^2*x^5)/
5
```

3.50 $\int (a+cx^2)^3 \, dx$

3.50.1 Optimal result

Integrand size = 9, antiderivative size = 35

$$\int (a+cx^2)^3 \, dx = a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

output
```
a^3*x+a^2*c*x^3+3/5*a*c^2*x^5+1/7*c^3*x^7
```

3.50.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+cx^2)^3 \, dx = a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

input
```
Integrate[(a + c*x^2)^3,x]
```

output
```
a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

3.50.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 210}$$

$$\int \left(a^3+3a^2cx^2+3ac^2x^4+c^3x^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}$$

input
```
Int[(a + c*x^2)^3,x]
```

output
```
a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

3.50.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.50.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
gosper	$a^3x + a^2c\,x^3 + \frac{3}{5}a\,c^2x^5 + \frac{1}{7}c^3x^7$	32
default	$a^3x + a^2c\,x^3 + \frac{3}{5}a\,c^2x^5 + \frac{1}{7}c^3x^7$	32
norman	$a^3x + a^2c\,x^3 + \frac{3}{5}a\,c^2x^5 + \frac{1}{7}c^3x^7$	32
risch	$a^3x + a^2c\,x^3 + \frac{3}{5}a\,c^2x^5 + \frac{1}{7}c^3x^7$	32
parallelrisch	$a^3x + a^2c\,x^3 + \frac{3}{5}a\,c^2x^5 + \frac{1}{7}c^3x^7$	32

input
```
int((c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+a^2*c*x^3+3/5*a*c^2*x^5+1/7*c^3*x^7
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a+cx^2)^3\,dx = \frac{1}{7}\,c^3x^7 + \frac{3}{5}\,ac^2x^5 + a^2cx^3 + a^3x$$

input
```
integrate((c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

3.50.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (a+cx^2)^3\,dx = a^3x + a^2cx^3 + \frac{3ac^2x^5}{5} + \frac{c^3x^7}{7}$$

input
```
integrate((c*x**2+a)**3,x)
```

output
```
a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a + cx^2)^3 \, dx = \frac{1}{7} c^3 x^7 + \frac{3}{5} a c^2 x^5 + a^2 c x^3 + a^3 x$$

input
```
integrate((c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a + cx^2)^3 \, dx = \frac{1}{7} c^3 x^7 + \frac{3}{5} a c^2 x^5 + a^2 c x^3 + a^3 x$$

input
```
integrate((c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a + cx^2)^3 \, dx = a^3 \, x + a^2 \, c \, x^3 + \frac{3 \, a \, c^2 \, x^5}{5} + \frac{c^3 \, x^7}{7}$$

input
```
int((a + c*x^2)^3,x)
```

output
```
a^3*x + (c^3*x^7)/7 + a^2*c*x^3 + (3*a*c^2*x^5)/5
```

3.51 $\int (a+cx^2)^2 \, dx$

3.51.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a+cx^2)^2 \, dx = a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

output
```
a^2*x+2/3*a*c*x^3+1/5*c^2*x^5
```

3.51.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+cx^2)^2 \, dx = a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

input
```
Integrate[(a + c*x^2)^2,x]
```

output
```
a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

3.51.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 dx$$

$$\Big\downarrow \; 210$$

$$\int \left(a^2+2acx^2+c^2x^4\right) dx$$

$$\Big\downarrow \; 2009$$

$$a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}$$

input
```
Int[(a + c*x^2)^2,x]
```

output
```
a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

3.51.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.51.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}x^5c^2$	22
default	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}x^5c^2$	22
norman	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}x^5c^2$	22
risch	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}x^5c^2$	22
parallelrisch	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}x^5c^2$	22

input
```
int((c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*c*x^3+1/5*x^5*c^2
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^2\right)^2\,dx=\frac{1}{5}\,c^2x^5+\frac{2}{3}\,acx^3+a^2x$$

input
```
integrate((c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(a+cx^2\right)^2\,dx=a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}$$

input
```
integrate((c*x**2+a)**2,x)
```

output
```
a**2*x + 2*a*c*x**3/3 + c**2*x**5/5
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^2\right)^2 \, dx = \frac{1}{5}\, c^2 x^5 + \frac{2}{3}\, acx^3 + a^2 x$$

input
```
integrate((c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^2\right)^2 \, dx = \frac{1}{5}\, c^2 x^5 + \frac{2}{3}\, acx^3 + a^2 x$$

input
```
integrate((c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^2\right)^2 \, dx = a^2\, x + \frac{2\, a\, c\, x^3}{3} + \frac{c^2\, x^5}{5}$$

input
```
int((a + c*x^2)^2,x)
```

output
```
a^2*x + (c^2*x^5)/5 + (2*a*c*x^3)/3
```

3.52 $\int (a+cx^2)\,dx$

3.52.1 Optimal result

Integrand size = 7, antiderivative size = 12

$$\int (a+cx^2)\,dx = ax+\frac{cx^3}{3}$$

output
```
a*x+1/3*c*x^3
```

3.52.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+cx^2)\,dx = ax+\frac{cx^3}{3}$$

input
```
Integrate[a + c*x^2,x]
```

output
```
a*x + (c*x^3)/3
```

3.52.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + cx^2)\, dx$$

$$\Big\downarrow \; 2009$$

$$ax + \frac{cx^3}{3}$$

input
```
Int[a + c*x^2,x]
```

output
```
a*x + (c*x^3)/3
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.52.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$ax + \frac{1}{3} c\, x^3$	11
default	$ax + \frac{1}{3} c\, x^3$	11
norman	$ax + \frac{1}{3} c\, x^3$	11
risch	$ax + \frac{1}{3} c\, x^3$	11
parallelrisch	$ax + \frac{1}{3} c\, x^3$	11
parts	$ax + \frac{1}{3} c\, x^3$	11

input
```
int(c*x^2+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*c*x^3
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + cx^2) \, dx = \frac{1}{3} cx^3 + ax$$

input
```
integrate(c*x^2+a,x, algorithm="fricas")
```

output
```
1/3*c*x^3 + a*x
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int (a + cx^2) \, dx = ax + \frac{cx^3}{3}$$

input
```
integrate(c*x**2+a,x)
```

output
```
a*x + c*x**3/3
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + cx^2) \, dx = \frac{1}{3} cx^3 + ax$$

input
```
integrate(c*x^2+a,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + a*x
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + cx^2) \, dx = \frac{1}{3}cx^3 + ax$$

input
```
integrate(c*x^2+a,x, algorithm="giac")
```

output
```
1/3*c*x^3 + a*x
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + cx^2) \, dx = \frac{c\,x^3}{3} + a\,x$$

input
```
int(a + c*x^2,x)
```

output
```
a*x + (c*x^3)/3
```

3.53 $\int \frac{1}{a+cx^2} dx$

3.53.1 Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+cx^2} dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

output
```
arctan(x*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+cx^2} dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

input
```
Integrate[(a + c*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

3.53.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+cx^2}\,dx$$
$$\Big\downarrow \; 218$$
$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

input
```
Int[(a + c*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

3.53.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.53.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$	16
risch	$-\frac{\ln\left(cx+\sqrt{-ac}\right)}{2\sqrt{-ac}}+\frac{\ln\left(-cx+\sqrt{-ac}\right)}{2\sqrt{-ac}}$	41

input
```
int(1/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{a+cx^2}\,dx = \left[-\frac{\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)}{2\,ac}, \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{ac}\right]$$

input
```
integrate(1/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))/(a*c), sqrt
(a*c)*arctan(sqrt(a*c)*x/a)/(a*c)]
```

3.53.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{a+cx^2}\,dx = -\frac{\sqrt{-\frac{1}{ac}}\log\left(-a\sqrt{-\frac{1}{ac}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ac}}\log\left(a\sqrt{-\frac{1}{ac}}+x\right)}{2}$$

input
```
integrate(1/(c*x**2+a),x)
```

output
```
-sqrt(-1/(a*c))*log(-a*sqrt(-1/(a*c)) + x)/2 + sqrt(-1/(a*c))*log(a*sqrt(-
1/(a*c)) + x)/2
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+cx^2}\,dx = \frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

input
```
integrate(1/(c*x^2+a),x, algorithm="maxima")
```

output
```
arctan(c*x/sqrt(a*c))/sqrt(a*c)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+cx^2}\,dx=\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

input
```
integrate(1/(c*x^2+a),x, algorithm="giac")
```

output
```
arctan(c*x/sqrt(a*c))/sqrt(a*c)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+cx^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

input
```
int(1/(a + c*x^2),x)
```

output
```
atan((c^(1/2)*x)/a^(1/2))/(a^(1/2)*c^(1/2))
```

3.54 $\int \frac{1}{(a+cx^2)^2}\,dx$

3.54.1 Optimal result

Integrand size = 9, antiderivative size = 45

$$\int \frac{1}{(a+cx^2)^2}\,dx = \frac{x}{2a(a+cx^2)} + \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

output
```
1/2*x/a/(c*x^2+a)+1/2*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+cx^2)^2}\,dx = \frac{x}{2a(a+cx^2)} + \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

input
```
Integrate[(a + c*x^2)^(-2),x]
```

output
```
x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

3.54.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^2} \, dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\int \frac{1}{cx^2+a}dx}{2a} + \frac{x}{2a\left(a+cx^2\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{x}{2a\left(a+cx^2\right)}$$

input
```
Int[(a + c*x^2)^(-2),x]
```

output
```
x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

3.54.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.54.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(c\,x^{2}+a)}+\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2a\sqrt{ac}}$	36
risch	$\frac{x}{2a(c\,x^{2}+a)}-\frac{\ln(cx+\sqrt{-ac})}{4\sqrt{-ac}\,a}+\frac{\ln(-cx+\sqrt{-ac})}{4\sqrt{-ac}\,a}$	62

input
```
int(1/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+cx^2)^2}\,dx = \left[\frac{2\,acx-(cx^2+a)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{4\,(a^2c^2x^2+a^3c)}, \frac{acx+(cx^2+a)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2\,(a^2c^2x^2+a^3c)}\right]$$

input
```
integrate(1/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*c*x - (c*x^2 + a)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c
*x^2 + a)))/(a^2*c^2*x^2 + a^3*c), 1/2*(a*c*x + (c*x^2 + a)*sqrt(a*c)*arct
an(sqrt(a*c)*x/a))/(a^2*c^2*x^2 + a^3*c)]
```

3.54.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+cx^2)^2}\,dx = \frac{x}{2a^2+2acx^2} - \frac{\sqrt{-\frac{1}{a^3c}}\log\left(-a^2\sqrt{-\frac{1}{a^3c}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c}}\log\left(a^2\sqrt{-\frac{1}{a^3c}}+x\right)}{4}$$

input
```
integrate(1/(c*x**2+a)**2,x)
```

output
```
x/(2*a**2 + 2*a*c*x**2) - sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c)) +
x)/4 + sqrt(-1/(a**3*c))*log(a**2*sqrt(-1/(a**3*c)) + x)/4
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+cx^2)^2}\,dx = \frac{x}{2\,(acx^2+a^2)} + \frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}a}$$

input
```
integrate(1/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*x/(a*c*x^2 + a^2) + 1/2*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+cx^2)^2}\,dx = \frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}a} + \frac{x}{2\,(cx^2+a)a}$$

input
```
integrate(1/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*x/((c*x^2 + a)*a)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+cx^2)^2}\,dx = \frac{x}{2\,a\,(c\,x^2+a)} + \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}}$$

input
```
int(1/(a + c*x^2)^2,x)
```

output
```
x/(2*a*(a + c*x^2)) + atan((c^(1/2)*x)/a^(1/2))/(2*a^(3/2)*c^(1/2))
```

3.55 $\int \frac{1}{(a+cx^2)^3} dx$

3.55.1 Optimal result

Integrand size = 9, antiderivative size = 62

$$\int \frac{1}{(a+cx^2)^3} dx = \frac{x}{4a(a+cx^2)^2} + \frac{3x}{8a^2(a+cx^2)} + \frac{3\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

output
```
1/4*x/a/(c*x^2+a)^2+3/8*x/a^2/(c*x^2+a)+3/8*arctan(x*c^(1/2)/a^(1/2))/a^(5
/2)/c^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+cx^2)^3} dx = \frac{5ax+3cx^3}{8a^2(a+cx^2)^2} + \frac{3\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

input
```
Integrate[(a + c*x^2)^(-3),x]
```

output
```
(5*a*x + 3*c*x^3)/(8*a^2*(a + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[c])
```

3.55.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^3}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{1}{(cx^2+a)^2}dx}{4a}+\frac{x}{4a(a+cx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a(a+cx^2)}\right)}{4a}+\frac{x}{4a(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a(a+cx^2)}\right)}{4a}+\frac{x}{4a(a+cx^2)^2}$$

input
```
Int[(a + c*x^2)^(-3),x]
```

output
```
x/(4*a*(a + c*x^2)^2) + (3*(x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[
a]]/(2*a^(3/2)*Sqrt[c])))/(4*a)
```

3.55.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.55.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x}{4a(c\,x^2+a)^2}+\frac{\frac{3x}{8a(c\,x^2+a)}+\frac{3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a\sqrt{ac}}}{a}$	57
risch	$\frac{\frac{3c\,x^3}{8a^2}+\frac{5x}{8a}}{(c\,x^2+a)^2}-\frac{3\ln\left(cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^2}+\frac{3\ln\left(-cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^2}$	73

input
```
int(1/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(c*x^2+a)^2+3/4/a*(1/2*x/a/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(c*x/
(a*c)^(1/2)))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.03

$$\int \frac{1}{(a+cx^2)^3}\,dx$$
$$=\left[\frac{6\,ac^2x^3+10\,a^2cx-3\,(c^2x^4+2\,acx^2+a^2)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)}{16\,(a^3c^3x^4+2\,a^4c^2x^2+a^5c)},\ \frac{3\,ac^2x^3+5\,a^2cx+3\,(c^2x^4+2\,acx\right.$$
$$8\,(a^3c^3x^4+2\,a^4c^2$$

input
```
integrate(1/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*a*c^2*x^3 + 10*a^2*c*x - 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(-a*c)
*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^3*c^3*x^4 + 2*a^4*c^2*x
^2 + a^5*c), 1/8*(3*a*c^2*x^3 + 5*a^2*c*x + 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*
sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c)]
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{1}{(a+cx^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16} + \frac{5ax+3cx^3}{8a^4+16a^3cx^2+8a^2c^2x^4}$$

input
```
integrate(1/(c*x**2+a)**3,x)
```

output
```
-3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt(-1/(a**5
*c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16 + (5*a*x + 3*c*x**3)/(8*a**4 + 16*
a**3*c*x**2 + 8*a**2*c**2*x**4)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+cx^2)^3}\,dx = \frac{3\,cx^3+5\,ax}{8\,(a^2c^2x^4+2\,a^3cx^2+a^4)} + \frac{3\,\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2}$$

input
```
integrate(1/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*c*x^3 + 5*a*x)/(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4) + 3/8*arctan(c*x/s
qrt(a*c))/(sqrt(a*c)*a^2)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+cx^2)^3}\,dx = \frac{3\,\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2} + \frac{3\,cx^3+5\,ax}{8\,(cx^2+a)^2a^2}$$

input
```
integrate(1/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/8*(3*c*x^3 + 5*a*x)/((c*x^2
+ a)^2*a^2)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+cx^2)^3}\,dx = \frac{\frac{5\,x}{8\,a}+\frac{3\,c\,x^3}{8\,a^2}}{a^2+2\,a\,c\,x^2+c^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}}$$

input
```
int(1/(a + c*x^2)^3,x)
```

output
```
((5*x)/(8*a) + (3*c*x^3)/(8*a^2))/(a^2 + c^2*x^4 + 2*a*c*x^2) + (3*atan((c
^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2))
```

3.56 $\int (a+cx^2)^{5/2}\,dx$

3.56.1 Optimal result

Integrand size = 11, antiderivative size = 84

$$\int (a+cx^2)^{5/2}\,dx = \frac{5}{16}a^2x\sqrt{a+cx^2} + \frac{5}{24}ax(a+cx^2)^{3/2} + \frac{1}{6}x(a+cx^2)^{5/2} + \frac{5a^3\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}$$

output
```
5/24*a*x*(c*x^2+a)^(3/2)+1/6*x*(c*x^2+a)^(5/2)+5/16*a^3*arctanh(x*c^(1/2)/
(c*x^2+a)^(1/2))/c^(1/2)+5/16*a^2*x*(c*x^2+a)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int (a+cx^2)^{5/2}\,dx = \frac{1}{48}\sqrt{a+cx^2}(33a^2x+26acx^3+8c^2x^5) - \frac{5a^3\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{16\sqrt{c}}$$

input
```
Integrate[(a + c*x^2)^(5/2),x]
```

output
```
(Sqrt[a + c*x^2]*(33*a^2*x + 26*a*c*x^3 + 8*c^2*x^5))/48 - (5*a^3*Log[-(Sq
rt[c]*x) + Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

3.56.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^{5/2} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\int \left(cx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{cx^2+a}dx+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

input
```
Int[(a + c*x^2)^(5/2),x]
```

output
```
(x*(a + c*x^2)^(5/2))/6 + (5*a*((x*(a + c*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
+ c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])))/4))/6
```

3.56.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.56.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{x\left(8x^{4}c^{2}+26ac\,x^{2}+33a^{2}\right)\sqrt{c\,x^{2}+a}}{48}+\frac{5a^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{16\sqrt{c}}$	59
pseudoelliptic	$\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{2}+a}}{x\sqrt{c}}\right)a^{3}}{16\sqrt{c}}+\frac{11\left(\frac{8c^{\frac{5}{2}}x^{4}}{33}+\frac{26a\,c^{\frac{3}{2}}x^{2}}{33}+a^{2}\sqrt{c}\right)x\sqrt{c\,x^{2}+a}}{16\sqrt{c}}$	67
default	$\frac{x\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{6}$	68

input
```
int((c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*c^2*x^4+26*a*c*x^2+33*a^2)*(c*x^2+a)^(1/2)+5/16*a^3*ln(c^(1/2)*x
+(c*x^2+a)^(1/2))/c^(1/2)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int (a+cx^2)^{5/2}\,dx = \left[\frac{15\,a^3\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\,(8\,c^3x^5+26\,ac^2x^3+33\,a^2cx)\sqrt{cx^2+a}}{96\,c}, -\frac{15\,a^3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(8\,c^3x^5+26\,ac^2x^3+33\,a^2cx)\sqrt{cx^2+a}}{48\,c}\right]$$

input
```
integrate((c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(15*a^3*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*
(8*c^3*x^5 + 26*a*c^2*x^3 + 33*a^2*c*x)*sqrt(c*x^2 + a))/c, -1/48*(15*a^3*
sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (8*c^3*x^5 + 26*a*c^2*x^3 +
33*a^2*c*x)*sqrt(c*x^2 + a))/c]
```

3.56.6 Sympy [A] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int (a+cx^2)^{5/2}\,dx = \frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}cx^3\sqrt{1+\frac{cx^2}{a}}}{24}+\frac{\sqrt{a}c^2x^5\sqrt{1+\frac{cx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16\sqrt{c}}$$

input
```
integrate((c*x**2+a)**(5/2),x)
```

output
```
11*a**(5/2)*x*sqrt(1 + c*x**2/a)/16 + 13*a**(3/2)*c*x**3*sqrt(1 + c*x**2/a
)/24 + sqrt(a)*c**2*x**5*sqrt(1 + c*x**2/a)/6 + 5*a**3*asinh(sqrt(c)*x/sqr
t(a))/(16*sqrt(c))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \left(a+cx^2\right)^{5/2} dx = \frac{1}{6}\left(cx^2+a\right)^{\frac{5}{2}}x + \frac{5}{24}\left(cx^2+a\right)^{\frac{3}{2}}ax + \frac{5}{16}\sqrt{cx^2+a}a^2x + \frac{5\,a^3\,\text{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{c}}$$

input
```
integrate((c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(c*x^2 + a)^(5/2)*x + 5/24*(c*x^2 + a)^(3/2)*a*x + 5/16*sqrt(c*x^2 + a
)*a^2*x + 5/16*a^3*arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \left(a+cx^2\right)^{5/2} dx = -\frac{5\,a^3\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,\sqrt{c}} + \frac{1}{48}\left(2\left(4\,c^2x^2+13\,ac\right)x^2+33\,a^2\right)\sqrt{cx^2+a}x$$

input
```
integrate((c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/16*a^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/48*(2*(4*c^2*
x^2 + 13*a*c)*x^2 + 33*a^2)*sqrt(c*x^2 + a)*x
```

3.56.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \left(a+cx^2\right)^{5/2} dx = \frac{x\left(c\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{c\,x^2}{a}\right)}{\left(\frac{c\,x^2}{a}+1\right)^{5/2}}$$

input
```
int((a + c*x^2)^(5/2),x)
```

output
```
(x*(a + c*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(c*x^2)/a))/((c*x^2)/a +
 1)^(5/2)
```

3.57 $\int (a+cx^2)^{3/2}\,dx$

3.57.1 Optimal result

Integrand size = 11, antiderivative size = 65

$$\int (a+cx^2)^{3/2}\,dx=\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x(a+cx^2)^{3/2}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}$$

output
```
1/4*x*(c*x^2+a)^(3/2)+3/8*a^2*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(1/2)+3
/8*a*x*(c*x^2+a)^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (a+cx^2)^{3/2}\,dx=\frac{1}{8}x\sqrt{a+cx^2}(5a+2cx^2)-\frac{3a^2\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{8\sqrt{c}}$$

input
```
Integrate[(a + c*x^2)^(3/2),x]
```

output
```
(x*Sqrt[a + c*x^2]*(5*a + 2*c*x^2))/8 - (3*a^2*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2]])/(8*Sqrt[c])
```

3.57.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}a\int \sqrt{cx^2+a}dx+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

input
```
Int[(a + c*x^2)^(3/2),x]
```

output
```
(x*(a + c*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])))/4
```

3.57.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.57.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{x\left(2c\,x^{2}+5a\right)\sqrt{c\,x^{2}+a}}{8}+\frac{3a^{2}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{8\sqrt{c}}$	48
default	$\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4}$	52
pseudoelliptic	$\frac{2\sqrt{c\,x^{2}+a}\,c^{\frac{3}{2}}x^{3}+5ax\sqrt{c\,x^{2}+a}\,\sqrt{c}+3\,\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{2}+a}}{x\sqrt{c}}\right)a^{2}}{8\sqrt{c}}$	62

input
```
int((c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*c*x^2+5*a)*(c*x^2+a)^(1/2)+3/8*a^2*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/
c^(1/2)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (a + cx^2)^{3/2}\,dx = \left[\frac{3\,a^2\sqrt{c}\log\left(-2\,cx^2 - 2\,\sqrt{cx^2+a}\sqrt{c}x - a\right) + 2\,(2\,c^2x^3 + 5\,acx)\sqrt{cx^2+a}}{16\,c}, -\frac{3\,a^2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right) - (2\,c^2x^3 + 5\,acx)\sqrt{cx^2+a}}{8\,c}\right]$$

input
```
integrate((c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*a^2*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(
2*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c, -1/8*(3*a^2*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) - (2*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c]
```

3.57.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int (a + cx^2)^{3/2}\,dx = \frac{5a^{\frac{3}{2}}x\sqrt{1 + \frac{cx^2}{a}}}{8} + \frac{\sqrt{a}cx^3\sqrt{1 + \frac{cx^2}{a}}}{4} + \frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8\sqrt{c}}$$

input
```
integrate((c*x**2+a)**(3/2),x)
```

output
```
5*a**(3/2)*x*sqrt(1 + c*x**2/a)/8 + sqrt(a)*c*x**3*sqrt(1 + c*x**2/a)/4 +
3*a**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \left(a+cx^2\right)^{3/2} dx = \frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{cx^2+a}ax + \frac{3\,a^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}}$$

input
```
integrate((c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(c*x^2 + a)^(3/2)*x + 3/8*sqrt(c*x^2 + a)*a*x + 3/8*a^2*arcsinh(c*x/sq
rt(a*c))/sqrt(c)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \left(a+cx^2\right)^{3/2} dx = \frac{1}{8}\left(2\,cx^2+5\,a\right)\sqrt{cx^2+a}x - \frac{3\,a^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\sqrt{c}}$$

input
```
integrate((c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*(2*c*x^2 + 5*a)*sqrt(c*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(c)*x + sqrt(
c*x^2 + a)))/sqrt(c)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

$$\int \left(a+cx^2\right)^{3/2} dx = \frac{x\left(c\,x^2+a\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{c\,x^2}{a}\right)}{\left(\frac{c\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + c*x^2)^(3/2),x)
```

output
```
(x*(a + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(c*x^2)/a))/((c*x^2)/a +
 1)^(3/2)
```

3.58 $\int \sqrt{a+cx^2}\,dx$

3.58.1 Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt{a+cx^2}\,dx = \frac{1}{2}x\sqrt{a+cx^2} + \frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

output
```
1/2*a*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(1/2)+1/2*x*(c*x^2+a)^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sqrt{a+cx^2}\,dx = \frac{1}{2}x\sqrt{a+cx^2} - \frac{a\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{2\sqrt{c}}$$

input
```
Integrate[Sqrt[a + c*x^2],x]
```

output
```
(x*Sqrt[a + c*x^2])/2 - (a*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*Sqrt[c]
)
```

3.58.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}a\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}$$

input
```
Int[Sqrt[a + c*x^2],x]
```

output
```
(x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c
])
```

3.58.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.58.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}$	36
risch	$\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}$	36
pseudoelliptic	$\frac{\sqrt{c\,x^{2}+a}\,x\sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{2}+a}}{x\sqrt{c}}\right)a}{2\sqrt{c}}$	40

input
```
int((c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int \sqrt{a+cx^2}\,dx$$
$$=\left[\frac{2\sqrt{cx^2+a}cx+a\sqrt{c}\log\left(-2\,cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)}{4\,c},\frac{\sqrt{cx^2+a}cx-a\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{2\,c}\right]$$

input
```
integrate((c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*x^2 + a)*c*x + a*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*s
qrt(c)*x - a))/c, 1/2*(sqrt(c*x^2 + a)*c*x - a*sqrt(-c)*arctan(sqrt(-c)*x/
sqrt(c*x^2 + a)))/c]
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \sqrt{a+cx^2}\,dx = \frac{\sqrt{a}x\sqrt{1+\frac{cx^2}{a}}}{2} + \frac{a\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}}$$

input
```
integrate((c*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*x*sqrt(1 + c*x**2/a)/2 + a*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c))
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \sqrt{a+cx^2}\,dx = \frac{1}{2}\sqrt{cx^2+a}x + \frac{a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}$$

input
```
integrate((c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + a)*x + 1/2*a*arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt{a+cx^2}\,dx = \frac{1}{2}\sqrt{cx^2+a}x - \frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}$$

input
```
integrate((c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + a)*x - 1/2*a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(
c)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sqrt{a+cx^2}\,dx = \frac{x\sqrt{c\,x^2+a}}{2} + \frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}$$

input
```
int((a + c*x^2)^(1/2),x)
```

output
```
(x*(a + c*x^2)^(1/2))/2 + (a*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/(2*c^(1/2
))
```

3.59 $\int \frac{1}{\sqrt{a+cx^2}} dx$

3.59.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{\sqrt{a+cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

output
```
arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

input
```
Integrate[1/Sqrt[a + c*x^2],x]
```

output
```
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

3.59.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}}\,dx$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{cx^2}{a+cx^2}}d\frac{x}{\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[a + c*x^2],x]
```

output
```
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

3.59.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.59.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{\sqrt{c}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{2}+a}}{x\sqrt{c}}\right)}{\sqrt{c}}$	22

input
```
int(1/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+cx^2}}\,dx = \left[\frac{\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{c}\right]$$

input
```
integrate(1/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)/sqrt(c), -sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a))/c]
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+cx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}}$$

input
```
integrate(1/(c*x**2+a)**(1/2),x)
```

output
```
asinh(sqrt(c)*x/sqrt(a))/sqrt(c)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+cx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}$$

input
```
integrate(1/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+cx^2}}\,dx = \frac{1}{2}\sqrt{cx^2+a}x - \frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}$$

input
```
integrate(1/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + a)*x - 1/2*a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(
c)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+cx^2}}\,dx = \frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}$$

input
```
int(1/(a + c*x^2)^(1/2),x)
```

output
```
log(c^(1/2)*x + (a + c*x^2)^(1/2))/c^(1/2)
```

3.60 $\int \frac{1}{(a+cx^2)^{3/2}}\,dx$

3.60.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{(a+cx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{a+cx^2}}$$

output
```
x/a/(c*x^2+a)^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+cx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{a+cx^2}}$$

input
```
Integrate[(a + c*x^2)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + c*x^2])
```

3.60.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 208}$$

$$\frac{x}{a\sqrt{a+cx^2}}$$

input
```
Int[(a + c*x^2)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + c*x^2])
```

3.60.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

3.60.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x}{a\sqrt{c\,x^{2}+a}}$	15
default	$\frac{x}{a\sqrt{c\,x^{2}+a}}$	15
trager	$\frac{x}{a\sqrt{c\,x^{2}+a}}$	15
pseudoelliptic	$\frac{x}{a\sqrt{c\,x^{2}+a}}$	15

input
```
int(1/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/a/(c*x^2+a)^(1/2)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+cx^2)^{3/2}}\,dx = \frac{\sqrt{cx^2+a}\,x}{acx^2+a^2}$$

input
```
integrate(1/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2 + a)*x/(a*c*x^2 + a^2)
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+cx^2)^{3/2}}\,dx = \frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

input
```
integrate(1/(c*x**2+a)**(3/2),x)
```

output
```
x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+cx^2)^{3/2}}\,dx = \frac{x}{\sqrt{cx^2+a}\,a}$$

input
```
integrate(1/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
x/(sqrt(c*x^2 + a)*a)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+cx^2)^{3/2}}\,dx = \frac{x}{\sqrt{cx^2+a}a}$$

input
```
integrate(1/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
x/(sqrt(c*x^2 + a)*a)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+cx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{c\,x^2+a}}$$

input
```
int(1/(a + c*x^2)^(3/2),x)
```

output
```
x/(a*(a + c*x^2)^(1/2))
```

3.61 $\int \frac{1}{(a+cx^2)^{5/2}} dx$

3.61.1 Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(a+cx^2)^{5/2}} dx = \frac{x}{3a(a+cx^2)^{3/2}} + \frac{2x}{3a^2\sqrt{a+cx^2}}$$

output
```
1/3*x/a/(c*x^2+a)^(3/2)+2/3*x/a^2/(c*x^2+a)^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+cx^2)^{5/2}} dx = \frac{3ax+2cx^3}{3a^2(a+cx^2)^{3/2}}$$

input
```
Integrate[(a + c*x^2)^(-5/2),x]
```

output
```
(3*a*x + 2*c*x^3)/(3*a^2*(a + c*x^2)^(3/2))
```

3.61.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 209$$

$$\frac{2\int \frac{1}{(cx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}$$

input
```
Int[(a + c*x^2)^(-5/2),x]
```

output
```
x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])
```

3.61.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

3.61.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x(2cx^2+3a)}{3(cx^2+a)^{\frac{3}{2}}a^2}$	26
trager	$\frac{x(2cx^2+3a)}{3(cx^2+a)^{\frac{3}{2}}a^2}$	26
pseudoelliptic	$\frac{x(2cx^2+3a)}{3(cx^2+a)^{\frac{3}{2}}a^2}$	26
default	$\frac{x}{3a(cx^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{cx^2+a}}$	32

input
```
int(1/(c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*c*x^2+3*a)/(c*x^2+a)^(3/2)/a^2
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+cx^2)^{5/2}}\,dx=\frac{(2\,cx^3+3\,ax)\sqrt{cx^2+a}}{3\,(a^2c^2x^4+2\,a^3cx^2+a^4)}$$

input
```
integrate(1/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*c*x^3 + 3*a*x)*sqrt(c*x^2 + a)/(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4)
```

3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(32) = 64.

Time = 0.49 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a+cx^2)^{5/2}}\,dx=\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}+\frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}$$

input
```
integrate(1/(c*x**2+a)**(5/2),x)
```

output
```
3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2/a
)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 +
c*x**2/a))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+cx^2)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{cx^2+a}a^2} + \frac{x}{3\,(cx^2+a)^{\frac{3}{2}}a}$$

input
```
integrate(1/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/(sqrt(c*x^2 + a)*a^2) + 1/3*x/((c*x^2 + a)^(3/2)*a)
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+cx^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,cx^2}{a^2}+\frac{3}{a}\right)}{3\,(cx^2+a)^{\frac{3}{2}}}$$

input
```
integrate(1/(c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*c*x^2/a^2 + 3/a)/(c*x^2 + a)^(3/2)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a+cx^2)^{5/2}}\,dx = \frac{2\,x\,(c\,x^2+a)+a\,x}{3\,a^2\,(c\,x^2+a)^{3/2}}$$

input
```
int(1/(a + c*x^2)^(5/2),x)
```

output
```
(2*x*(a + c*x^2) + a*x)/(3*a^2*(a + c*x^2)^(3/2))
```

3.62 $\int \frac{1}{(a+cx^2)^{7/2}}\,dx$

3.62.1 Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{1}{(a+cx^2)^{7/2}}\,dx = \frac{x}{5a(a+cx^2)^{5/2}} + \frac{4x}{15a^2(a+cx^2)^{3/2}} + \frac{8x}{15a^3\sqrt{a+cx^2}}$$

output
```
1/5*x/a/(c*x^2+a)^(5/2)+4/15*x/a^2/(c*x^2+a)^(3/2)+8/15*x/a^3/(c*x^2+a)^(1
/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+cx^2)^{7/2}}\,dx = \frac{15a^2x + 20acx^3 + 8c^2x^5}{15a^3(a+cx^2)^{5/2}}$$

input
```
Integrate[(a + c*x^2)^(-7/2),x]
```

output
```
(15*a^2*x + 20*a*c*x^3 + 8*c^2*x^5)/(15*a^3*(a + c*x^2)^(5/2))
```

3.62.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{209}$$

$$\frac{4\int \frac{1}{(cx^2+a)^{5/2}}dx}{5a}+\frac{x}{5a(a+cx^2)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4\left(\frac{2\int \frac{1}{(cx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a(a+cx^2)^{3/2}}\right)}{5a}+\frac{x}{5a(a+cx^2)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{4\left(\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a(a+cx^2)^{3/2}}\right)}{5a}+\frac{x}{5a(a+cx^2)^{5/2}}$$

input
```
Int[(a + c*x^2)^(-7/2),x]
```

output
```
x/(5*a*(a + c*x^2)^(5/2)) + (4*(x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*S
qrt[a + c*x^2])))/(5*a)
```

3.62.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

3.62.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(8x^4c^2+20acx^2+15a^2)}{15(cx^2+a)^{\frac{5}{2}}a^3}$	37
trager	$\frac{x(8x^4c^2+20acx^2+15a^2)}{15(cx^2+a)^{\frac{5}{2}}a^3}$	37
pseudoelliptic	$\frac{x(8x^4c^2+20acx^2+15a^2)}{15(cx^2+a)^{\frac{5}{2}}a^3}$	37
default	$\frac{x}{5a(cx^2+a)^{\frac{5}{2}}}+\frac{\frac{4x}{15a(cx^2+a)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{cx^2+a}}}{a}$	53

input
```
int(1/(c*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(8*c^2*x^4+20*a*c*x^2+15*a^2)/(c*x^2+a)^(5/2)/a^3
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{(a+cx^2)^{7/2}}\,dx = \frac{(8\,c^2x^5+20\,acx^3+15\,a^2x)\sqrt{cx^2+a}}{15\,(a^3c^3x^6+3\,a^4c^2x^4+3\,a^5cx^2+a^6)}$$

input
```
integrate(1/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(8*c^2*x^5 + 20*a*c*x^3 + 15*a^2*x)*sqrt(c*x^2 + a)/(a^3*c^3*x^6 + 3*
a^4*c^2*x^4 + 3*a^5*c*x^2 + a^6)
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(51) = 102$.

Time = 0.80 (sec) , antiderivative size = 413, normalized size of antiderivative = 7.12

$$\int \frac{1}{(a+cx^2)^{7/2}}\,dx = \frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$
$$+\frac{35a^4cx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$
$$+\frac{28a^3c^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$
$$+\frac{8a^2c^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$

input
```
integrate(1/(c*x**2+a)**(7/2),x)
```

output
```
15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 +
c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*
x**6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a)
 + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1
+ c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x*
*5/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/
a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sq
rt(1 + c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45
*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x
**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+cx^2)^{7/2}}\,dx = \frac{8\,x}{15\,\sqrt{cx^2+a}a^3}+\frac{4\,x}{15\,(cx^2+a)^{\frac{3}{2}}a^2}+\frac{x}{5\,(cx^2+a)^{\frac{5}{2}}a}$$

input
```
integrate(1/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
8/15*x/(sqrt(c*x^2 + a)*a^3) + 4/15*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*x/((c*
x^2 + a)^(5/2)*a)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx = \frac{\left(4\,x^2\left(\frac{2\,c^2x^2}{a^3}+\frac{5\,c}{a^2}\right)+\frac{15}{a}\right)x}{15\left(cx^2+a\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(c*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*(4*x^2*(2*c^2*x^2/a^3 + 5*c/a^2) + 15/a)*x/(c*x^2 + a)^(5/2)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx = \frac{8\,x\left(c\,x^2+a\right)^2+3\,a^2\,x+4\,a\,x\left(c\,x^2+a\right)}{15\,a^3\left(c\,x^2+a\right)^{5/2}}$$

input
```
int(1/(a + c*x^2)^(7/2),x)
```

output
```
(8*x*(a + c*x^2)^2 + 3*a^2*x + 4*a*x*(a + c*x^2))/(15*a^3*(a + c*x^2)^(5/2
))
```

3.63 $\int \frac{1}{(a+cx^2)^{9/2}}\,dx$

3.63.1 Optimal result

Integrand size = 11, antiderivative size = 77

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx=\frac{x}{7a\,(a+cx^2)^{7/2}}+\frac{6x}{35a^2\,(a+cx^2)^{5/2}}+\frac{8x}{35a^3\,(a+cx^2)^{3/2}}+\frac{16x}{35a^4\sqrt{a+cx^2}}$$

output
```
1/7*x/a/(c*x^2+a)^(7/2)+6/35*x/a^2/(c*x^2+a)^(5/2)+8/35*x/a^3/(c*x^2+a)^(3
/2)+16/35*x/a^4/(c*x^2+a)^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx=\frac{35a^3x+70a^2cx^3+56ac^2x^5+16c^3x^7}{35a^4\,(a+cx^2)^{7/2}}$$

input
```
Integrate[(a + c*x^2)^(-9/2),x]
```

output
```
(35*a^3*x + 70*a^2*c*x^3 + 56*a*c^2*x^5 + 16*c^3*x^7)/(35*a^4*(a + c*x^2)^
(7/2))
```

3.63.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{209}$$

$$\frac{6\int \frac{1}{(cx^2+a)^{7/2}}dx}{7a}+\frac{x}{7a\,(a+cx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6\left(\frac{4\int \frac{1}{(cx^2+a)^{5/2}}dx}{5a}+\frac{x}{5a(a+cx^2)^{5/2}}\right)}{7a}+\frac{x}{7a\,(a+cx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{(cx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+cx^2)^{5/2}}\right)}{7a}+\frac{x}{7a\,(a+cx^2)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+cx^2)^{5/2}}\right)}{7a}+\frac{x}{7a\,(a+cx^2)^{7/2}}$$

input
```
Int[(a + c*x^2)^(-9/2),x]
```

output
```
x/(7*a*(a + c*x^2)^(7/2)) + (6*(x/(5*a*(a + c*x^2)^(5/2)) + (4*(x/(3*a*(a
+ c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])))/(5*a)))/(7*a)
```

3.63.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

3.63.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x\left(16c^{3}x^{6}+56a\,x^{4}c^{2}+70a^{2}c\,x^{2}+35a^{3}\right)}{35\left(c\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
trager	$\frac{x\left(16c^{3}x^{6}+56a\,x^{4}c^{2}+70a^{2}c\,x^{2}+35a^{3}\right)}{35\left(c\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
pseudoelliptic	$\frac{x\left(16c^{3}x^{6}+56a\,x^{4}c^{2}+70a^{2}c\,x^{2}+35a^{3}\right)}{35\left(c\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
default	$\frac{x}{7a\left(c\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(c\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{c\,x^{2}+a}}\right)}{7a}}{a}$	74

input
```
int(1/(c*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(16*c^3*x^6+56*a*c^2*x^4+70*a^2*c*x^2+35*a^3)/(c*x^2+a)^(7/2)/a^4
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx = \frac{\left(16\,c^3x^7+56\,ac^2x^5+70\,a^2cx^3+35\,a^3x\right)\sqrt{cx^2+a}}{35\,\left(a^4c^4x^8+4\,a^5c^3x^6+6\,a^6c^2x^4+4\,a^7cx^2+a^8\right)}$$

input
```
integrate(1/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(16*c^3*x^7 + 56*a*c^2*x^5 + 70*a^2*c*x^3 + 35*a^3*x)*sqrt(c*x^2 + a)
/(a^4*c^4*x^8 + 4*a^5*c^3*x^6 + 6*a^6*c^2*x^4 + 4*a^7*c*x^2 + a^8)
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(70) = 140$.

Time = 1.23 (sec) , antiderivative size = 1265, normalized size of antiderivative = 16.43

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x**2+a)**(9/2),x)
```

output
```
35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1
+ c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c
**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) +
 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqr
t(1 + c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210
*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*
x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*
x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35
*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37
/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**
(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x
**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x
**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 42
9*a**11*c**3*x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*
sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(
31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x*
*2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x*
*12*sqrt(1 + c*x**2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**
2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*s
qrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a*...
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx = \frac{16\,x}{35\,\sqrt{cx^2+a}a^4} + \frac{8\,x}{35\,(cx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,x}{35\,(cx^2+a)^{\frac{5}{2}}a^2} + \frac{x}{7\,(cx^2+a)^{\frac{7}{2}}a}$$

input
```
integrate(1/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*x/(sqrt(c*x^2 + a)*a^4) + 8/35*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*x/((
c*x^2 + a)^(5/2)*a^2) + 1/7*x/((c*x^2 + a)^(7/2)*a)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx = \frac{\left(2\left(4\,x^2\left(\frac{2\,c^3x^2}{a^4}+\frac{7\,c^2}{a^3}\right)+\frac{35\,c}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\,(cx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(1/(c*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/35*(2*(4*x^2*(2*c^3*x^2/a^4 + 7*c^2/a^3) + 35*c/a^2)*x^2 + 35/a)*x/(c*x^
2 + a)^(7/2)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx = \frac{16\,x}{35\,a^4\,\sqrt{c\,x^2+a}} + \frac{8\,x}{35\,a^3\,(c\,x^2+a)^{3/2}} + \frac{6\,x}{35\,a^2\,(c\,x^2+a)^{5/2}} + \frac{x}{7\,a\,(c\,x^2+a)^{7/2}}$$

input
```
int(1/(a + c*x^2)^(9/2),x)
```

output
```
(16*x)/(35*a^4*(a + c*x^2)^(1/2)) + (8*x)/(35*a^3*(a + c*x^2)^(3/2)) + (6*
x)/(35*a^2*(a + c*x^2)^(5/2)) + x/(7*a*(a + c*x^2)^(7/2))
```

3.64 $\int (4+12x+9x^2)^{3/2}\,dx$

3.64.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx=\frac{1}{12}(2+3x)\left(4+12x+9x^2\right)^{3/2}$$

output
```
1/12*(2+3*x)*(9*x^2+12*x+4)^(3/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx=\frac{1}{12}(2+3x)\left((2+3x)^2\right)^{3/2}$$

input
```
Integrate[(4 + 12*x + 9*x^2)^(3/2),x]
```

output
```
((2 + 3*x)*((2 + 3*x)^2)^(3/2))/12
```

3.64.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(9x^2+12x+4\right)^{3/2}dx$$

$$\Big\downarrow \ 1079$$

$$\frac{\sqrt{9x^2+12x+4}\int (9x+6)^3dx}{27(3x+2)}$$

$$\Big\downarrow \ 17$$

$$\frac{1}{12}(3x+2)^3\sqrt{9x^2+12x+4}$$

input
```
Int[(4 + 12*x + 9*x^2)^(3/2),x]
```

output
```
((2 + 3*x)^3*Sqrt[4 + 12*x + 9*x^2])/12
```

3.64.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.64.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{(2+3x)\left((2+3x)^2\right)^{\frac{3}{2}}}{12}$	17
risch	$\frac{\sqrt{(2+3x)^2}\,(2+3x)^3}{12}$	19
gosper	$\frac{x(27x^3+72x^2+72x+32)\left((2+3x)^2\right)^{\frac{3}{2}}}{4(2+3x)^3}$	35

input
```
int((9*x^2+12*x+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(2+3*x)*((2+3*x)^2)^(3/2)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx=\frac{27}{4}\,x^4+18\,x^3+18\,x^2+8\,x$$

input
```
integrate((9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
```

output
```
27/4*x^4 + 18*x^3 + 18*x^2 + 8*x
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(19) = 38.

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx=4\left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{9x^2+12x+4}$$
$$+12\left(\frac{x^2}{3}+\frac{x}{9}-\frac{2}{27}\right)\sqrt{9x^2+12x+4}+9\sqrt{9x^2+12x+4}\left(\frac{x^3}{4}+\frac{x^2}{18}-\frac{x}{27}+\frac{2}{81}\right)$$

input
```
integrate((9*x**2+12*x+4)**(3/2),x)
```

output
```
4*(x/2 + 1/3)*sqrt(9*x**2 + 12*x + 4) + 12*(x**2/3 + x/9 - 2/27)*sqrt(9*x*
*2 + 12*x + 4) + 9*sqrt(9*x**2 + 12*x + 4)*(x**3/4 + x**2/18 - x/27 + 2/81
)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx = \frac{1}{4}\left(9\,x^2+12\,x+4\right)^{\frac{3}{2}}x+\frac{1}{6}\left(9\,x^2+12\,x+4\right)^{\frac{3}{2}}$$

input
```
integrate((9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(9*x^2 + 12*x + 4)^(3/2)*x + 1/6*(9*x^2 + 12*x + 4)^(3/2)
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx = \frac{3}{4}\left(3\,x^2+4\,x\right)^2\mathrm{sgn}(3\,x+2) + 2\left(3\,x^2+4\,x\right)\mathrm{sgn}(3\,x+2)+\frac{4}{3}\,\mathrm{sgn}(3\,x+2)$$

input
```
integrate((9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

output
```
3/4*(3*x^2 + 4*x)^2*sgn(3*x + 2) + 2*(3*x^2 + 4*x)*sgn(3*x + 2) + 4/3*sgn(
3*x + 2)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(4+12x+9x^2\right)^{3/2} \, dx = \frac{(9\,x+6)\,\left(9\,x^2+12\,x+4\right)^{3/2}}{36}$$

input
```
int((12*x + 9*x^2 + 4)^(3/2),x)
```

output
```
((9*x + 6)*(12*x + 9*x^2 + 4)^(3/2))/36
```

3.65 $\int \sqrt{4+12x+9x^2}\,dx$

3.65.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{4+12x+9x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{4+12x+9x^2}$$

output
```
1/6*(2+3*x)*((2+3*x)^2)^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{4+12x+9x^2}\,dx=\frac{x\sqrt{(2+3x)^2}(4+3x)}{4+6x}$$

input
```
Integrate[Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
(x*Sqrt[(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

3.65.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^2+12x+4}\,dx$$

$$\Big\downarrow \; 1079$$

$$\frac{\sqrt{9x^2+12x+4}\int (9x+6)dx}{3(3x+2)}$$

$$\Big\downarrow \; 17$$

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

input
```
Int[Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6
```

3.65.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.65.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 2.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\operatorname{csgn}(2+3x)(2+3x)^2}{6}$	16
gosper	$\frac{x(4+3x)\sqrt{(2+3x)^2}}{4+6x}$	25
risch	$\frac{3\sqrt{(2+3x)^2}\,x^2}{2(2+3x)}+\frac{2\sqrt{(2+3x)^2}\,x}{2+3x}$	42

input
```
int((9*x^2+12*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*csgn(2+3*x)*(2+3*x)^2
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{3}{2}\,x^2 + 2\,x$$

input
```
integrate((9*x^2+12*x+4)^(1/2),x, algorithm="fricas")
```

output
```
3/2*x^2 + 2*x
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{4+12x+9x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{9x^2+12x+4}$$

input
```
integrate((9*x**2+12*x+4)**(1/2),x)
```

output
```
(x/2 + 1/3)*sqrt(9*x**2 + 12*x + 4)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{1}{2}\sqrt{9\,x^2+12\,x+4}x + \frac{1}{3}\sqrt{9\,x^2+12\,x+4}$$

input
```
integrate((9*x^2+12*x+4)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(9*x^2 + 12*x + 4)*x + 1/3*sqrt(9*x^2 + 12*x + 4)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{1}{2}\left(3\,x^2+4\,x\right)\mathrm{sgn}(3\,x+2) + \frac{2}{3}\,\mathrm{sgn}(3\,x+2)$$

input
```
integrate((9*x^2+12*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/2*(3*x^2 + 4*x)*sgn(3*x + 2) + 2/3*sgn(3*x + 2)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{(3\,x+2)\,\sqrt{9\,x^2+12\,x+4}}{6}$$

input
```
int((12*x + 9*x^2 + 4)^(1/2),x)
```

output
```
((3*x + 2)*(12*x + 9*x^2 + 4)^(1/2))/6
```

3.66 $\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx$

3.66.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{(2+3x)\log(2+3x)}{3\sqrt{4+12x+9x^2}}$$

output
```
1/3*(2+3*x)*ln(2+3*x)/((2+3*x)^2)^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{(2+3x)\log(2+3x)}{3\sqrt{(2+3x)^2}}$$

input
```
Integrate[1/Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[(2 + 3*x)^2])
```

3.66.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9x^2+12x+4}}dx$$

$$\Big\downarrow \ 1079$$

$$\frac{3(3x+2)\int \frac{1}{9x+6}dx}{\sqrt{9x^2+12x+4}}$$

$$\Big\downarrow \ 16$$

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

input
```
Int[1/Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])
```

3.66.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.66.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.31

method	result	size
meijerg	$\frac{\ln\left(1+\frac{3x}{2}\right)}{3}$	9
default	$\frac{(2+3x)\ln(2+3x)}{3\sqrt{(2+3x)^{2}}}$	23
risch	$\frac{\sqrt{(2+3x)^{2}}\,\ln(2+3x)}{9x+6}$	25

input
```
int(1/(9*x^2+12*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1+3/2*x)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{1}{3}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(3*x + 2)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{\left(x+\frac{2}{3}\right)\log\left(x+\frac{2}{3}\right)}{3\sqrt{\left(x+\frac{2}{3}\right)^{2}}}$$

input
```
integrate(1/(9*x**2+12*x+4)**(1/2),x)
```

output
```
(x + 2/3)*log(x + 2/3)/(3*sqrt((x + 2/3)**2))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{1}{3}\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log(x + 2/3)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{\log\left(|3\,x+2||\mathrm{sgn}(3\,x+2)|\right)}{3\,\mathrm{sgn}\left(3\,x+2\right)}$$

input
```
integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/3*log(abs(3*x + 2)*abs(sgn(3*x + 2)))/sgn(3*x + 2)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{\ln\left(9\,x+6\right)\,\mathrm{sign}(18\,x+12)}{3}$$

input
```
int(1/(12*x + 9*x^2 + 4)^(1/2),x)
```

output
```
(log(9*x + 6)*sign(18*x + 12))/3
```

3.67 $\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx$

3.67.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6(2+3x)\sqrt{4+12x+9x^2}}$$

output
```
-1/6/(2+3*x)/((2+3*x)^2)^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{2+3x}{6\left((2+3x)^2\right)^{3/2}}$$

input
```
Integrate[(4 + 12*x + 9*x^2)^(-3/2),x]
```

output
```
-1/6*(2 + 3*x)/((2 + 3*x)^2)^(3/2)
```

3.67.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(9x^2+12x+4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1078}$$

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

input
```
Int[(4 + 12*x + 9*x^2)^(-3/2),x]
```

output
```
-1/6*1/((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])
```

3.67.3.1 Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.67.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

method	result	size
meijerg	$\frac{x\left(\frac{3x}{2}+2\right)}{16\left(1+\frac{3x}{2}\right)^{2}}$	16
gosper	$-\frac{2+3x}{6\left((2+3x)^{2}\right)^{\frac{3}{2}}}$	17
default	$-\frac{2+3x}{6\left((2+3x)^{2}\right)^{\frac{3}{2}}}$	17
risch	$-\frac{\sqrt{(2+3x)^{2}}}{6(2+3x)^{3}}$	19

input
```
int(1/(9*x^2+12*x+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*x*(3/2*x+2)/(1+3/2*x)^2
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6\,(9\,x^2+12\,x+4)}$$

input
```
integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
```

output
```
-1/6/(9*x^2 + 12*x + 4)
```

3.67.6 Sympy [F]

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = \int \frac{1}{(9x^2+12x+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(9*x**2+12*x+4)**(3/2),x)
```

output
```
Integral((9*x**2 + 12*x + 4)**(-3/2), x)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.36

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6\,(3\,x+2)^2}$$

input
```
integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

output
```
-1/6/(3*x + 2)^2
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6\,(3\,x+2)^2\mathrm{sgn}\,(3\,x+2)}$$

input
```
integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

output
```
-1/6/((3*x + 2)^2*sgn(3*x + 2))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{\sqrt{9\,x^2+12\,x+4}}{6\,(3\,x+2)^3}$$

input
```
int(1/(12*x + 9*x^2 + 4)^(3/2),x)
```

output
```
-(12*x + 9*x^2 + 4)^(1/2)/(6*(3*x + 2)^3)
```

3.68 $\int \sqrt{4-12x+9x^2}\,dx$

3.68.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{4-12x+9x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{4-12x+9x^2}$$

output
```
-1/6*(2-3*x)*((-2+3*x)^2)^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{\sqrt{(2-3x)^2}x(-4+3x)}{-4+6x}$$

input
```
Integrate[Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
(Sqrt[(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

3.68.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^2-12x+4}\,dx$$

$$\Big\downarrow \; 1079$$

$$-\frac{\sqrt{9x^2-12x+4}\int (9x-6)dx}{3(2-3x)}$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

input
```
Int[Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
-1/6*((2 - 3*x)*Sqrt[4 - 12*x + 9*x^2])
```

3.68.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.68.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\operatorname{csgn}(-2+3x)(-2+3x)^2}{6}$	16
gosper	$\frac{x(3x-4)\sqrt{(-2+3x)^2}}{-4+6x}$	25
risch	$\frac{3\sqrt{(-2+3x)^2}\,x^2}{2(-2+3x)}-\frac{2\sqrt{(-2+3x)^2}\,x}{-2+3x}$	42

input
```
int(((-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*csgn(-2+3*x)*(-2+3*x)^2
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{3}{2}\,x^2 - 2\,x$$

input
```
integrate(((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
3/2*x^2 - 2*x
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{4-12x+9x^2}\,dx = \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{9x^2-12x+4}$$

input
```
integrate(((-2+3*x)**2)**(1/2),x)
```

output
```
(x/2 - 1/3)*sqrt(9*x**2 - 12*x + 4)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{1}{2}\sqrt{9\,x^2-12\,x+4}x - \frac{1}{3}\sqrt{9\,x^2-12\,x+4}$$

input
```
integrate(((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(9*x^2 - 12*x + 4)*x - 1/3*sqrt(9*x^2 - 12*x + 4)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{1}{2}\left(3\,x^2-4\,x\right)\mathrm{sgn}(3\,x-2) + \frac{2}{3}\,\mathrm{sgn}(3\,x-2)$$

input
```
integrate(((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(3*x^2 - 4*x)*sgn(3*x - 2) + 2/3*sgn(3*x - 2)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{|3\,x-2|\,\left(3\,x-2\right)}{6}$$

input
```
int(((3*x - 2)^2)^(1/2),x)
```

output
```
(abs(3*x - 2)*(3*x - 2))/6
```

3.69 $\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx$

3.69.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = -\frac{(2-3x)\log(2-3x)}{3\sqrt{4-12x+9x^2}}$$

output
```
-1/3*(2-3*x)*ln(2-3*x)/((-2+3*x)^2)^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = -\frac{(2-3x)\log(2-3x)}{3\sqrt{(2-3x)^2}}$$

input
```
Integrate[1/Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
-1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[(2 - 3*x)^2]
```

3.69.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9x^2-12x+4}}dx$$
$$\Big\downarrow \text{1079}$$
$$-\frac{3(2-3x)\int \frac{1}{9x-6}dx}{\sqrt{9x^2-12x+4}}$$
$$\Big\downarrow \text{16}$$
$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

input
```
Int[1/Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
-1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[4 - 12*x + 9*x^2]
```

3.69.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.69.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{(-2+3x)^{2}}}$	23
risch	$\frac{\sqrt{(-2+3x)^{2}}\,\ln(-2+3x)}{-6+9x}$	25
meijerg	$-\frac{2\ln\left(1-\frac{3x}{2}\right)}{3\sqrt{(-2+3x)^{2}}}+\frac{x\ln\left(1-\frac{3x}{2}\right)}{\sqrt{(-2+3x)^{2}}}$	36

input
```
int(1/((-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((-2+3*x)^2)^(1/2)*(-2+3*x)*ln(-2+3*x)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{1}{3}\log\left(3\,x-2\right)$$

input
```
integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(3*x - 2)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{\left(x-\frac{2}{3}\right)\log\left(x-\frac{2}{3}\right)}{3\sqrt{\left(x-\frac{2}{3}\right)^{2}}}$$

input
```
integrate(1/((-2+3*x)**2)**(1/2),x)
```

output
```
(x - 2/3)*log(x - 2/3)/(3*sqrt((x - 2/3)**2))
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{1}{3}\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log(x - 2/3)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{1}{3}\log\left(|3\,x-2|\right)\operatorname{sgn}(3\,x-2)$$

input
```
integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*log(abs(3*x - 2))*sgn(3*x - 2)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{\ln\left(3\,x-2\right)\,\operatorname{sign}(3\,x-2)}{3}$$

input
```
int(1/((3*x - 2)^2)^(1/2),x)
```

output
```
(log(3*x - 2)*sign(3*x - 2))/3
```

3.70 $\int \sqrt{-4+12x-9x^2}\,dx$

3.70.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{-4+12x-9x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{-4+12x-9x^2}$$

output
```
-1/6*(2-3*x)*(-(-2+3*x)^2)^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sqrt{-4+12x-9x^2}\,dx = \frac{\sqrt{-(2-3x)^2}x(-4+3x)}{-4+6x}$$

input
```
Integrate[Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
(Sqrt[-(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

3.70.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-9x^2+12x-4}\,dx$$

$$\Big\downarrow \; 1079$$

$$\frac{\sqrt{-9x^2+12x-4}\int (6-9x)dx}{3(2-3x)}$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

input
```
Int[Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
-1/6*((2 - 3*x)*Sqrt[-4 + 12*x - 9*x^2])
```

3.70.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.70.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
gosper	$\frac{x(3x-4)\sqrt{-(-2+3x)^2}}{-4+6x}$	27
default	$\frac{x(3x-4)\sqrt{-(-2+3x)^2}}{-4+6x}$	27
risch	$-\frac{2\sqrt{-(-2+3x)^2}\,x}{-2+3x}+\frac{3\sqrt{-(-2+3x)^2}\,x^2}{2(-2+3x)}$	46

input
```
int((-(-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(3*x-4)*(-(-2+3*x)^2)^(1/2)/(-2+3*x)
```

3.70.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \sqrt{-4+12x-9x^2}\,dx = \frac{3}{2}i\,x^2 - 2i\,x$$

input
```
integrate((-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
3/2*I*x^2 - 2*I*x
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{-4+12x-9x^2}\,dx = \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-9x^2+12x-4}$$

input
```
integrate((-(-2+3*x)**2)**(1/2),x)
```

output
```
(x/2 - 1/3)*sqrt(-9*x**2 + 12*x - 4)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \sqrt{-4+12x-9x^2}\,dx = \frac{1}{2}\sqrt{-9\,x^2+12\,x-4}x - \frac{1}{3}\sqrt{-9\,x^2+12\,x-4}$$

input
```
integrate((-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-9*x^2 + 12*x - 4)*x - 1/3*sqrt(-9*x^2 + 12*x - 4)
```

3.70.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sqrt{-4+12x-9x^2}\,dx = -\frac{1}{2}i\,(3\,x^2-4\,x)\mathrm{sgn}(-3\,x+2) - \frac{2}{3}i\,\mathrm{sgn}(-3\,x+2)$$

input
```
integrate((-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*(3*x^2 - 4*x)*sgn(-3*x + 2) - 2/3*I*sgn(-3*x + 2)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \sqrt{-4+12x-9x^2}\,dx = \frac{(3\,x-2)\,\sqrt{-(3\,x-2)^2}}{6}$$

input
```
int((-(3*x - 2)^2)^(1/2),x)
```

output
```
((3*x - 2)*(-(3*x - 2)^2)^(1/2))/6
```

3.71 $\int \frac{1}{\sqrt{-4+12x-9x^2}} dx$

3.71.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{1}{\sqrt{-4+12x-9x^2}} dx = -\frac{(2-3x)\log(2-3x)}{3\sqrt{-4+12x-9x^2}}$$

output
```
-1/3*(2-3*x)*ln(2-3*x)/(-(-2+3*x)^2)^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{-4+12x-9x^2}} dx = -\frac{(2-3x)\log(2-3x)}{3\sqrt{-(2-3x)^2}}$$

input
```
Integrate[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
-1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[-(2 - 3*x)^2]
```

3.71.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-9x^2+12x-4}}dx$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{3(2-3x)\int \frac{1}{6-9x}dx}{\sqrt{-9x^2+12x-4}}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

input
```
Int[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
-1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[-4 + 12*x - 9*x^2]
```

3.71.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.71.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

method	result	size
meijerg	$-\frac{i \ln\left(1-\frac{3x}{2}\right)}{3}$	10
default	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{-(-2+3x)^2}}$	25
risch	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{-(-2+3x)^2}}$	25

input
```
int(1/(-(-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*ln(1-3/2*x)
```

3.71.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = -\frac{1}{3}i\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*I*log(x - 2/3)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = \frac{\left(x-\frac{2}{3}\right)\log\left(x-\frac{2}{3}\right)}{3\sqrt{-\left(x-\frac{2}{3}\right)^2}}$$

input
```
integrate(1/(-(-2+3*x)**2)**(1/2),x)
```

output
```
(x - 2/3)*log(x - 2/3)/(3*sqrt(-(x - 2/3)**2))
```

3.71.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = \frac{1}{3}i\,\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*I*log(x - 2/3)
```

3.71.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = \frac{i\,\log\left((-3i\,x+2i)\mathrm{sgn}(-3\,x+2)\right)}{3\,\mathrm{sgn}\,(-3\,x+2)}$$

input
```
integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*I*log((-3*I*x + 2*I)*sgn(-3*x + 2))/sgn(-3*x + 2)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = -\frac{\ln(2-3\,x)\ \mathrm{sign}(3\,x-2)\ 1\mathrm{i}}{3}$$

input
```
int(1/(-(3*x - 2)^2)^(1/2),x)
```

output
```
-(log(2 - 3*x)*sign(3*x - 2)*1i)/3
```

3.72 $\int \sqrt{-4-12x-9x^2}\,dx$

3.72.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{-4-12x-9x^2}$$

output
```
1/6*(2+3*x)*(-(2+3*x)^2)^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{x\sqrt{-(2+3x)^2}(4+3x)}{4+6x}$$

input
```
Integrate[Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
(x*Sqrt[-(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

3.72.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-9x^2-12x-4}\,dx$$

$$\Big\downarrow \text{1079}$$

$$-\frac{\sqrt{-9x^2-12x-4}\int (-9x-6)dx}{3(3x+2)}$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

input
```
Int[Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6
```

3.72.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.72.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
gosper	$\frac{x(4+3x)\sqrt{-(2+3x)^2}}{4+6x}$	27
default	$\frac{x(4+3x)\sqrt{-(2+3x)^2}}{4+6x}$	27
risch	$\frac{2\sqrt{-(2+3x)^2}\,x}{2+3x}+\frac{3\sqrt{-(2+3x)^2}\,x^2}{2(2+3x)}$	46

input
```
int((-(2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(4+3*x)*(-(2+3*x)^2)^(1/2)/(2+3*x)
```

3.72.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \sqrt{-4-12x-9x^2}\,dx=\frac{3}{2}i\,x^2+2i\,x$$

input
```
integrate((-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
3/2*I*x^2 + 2*I*x
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \sqrt{-4-12x-9x^2}\,dx=\left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{-9x^2-12x-4}$$

input
```
integrate((-(2+3*x)**2)**(1/2),x)
```

output
```
(x/2 + 1/3)*sqrt(-9*x**2 - 12*x - 4)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{1}{2}\sqrt{-9\,x^2-12\,x-4}x + \frac{1}{3}\sqrt{-9\,x^2-12\,x-4}$$

input
```
integrate((-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-9*x^2 - 12*x - 4)*x + 1/3*sqrt(-9*x^2 - 12*x - 4)
```

3.72.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sqrt{-4-12x-9x^2}\,dx = -\frac{1}{2}i\,(3\,x^2+4\,x)\mathrm{sgn}(-3\,x-2) - \frac{2}{3}i\,\mathrm{sgn}(-3\,x-2)$$

input
```
integrate((-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*(3*x^2 + 4*x)*sgn(-3*x - 2) - 2/3*I*sgn(-3*x - 2)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{(3\,x+2)\,\sqrt{-(3\,x+2)^2}}{6}$$

input
```
int((-(3*x + 2)^2)^(1/2),x)
```

output
```
((3*x + 2)*(-(3*x + 2)^2)^(1/2))/6
```

3.73 $\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx$

3.73.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx=\frac{(2+3x)\log(2+3x)}{3\sqrt{-4-12x-9x^2}}$$

output
```
1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx=\frac{(2+3x)\log(2+3x)}{3\sqrt{-(2+3x)^2}}$$

input
```
Integrate[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

3.73.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-9x^2-12x-4}}\,dx$$
$$\Big\downarrow \; 1079$$
$$-\frac{3(3x+2)\int \frac{1}{-9x-6}dx}{\sqrt{-9x^2-12x-4}}$$
$$\Big\downarrow \; 16$$
$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

input
```
Int[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])
```

3.73.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

3.73.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

method	result	size
meijerg	$-\frac{i \ln\left(1+\frac{3x}{2}\right)}{3}$	10
default	$\frac{(2+3x)\ln(2+3x)}{3\sqrt{-(2+3x)^2}}$	25
risch	$\frac{(2+3x)\ln(2+3x)}{3\sqrt{-(2+3x)^2}}$	25

input
```
int(1/(-(2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*ln(1+3/2*x)
```

3.73.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*I*log(x + 2/3)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx = \frac{\left(x+\frac{2}{3}\right)\log\left(x+\frac{2}{3}\right)}{3\sqrt{-\left(x+\frac{2}{3}\right)^2}}$$

input
```
integrate(1/(-(2+3*x)**2)**(1/2),x)
```

output
```
(x + 2/3)*log(x + 2/3)/(3*sqrt(-(x + 2/3)**2))
```

3.73.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx = \frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*I*log(x + 2/3)
```

3.73.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx = \frac{i\log\left((-3i\,x-2i)\mathrm{sgn}(-3\,x-2)\right)}{3\,\mathrm{sgn}\left(-3\,x-2\right)}$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*I*log((-3*I*x - 2*I)*sgn(-3*x - 2))/sgn(-3*x - 2)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{\ln\left(-3\,x-2\right)\,\mathrm{sign}(3\,x+2)\,1\mathrm{i}}{3}$$

input
```
int(1/(-(3*x + 2)^2)^(1/2),x)
```

output
```
-(log(- 3*x - 2)*sign(3*x + 2)*1i)/3
```

3.74 $\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 \, dx$

3.74.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(1-b-2cx)^6}{384c^6}-\frac{5(1-b-2cx)^7}{896c^6}+\frac{5(1-b-2cx)^8}{1024c^6} -\frac{5(1-b-2cx)^9}{2304c^6}+\frac{(1-b-2cx)^{10}}{2048c^6}-\frac{(1-b-2cx)^{11}}{22528c^6}$$

output
```
1/384*(-2*c*x-b+1)^6/c^6-5/896*(-2*c*x-b+1)^7/c^6+5/1024*(-2*c*x-b+1)^8/c^
6-5/2304*(-2*c*x-b+1)^9/c^6+1/2048*(-2*c*x-b+1)^10/c^6-1/22528*(-2*c*x-b+1
)^11/c^6
```

3.74.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.89

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(-1+b^2)^5 x}{1024c^5}+\frac{5b(-1+b^2)^4 x^2}{512c^4} +\frac{5(-1+b^2)^3(-1+9b^2)x^3}{768c^3}+\frac{5b(-1+b^2)^2(-1+3b^2)x^4}{64c^2} +\frac{(-1+b^2)(1-14b^2+21b^4)x^5}{32c}+\frac{1}{48}b(15-70b^2+63b^4)x^6 +\frac{5}{56}(1-14b^2+21b^4)cx^7+\frac{5}{8}b(-1+3b^2)c^2x^8 +\frac{5}{36}(-1+9b^2)c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

input
```
Integrate[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((-1 + b^2)^5*x)/(1024*c^5) + (5*b*(-1 + b^2)^4*x^2)/(512*c^4) + (5*(-1 +
b^2)^3*(-1 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-1 + b^2)^2*(-1 + 3*b^2)*x^4)/(
64*c^2) + ((-1 + b^2)*(1 - 14*b^2 + 21*b^4)*x^5)/(32*c) + (b*(15 - 70*b^2
+ 63*b^4)*x^6)/48 + (5*(1 - 14*b^2 + 21*b^4)*c*x^7)/56 + (5*b*(-1 + 3*b^2)
*c^2*x^8)/8 + (5*(-1 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

3.74.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2-1}{4c}+bx+cx^2\right)^5 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left(\left(\frac{b-1}{2}+cx\right)^{10}+\left(\frac{b-1}{2}+cx\right)^5-\frac{5}{512}(-b-2cx+1)^9+\frac{5}{128}(-b-2cx+1)^8-\frac{5}{64}(-b-2cx+1)^7+\frac{5}{64}(-b-2cx+\right.}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(-b-2cx+1)^{11}}{22528c}+\frac{(-b-2cx+1)^{10}}{2048c}-\frac{5(-b-2cx+1)^9}{2304c}+\frac{5(-b-2cx+1)^8}{1024c}-\frac{5(-b-2cx+1)^7}{896c}+\frac{(-b-2cx+1)^6}{384c}}{c^5}$$

input
```
Int[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((1 - b - 2*c*x)^6/(384*c) - (5*(1 - b - 2*c*x)^7)/(896*c) + (5*(1 - b - 2
*c*x)^8)/(1024*c) - (5*(1 - b - 2*c*x)^9)/(2304*c) + (1 - b - 2*c*x)^10/(2
048*c) - (1 - b - 2*c*x)^11/(22528*c))/c^5
```

3.74.3.1 Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(97) = 194$.

Time = 2.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.50

method	result
norman	$\left(\frac{5}{4}b^2c^7-\frac{5}{36}c^7\right)x^9+\left(\frac{15}{8}b^3c^6-\frac{5}{8}bc^6\right)x^8+\left(\frac{15}{8}b^4c^5-\frac{5}{4}b^2c^5+\frac{5}{56}c^5\right)x^7+\left(\frac{21}{16}b^5c^4-\frac{35}{24}c^4b^3+\frac{5}{16}bc^4\right)x^6+\left(\frac{15}{64}b^7c^2-\frac{35}{64}b^5c^2+\frac{25}{64}c^2b^3-\right.$
gosper	$x\left(64512c^{10}x^{10}+354816c^9bx^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-98560x^8c^8+931392x^5b^5c^5-443520bc^7x^7+4\right.$
parallelrisch	$64512c^{10}x^{11}+354816c^9bx^{10}+887040x^9b^2c^8+1330560b^3c^7x^8+1330560x^7b^4c^6-98560x^9c^8+931392x^6b^5c^5-443520bc^7x^8+46$
risch	$\frac{5bx^6}{16}-\frac{35b^3x^6}{24}-\frac{x}{1024c^5}+\frac{21b^5x^6}{16}-\frac{x^5}{32c}+\frac{15b^4cx^7}{8}+\frac{21b^6x^5}{32c}+\frac{15b^2x^5}{32c}+\frac{5c^3x^9b^2}{4}+\frac{15b^7x^4}{64c^2}-\frac{35b^5x^4}{64c^2}+$
default	$\frac{c^5x^{11}}{11}+\frac{bc^4x^{10}}{2}+\frac{\left(256\left(b^2-1\right)c^3+4096b^2c^3+4c\left(32\left(24b^2-8\right)c^2+1024b^2c^2\right)\right)x^9}{9216}+\frac{\left(1024\left(b^2-1\right)c^2b+4b\left(32\left(24b^2-8\right)c^2+10\right.\right.}{}$

input
```
int((1/4*(b^2-1)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
((5/4*b^2*c^7-5/36*c^7)*x^9+(15/8*b^3*c^6-5/8*b*c^6)*x^8+(15/8*b^4*c^5-5/4
*b^2*c^5+5/56*c^5)*x^7+(21/16*b^5*c^4-35/24*c^4*b^3+5/16*b*c^4)*x^6+(15/64
*b^7*c^2-35/64*b^5*c^2+25/64*c^2*b^3-5/64*b*c^2)*x^4+(21/32*c^3*b^6-35/32*
b^4*c^3+15/32*b^2*c^3-1/32*c^3)*x^5+(5/512*b^9-5/128*b^7+15/256*b^5-5/128*
b^3+5/512*b)*x^2+(15/256*b^8*c-35/192*b^6*c+25/128*b^4*c-5/64*b^2*c+5/768*
c)*x^3+1/11*c^9*x^11+1/2*b*c^8*x^10+1/1024*(b^10-5*b^8+10*b^6-10*b^4+5*b^2
-1)/c*x)/c^4
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(85) = 170.

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.14

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\,(9\,b^2-1)c^8x^9+443520\,(3\,b^3-b)c^7x^8+63360\,(21\,b^4-14\,b^2+1)c^6}{}$$

input
```
integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 1)*c^8*x^9
+ 443520*(3*b^3 - b)*c^7*x^8 + 63360*(21*b^4 - 14*b^2 + 1)*c^6*x^7 + 14784
*(63*b^5 - 70*b^3 + 15*b)*c^5*x^6 + 22176*(21*b^6 - 35*b^4 + 15*b^2 - 1)*c
^4*x^5 + 55440*(3*b^7 - 7*b^5 + 5*b^3 - b)*c^3*x^4 + 4620*(9*b^8 - 28*b^6
+ 30*b^4 - 12*b^2 + 1)*c^2*x^3 + 6930*(b^9 - 4*b^7 + 6*b^5 - 4*b^3 + b)*c*
x^2 + 693*(b^10 - 5*b^8 + 10*b^6 - 10*b^4 + 5*b^2 - 1)*x)/c^5
```

3.74.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(95) = 190.

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.32

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\cdot\left(\frac{5b^2c^3}{4}-\frac{5c^3}{36}\right)+x^8 \cdot\left(\frac{15b^3c^2}{8}-\frac{5bc^2}{8}\right)+x^7\cdot\left(\frac{15b^4c}{8}-\frac{5b^2c}{4}+\frac{5c}{56}\right)+x^6 \cdot\left(\frac{21b^5}{16}-\frac{35b^3}{24}+\frac{5b}{16}\right)+\frac{x^5\cdot(21b^6-35b^4+15b^2-1)}{32c} + \frac{x^4\cdot(15b^7-35b^5+25b^3-5b)}{64c^2} + \frac{x^3\cdot(45b^8-140b^6+150b^4-60b^2+5)}{768c^3} + \frac{x^2\cdot(5b^9-20b^7+30b^5-20b^3+5b)}{512c^4} + \frac{x(b^{10}-5b^8+10b^6-10b^4+5b^2-1)}{1024c^5}$$

input
```
integrate((1/4*(b**2-1)/c+b*x+c*x**2)**5,x)
```

output
```
b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/36) + x**8*(
15*b**3*c**2/8 - 5*b*c**2/8) + x**7*(15*b**4*c/8 - 5*b**2*c/4 + 5*c/56) +
x**6*(21*b**5/16 - 35*b**3/24 + 5*b/16) + x**5*(21*b**6 - 35*b**4 + 15*b**
2 - 1)/(32*c) + x**4*(15*b**7 - 35*b**5 + 25*b**3 - 5*b)/(64*c**2) + x**3*
(45*b**8 - 140*b**6 + 150*b**4 - 60*b**2 + 5)/(768*c**3) + x**2*(5*b**9 -
20*b**7 + 30*b**5 - 20*b**3 + 5*b)/(512*c**4) + x*(b**10 - 5*b**8 + 10*b**
6 - 10*b**4 + 5*b**2 - 1)/(1024*c**5)
```

3.74.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{1}{11}c^5x^{11}+\frac{1}{2}bc^4x^{10}+\frac{10}{9}b^2c^3x^9+\frac{5}{4}b^3c^2x^8+\frac{5}{7}b^4cx^7+\frac{1}{6}b^5x^6 + \frac{5\,(2\,cx^3+3\,bx^2)(b^2-1)^4}{1536\,c^4}+\frac{(6\,c^2x^5+15\,bcx^4+10\,b^2x^3)(b^2-1)^3}{192\,c^3} + \frac{(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4)(b^2-1)^2}{224\,c^2} + \frac{(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5)(b^2-1)}{504\,c}+\frac{(b^2-1)^5x}{1024\,c^5}$$

input
```
integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

output
```
1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*
b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 1)^4/c^4 + 1/1
92*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 1)^3/c^3 + 1/224*(20*c^3*x
^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 1)^2/c^2 + 1/504*(70
*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*
(b^2 - 1)/c + 1/1024*(b^2 - 1)^5*x/c^5
```

3.74.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(85) = 170.

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.06

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+887040\,b^2c^8x^9+1330560\,b^3c^7x^8+1330560\,b^4c^6x^7-98560\,c^8x^9+931392}{}$$

input
```
integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 13305
60*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 98560*c^8*x^9 + 931392*b^5*c^5*x^6
- 443520*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 166320*b^7*
c^3*x^4 - 1034880*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 6
3360*c^6*x^7 + 6930*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 221760*b*c^5*x^6 + 69
3*b^10*x - 129360*b^6*c^2*x^3 + 332640*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 277
200*b^3*c^3*x^4 - 3465*b^8*x + 138600*b^4*c^2*x^3 - 22176*c^4*x^5 + 41580*
b^5*c*x^2 - 55440*b*c^3*x^4 + 6930*b^6*x - 55440*b^2*c^2*x^3 - 27720*b^3*c
*x^2 - 6930*b^4*x + 4620*c^2*x^3 + 6930*b*c*x^2 + 3465*b^2*x - 693*x)/c^5
```

3.74.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx &= \frac{c^5\,x^{11}}{11}+\frac{x\,{\left(b^2-1\right)}^5}{1024\,c^5}+\frac{b\,x^6\,\left(63\,b^4-70\,b^2+15\right)}{48}\\ &+\frac{5\,c\,x^7\,\left(21\,b^4-14\,b^2+1\right)}{56}+\frac{b\,c^4\,x^{10}}{2}\\ &+\frac{5\,c^3\,x^9\,\left(9\,b^2-1\right)}{36}+\frac{x^5\,\left(21\,b^6-35\,b^4+15\,b^2-1\right)}{32\,c}\\ &+\frac{5\,b\,c^2\,x^8\,\left(3\,b^2-1\right)}{8}+\frac{5\,b\,x^2\,{\left(b^2-1\right)}^4}{512\,c^4}\\ &+\frac{5\,x^3\,{\left(b^2-1\right)}^3\,\left(9\,b^2-1\right)}{768\,c^3}+\frac{5\,b\,x^4\,{\left(b^2-1\right)}^2\,\left(3\,b^2-1\right)}{64\,c^2}\end{aligned}$$

input
```
int((b*x + c*x^2 + (b^2/4 - 1/4)/c)^5,x)
```

output
```
(c^5*x^11)/11 + (x*(b^2 - 1)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 70*b^2 + 15)
)/48 + (5*c*x^7*(21*b^4 - 14*b^2 + 1))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9*(9
*b^2 - 1))/36 + (x^5*(15*b^2 - 35*b^4 + 21*b^6 - 1))/(32*c) + (5*b*c^2*x^8
*(3*b^2 - 1))/8 + (5*b*x^2*(b^2 - 1)^4)/(512*c^4) + (5*x^3*(b^2 - 1)^3*(9*
b^2 - 1))/(768*c^3) + (5*b*x^4*(b^2 - 1)^2*(3*b^2 - 1))/(64*c^2)
```

3.74. $\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx$

3.75 $\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx$

3.75.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(2-b-2cx)^6}{12c^6}-\frac{5(2-b-2cx)^7}{56c^6}+\frac{5(2-b-2cx)^8}{128c^6}-\frac{5(2-b-2cx)^9}{576c^6}+\frac{(2-b-2cx)^{10}}{1024c^6}-\frac{(2-b-2cx)^{11}}{22528c^6}$$

output
```
1/12*(-2*c*x-b+2)^6/c^6-5/56*(-2*c*x-b+2)^7/c^6+5/128*(-2*c*x-b+2)^8/c^6-5
/576*(-2*c*x-b+2)^9/c^6+1/1024*(-2*c*x-b+2)^10/c^6-1/22528*(-2*c*x-b+2)^11
/c^6
```

3.75.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.89

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(-4+b^2)^5 x}{1024c^5}+\frac{5b(-4+b^2)^4 x^2}{512c^4}+\frac{5(-4+b^2)^3(-4+9b^2)x^3}{768c^3}+\frac{5b(-4+b^2)^2(-4+3b^2)x^4}{64c^2}+\frac{(-4+b^2)(16-56b^2+21b^4)x^5}{32c}+\frac{1}{48}b(240-280b^2+63b^4)x^6+\frac{5}{56}(16-56b^2+21b^4)cx^7+\frac{5}{8}b(-4+3b^2)c^2x^8+\frac{5}{36}(-4+9b^2)c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

input
```
Integrate[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((-4 + b^2)^5*x)/(1024*c^5) + (5*b*(-4 + b^2)^4*x^2)/(512*c^4) + (5*(-4 +
b^2)^3*(-4 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-4 + b^2)^2*(-4 + 3*b^2)*x^4)/(
64*c^2) + ((-4 + b^2)*(16 - 56*b^2 + 21*b^4)*x^5)/(32*c) + (b*(240 - 280*b
^2 + 63*b^4)*x^6)/48 + (5*(16 - 56*b^2 + 21*b^4)*c*x^7)/56 + (5*b*(-4 + 3*
b^2)*c^2*x^8)/8 + (5*(-4 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11
)/11
```

3.75.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2-4}{4c}+bx+cx^2\right)^5 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left(\left(\frac{b-2}{2}+cx\right)^{10}-\frac{5}{256}(-b-2cx+2)^9+\frac{5}{32}(-b-2cx+2)^8-\frac{5}{8}(-b-2cx+2)^7+\frac{5}{4}(-b-2cx+2)^6+(b+2cx-\right.}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(-b-2cx+2)^{11}}{22528c}+\frac{(-b-2cx+2)^{10}}{1024c}-\frac{5(-b-2cx+2)^9}{576c}+\frac{5(-b-2cx+2)^8}{128c}-\frac{5(-b-2cx+2)^7}{56c}+\frac{(-b-2cx+2)^6}{12c}}{c^5}$$

input
```
Int[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((2 - b - 2*c*x)^6/(12*c) - (5*(2 - b - 2*c*x)^7)/(56*c) + (5*(2 - b - 2*c
*x)^8)/(128*c) - (5*(2 - b - 2*c*x)^9)/(576*c) + (2 - b - 2*c*x)^10/(1024*
c) - (2 - b - 2*c*x)^11/(22528*c))/c^5
```

3.75.3.1 Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(97) = 194$.

Time = 2.23 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.50

method	result
norman	$\frac{\left(\frac{5}{4}b^2c^7-\frac{5}{9}c^7\right)x^9+\left(\frac{15}{8}b^3c^6-\frac{5}{2}b\,c^6\right)x^8+\left(\frac{15}{8}b^4c^5-5b^2c^5+\frac{10}{7}c^5\right)x^7+\left(\frac{21}{16}b^5c^4-\frac{35}{6}c^4b^3+5b\,c^4\right)x^6+\left(\frac{15}{64}b^7c^2-\frac{35}{16}b^5c^2+\frac{25}{4}c^2b^3-5b\right.}{}$
gosper	$\frac{x\left(64512c^{10}x^{10}+354816c^9b\,x^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-394240x^8c^8+931392x^5b^5c^5-1774080b\,c^7x^7-\right.}{}$
parallelrisch	$\frac{64512c^{10}x^{11}+354816c^9b\,x^{10}+887040x^9b^2c^8+1330560b^3c^7x^8+1330560x^7b^4c^6-394240x^9c^8+931392x^6b^5c^5-1774080b\,c^7x^8+}{}$
risch	$5b\,x^6-\frac{35b^3x^6}{6}-\frac{x}{c^5}+\frac{21b^5x^6}{16}-\frac{2x^5}{c}+\frac{15b^4c\,x^7}{8}+\frac{21b^6x^5}{32c}+\frac{15b^2x^5}{2c}+\frac{5c^3x^9b^2}{4}+\frac{15b^7x^4}{64c^2}-\frac{35b^5x^4}{16c^2}+\frac{5b^9}{512}$
default	$\frac{c^5x^{11}}{11}+\frac{b\,c^4x^{10}}{2}+\frac{\left(256\left(b^2-4\right)c^3+4096b^2c^3+4c\left(32\left(24b^2-32\right)c^2+1024b^2c^2\right)\right)x^9}{9216}+\frac{\left(1024\left(b^2-4\right)c^2b+4b\left(32\left(24b^2-32\right)c^2+\right.\right.}{}$

input
```
int((1/4*(b^2-4)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
((5/4*b^2*c^7-5/9*c^7)*x^9+(15/8*b^3*c^6-5/2*b*c^6)*x^8+(15/8*b^4*c^5-5*b^
2*c^5+10/7*c^5)*x^7+(21/16*b^5*c^4-35/6*c^4*b^3+5*b*c^4)*x^6+(15/64*b^7*c^
2-35/16*b^5*c^2+25/4*c^2*b^3-5*b*c^2)*x^4+(21/32*c^3*b^6-35/8*b^4*c^3+15/2
*b^2*c^3-2*c^3)*x^5+(5/512*b^9-5/32*b^7+15/16*b^5-5/2*b^3+5/2*b)*x^2+(15/2
56*b^8*c-35/48*b^6*c+25/8*b^4*c-5*b^2*c+5/3*c)*x^3+1/11*c^9*x^11+1/2*b*c^8
*x^10+1/1024*(b^10-20*b^8+160*b^6-640*b^4+1280*b^2-1024)/c*x)/c^4
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(85) = 170$.

Time = 0.45 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.16

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\,(9\,b^2-4)c^8x^9+443520\,(3\,b^3-4\,b)c^7x^8+63360\,(21\,b^4-56\,b^2+16}{}$$

input
```
integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 4)*c^8*x^9
+ 443520*(3*b^3 - 4*b)*c^7*x^8 + 63360*(21*b^4 - 56*b^2 + 16)*c^6*x^7 + 14
784*(63*b^5 - 280*b^3 + 240*b)*c^5*x^6 + 22176*(21*b^6 - 140*b^4 + 240*b^2
 - 64)*c^4*x^5 + 55440*(3*b^7 - 28*b^5 + 80*b^3 - 64*b)*c^3*x^4 + 4620*(9*
b^8 - 112*b^6 + 480*b^4 - 768*b^2 + 256)*c^2*x^3 + 6930*(b^9 - 16*b^7 + 96
*b^5 - 256*b^3 + 256*b)*c*x^2 + 693*(b^10 - 20*b^8 + 160*b^6 - 640*b^4 + 1
280*b^2 - 1024)*x)/c^5
```

3.75.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.

Time = 0.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.29

$$\begin{aligned}\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx &= \frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\cdot\left(\frac{5b^2c^3}{4}-\frac{5c^3}{9}\right)+x^8\\ &\cdot\left(\frac{15b^3c^2}{8}-\frac{5bc^2}{2}\right)+x^7\cdot\left(\frac{15b^4c}{8}-5b^2c+\frac{10c}{7}\right)+x^6\\ &\cdot\left(\frac{21b^5}{16}-\frac{35b^3}{6}+5b\right)+\frac{x^5\cdot(21b^6-140b^4+240b^2-64)}{32c}\\ &+\frac{x^4\cdot(15b^7-140b^5+400b^3-320b)}{64c^2}\\ &+\frac{x^3\cdot(45b^8-560b^6+2400b^4-3840b^2+1280)}{768c^3}\\ &+\frac{x^2\cdot(5b^9-80b^7+480b^5-1280b^3+1280b)}{512c^4}\\ &+\frac{x(b^{10}-20b^8+160b^6-640b^4+1280b^2-1024)}{1024c^5}\end{aligned}$$

input
```
integrate((1/4*(b**2-4)/c+b*x+c*x**2)**5,x)
```

output
```
b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/9) + x**8*(1
5*b**3*c**2/8 - 5*b*c**2/2) + x**7*(15*b**4*c/8 - 5*b**2*c + 10*c/7) + x**
6*(21*b**5/16 - 35*b**3/6 + 5*b) + x**5*(21*b**6 - 140*b**4 + 240*b**2 - 6
4)/(32*c) + x**4*(15*b**7 - 140*b**5 + 400*b**3 - 320*b)/(64*c**2) + x**3*
(45*b**8 - 560*b**6 + 2400*b**4 - 3840*b**2 + 1280)/(768*c**3) + x**2*(5*b
**9 - 80*b**7 + 480*b**5 - 1280*b**3 + 1280*b)/(512*c**4) + x*(b**10 - 20*
b**8 + 160*b**6 - 640*b**4 + 1280*b**2 - 1024)/(1024*c**5)
```

3.75.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\begin{aligned}\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx &= \frac{1}{11}c^5x^{11}+\frac{1}{2}bc^4x^{10}+\frac{10}{9}b^2c^3x^9+\frac{5}{4}b^3c^2x^8+\frac{5}{7}b^4cx^7+\frac{1}{6}b^5x^6 \\ &\quad+\frac{5\,(2\,cx^3+3\,bx^2)(b^2-4)^4}{1536\,c^4}+\frac{(6\,c^2x^5+15\,bcx^4+10\,b^2x^3)(b^2-4)^3}{192\,c^3} \\ &\quad+\frac{(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4)(b^2-4)^2}{224\,c^2} \\ &\quad+\frac{(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5)(b^2-4)}{504\,c}+\frac{(b^2-4)^5x}{1024\,c^5}\end{aligned}$$

input
```
integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

output
```
1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*
b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 4)^4/c^4 + 1/1
92*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 4)^3/c^3 + 1/224*(20*c^3*x
^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 4)^2/c^2 + 1/504*(70
*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*
(b^2 - 4)/c + 1/1024*(b^2 - 4)^5*x/c^5
```

3.75.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(85) = 170.

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.06

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+887040\,b^2c^8x^9+1330560\,b^3c^7x^8+1330560\,b^4c^6x^7-394240\,c^8x^9+931392}{}$$

input
```
integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 13305
60*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 394240*c^8*x^9 + 931392*b^5*c^5*x^6
 - 1774080*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 3548160*b^2*c^6*x^7 + 166320*b
^7*c^3*x^4 - 4139520*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 3104640*b^4*c^4*x^5
 + 1013760*c^6*x^7 + 6930*b^9*c*x^2 - 1552320*b^5*c^3*x^4 + 3548160*b*c^5*
x^6 + 693*b^10*x - 517440*b^6*c^2*x^3 + 5322240*b^2*c^4*x^5 - 110880*b^7*c
*x^2 + 4435200*b^3*c^3*x^4 - 13860*b^8*x + 2217600*b^4*c^2*x^3 - 1419264*c
^4*x^5 + 665280*b^5*c*x^2 - 3548160*b*c^3*x^4 + 110880*b^6*x - 3548160*b^2
*c^2*x^3 - 1774080*b^3*c*x^2 - 443520*b^4*x + 1182720*c^2*x^3 + 1774080*b*
c*x^2 + 887040*b^2*x - 709632*x)/c^5
```

3.75.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.69

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{c^5\,x^{11}}{11}+\frac{x\,\left(b^2-4\right)^5}{1024\,c^5}+\frac{b\,x^6\,\left(63\,b^4-280\,b^2+240\right)}{48} + \frac{5\,c\,x^7\,\left(21\,b^4-56\,b^2+16\right)}{56}+\frac{b\,c^4\,x^{10}}{2} + \frac{5\,c^3\,x^9\,\left(9\,b^2-4\right)}{36}+\frac{x^5\,\left(21\,b^6-140\,b^4+240\,b^2-64\right)}{32\,c} + \frac{5\,b\,c^2\,x^8\,\left(3\,b^2-4\right)}{8}+\frac{5\,b\,x^2\,\left(b^2-4\right)^4}{512\,c^4} + \frac{5\,x^3\,\left(b^2-4\right)^3\,\left(9\,b^2-4\right)}{768\,c^3}+\frac{5\,b\,x^4\,\left(b^2-4\right)^2\,\left(3\,b^2-4\right)}{64\,c^2}$$

input
```
int((b*x + c*x^2 + (b^2/4 - 1)/c)^5,x)
```

output
```
(c^5*x^11)/11 + (x*(b^2 - 4)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 280*b^2 + 24
0))/48 + (5*c*x^7*(21*b^4 - 56*b^2 + 16))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9
*(9*b^2 - 4))/36 + (x^5*(240*b^2 - 140*b^4 + 21*b^6 - 64))/(32*c) + (5*b*c
^2*x^8*(3*b^2 - 4))/8 + (5*b*x^2*(b^2 - 4)^4)/(512*c^4) + (5*x^3*(b^2 - 4)
^3*(9*b^2 - 4))/(768*c^3) + (5*b*x^4*(b^2 - 4)^2*(3*b^2 - 4))/(64*c^2)
```

3.75. $\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx$

3.76 $\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx$

3.76.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{81(3-b-2cx)^6}{128c^6}-\frac{405(3-b-2cx)^7}{896c^6}+\frac{135(3-b-2cx)^8}{1024c^6} - \frac{5(3-b-2cx)^9}{256c^6}+\frac{3(3-b-2cx)^{10}}{2048c^6}-\frac{(3-b-2cx)^{11}}{22528c^6}$$

output
```
81/128*(-2*c*x-b+3)^6/c^6-405/896*(-2*c*x-b+3)^7/c^6+135/1024*(-2*c*x-b+3)
^8/c^6-5/256*(-2*c*x-b+3)^9/c^6+3/2048*(-2*c*x-b+3)^10/c^6-1/22528*(-2*c*x
-b+3)^11/c^6
```

3.76.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.82

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(-9+b^2)^5 x}{1024c^5}+\frac{5b(-9+b^2)^4 x^2}{512c^4} + \frac{15(-9+b^2)^3(-1+b^2)x^3}{256c^3}+\frac{15b(-9+b^2)^2(-3+b^2)x^4}{64c^2} + \frac{3(-9+b^2)(27-42b^2+7b^4)x^5}{32c} + \frac{3}{16}b(135-70b^2+7b^4)x^6+\frac{15}{56}(27-42b^2+7b^4)cx^7 + \frac{15}{8}b(-3+b^2)c^2x^8+\frac{5}{4}(-1+b^2)c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

input
```
Integrate[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((-9 + b^2)^5*x)/(1024*c^5) + (5*b*(-9 + b^2)^4*x^2)/(512*c^4) + (15*(-9 +
 b^2)^3*(-1 + b^2)*x^3)/(256*c^3) + (15*b*(-9 + b^2)^2*(-3 + b^2)*x^4)/(64
*c^2) + (3*(-9 + b^2)*(27 - 42*b^2 + 7*b^4)*x^5)/(32*c) + (3*b*(135 - 70*b
^2 + 7*b^4)*x^6)/16 + (15*(27 - 42*b^2 + 7*b^4)*c*x^7)/56 + (15*b*(-3 + b^
2)*c^2*x^8)/8 + (5*(-1 + b^2)*c^3*x^9)/4 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

3.76.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2-9}{4c}+bx+cx^2\right)^5 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left(\left(\frac{b-3}{2}+cx\right)^{10}-\frac{15}{512}(-b-2cx+3)^9+\frac{45}{128}(-b-2cx+3)^8-\frac{135}{64}(-b-2cx+3)^7+\frac{405}{64}(-b-2cx+3)^6-\frac{243}{32}(-\right.}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(-b-2cx+3)^{11}}{22528c}+\frac{3(-b-2cx+3)^{10}}{2048c}-\frac{5(-b-2cx+3)^9}{256c}+\frac{135(-b-2cx+3)^8}{1024c}-\frac{405(-b-2cx+3)^7}{896c}+\frac{81(-b-2cx+3)^6}{128c}}{c^5}$$

input
```
Int[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((81*(3 - b - 2*c*x)^6)/(128*c) - (405*(3 - b - 2*c*x)^7)/(896*c) + (135*(
3 - b - 2*c*x)^8)/(1024*c) - (5*(3 - b - 2*c*x)^9)/(256*c) + (3*(3 - b - 2
*c*x)^10)/(2048*c) - (3 - b - 2*c*x)^11/(22528*c))/c^5
```

3.76.3.1 Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(97) = 194$.

Time = 2.26 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.50

method	result
norman	$\frac{\left(\frac{5}{4}b^2c^7-\frac{5}{4}c^7\right)x^9+\left(\frac{15}{8}b^3c^6-\frac{45}{8}b\,c^6\right)x^8+\left(\frac{15}{8}b^4c^5-\frac{45}{4}b^2c^5+\frac{405}{56}c^5\right)x^7+\left(\frac{21}{16}b^5c^4-\frac{105}{8}c^4b^3+\frac{405}{16}b\,c^4\right)x^6+\left(\frac{15}{64}b^7c^2-\frac{315}{64}b^5c^2+\frac{202}{64}\right.}{}$
gosper	$\frac{x\left(7168c^{10}x^{10}+39424c^9b\,x^9+98560x^8b^2c^8+147840b^3c^7x^7+147840x^6b^4c^6-98560x^8c^8+103488x^5b^5c^5-443520b\,c^7x^7+51744b\right.}{}$
parallelrisch	$\frac{7168c^{10}x^{11}+39424c^9b\,x^{10}+98560x^9b^2c^8+147840b^3c^7x^8+147840x^7b^4c^6-98560x^9c^8+103488x^6b^5c^5-443520b\,c^7x^8+51744b^6c}{}$
risch	$\frac{405b\,x^6}{16}-\frac{105b^3x^6}{8}-\frac{59049x}{1024c^5}+\frac{21b^5x^6}{16}-\frac{729x^5}{32c}+\frac{15b^4c\,x^7}{8}+\frac{21b^6x^5}{32c}+\frac{1215b^2x^5}{32c}+\frac{5c^3x^9b^2}{4}+\frac{15b^7x^4}{64c^2}-\frac{315}{6}$
default	$\frac{c^5x^{11}}{11}+\frac{b\,c^4x^{10}}{2}+\frac{\left(256\left(b^2-9\right)c^3+4096b^2c^3+4c\left(32\left(24b^2-72\right)c^2+1024b^2c^2\right)\right)x^9}{9216}+\frac{\left(1024\left(b^2-9\right)c^2b+4b\left(32\left(24b^2-72\right)c^2+\right.\right.}{}$

input
```
int((1/4*(b^2-9)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
((5/4*b^2*c^7-5/4*c^7)*x^9+(15/8*b^3*c^6-45/8*b*c^6)*x^8+(15/8*b^4*c^5-45/
4*b^2*c^5+405/56*c^5)*x^7+(21/16*b^5*c^4-105/8*c^4*b^3+405/16*b*c^4)*x^6+(
15/64*b^7*c^2-315/64*b^5*c^2+2025/64*c^2*b^3-3645/64*b*c^2)*x^4+(21/32*c^3
*b^6-315/32*b^4*c^3+1215/32*b^2*c^3-729/32*c^3)*x^5+(5/512*b^9-45/128*b^7+
1215/256*b^5-3645/128*b^3+32805/512*b)*x^2+(15/256*b^8*c-105/64*b^6*c+2025
/128*b^4*c-3645/64*b^2*c+10935/256*c)*x^3+1/11*c^9*x^11+1/2*b*c^8*x^10+1/1
024*(b^10-45*b^8+810*b^6-7290*b^4+32805*b^2-59049)/c*x)/c^4
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(85) = 170$.

Time = 0.43 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.08

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{7168\,c^{10}x^{11}+39424\,bc^9x^{10}+98560\,(b^2-1)c^8x^9+147840\,(b^3-3\,b)c^7x^8+21120\,(7\,b^4-42\,b^2+27)c^6x^7 -}{}$$

input
```
integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

output
```
1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*(b^2 - 1)*c^8*x^9 + 147
840*(b^3 - 3*b)*c^7*x^8 + 21120*(7*b^4 - 42*b^2 + 27)*c^6*x^7 + 14784*(7*b
^5 - 70*b^3 + 135*b)*c^5*x^6 + 7392*(7*b^6 - 105*b^4 + 405*b^2 - 243)*c^4*
x^5 + 18480*(b^7 - 21*b^5 + 135*b^3 - 243*b)*c^3*x^4 + 4620*(b^8 - 28*b^6
+ 270*b^4 - 972*b^2 + 729)*c^2*x^3 + 770*(b^9 - 36*b^7 + 486*b^5 - 2916*b^
3 + 6561*b)*c*x^2 + 77*(b^10 - 45*b^8 + 810*b^6 - 7290*b^4 + 32805*b^2 - 5
9049)*x)/c^5
```

3.76.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(99) = 198$.

Time = 0.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.32

$$\begin{aligned}\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = {} & \frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\cdot\left(\frac{5b^2c^3}{4}-\frac{5c^3}{4}\right)+x^8\cdot\left(\frac{15b^3c^2}{8}-\frac{45bc^2}{8}\right)\\ & +x^7\cdot\left(\frac{15b^4c}{8}-\frac{45b^2c}{4}+\frac{405c}{56}\right)+x^6\cdot\left(\frac{21b^5}{16}-\frac{105b^3}{8}+\frac{405b}{16}\right)\\ & +\frac{x^5\cdot(21b^6-315b^4+1215b^2-729)}{32c}\\ & +\frac{x^4\cdot(15b^7-315b^5+2025b^3-3645b)}{64c^2}\\ & +\frac{x^3\cdot(15b^8-420b^6+4050b^4-14580b^2+10935)}{256c^3}\\ & +\frac{x^2\cdot(5b^9-180b^7+2430b^5-14580b^3+32805b)}{512c^4}\\ & +\frac{x(b^{10}-45b^8+810b^6-7290b^4+32805b^2-59049)}{1024c^5}\end{aligned}$$

input
```
integrate((1/4*(b**2-9)/c+b*x+c*x**2)**5,x)
```

output
```
b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/4) + x**8*(1
5*b**3*c**2/8 - 45*b*c**2/8) + x**7*(15*b**4*c/8 - 45*b**2*c/4 + 405*c/56)
 + x**6*(21*b**5/16 - 105*b**3/8 + 405*b/16) + x**5*(21*b**6 - 315*b**4 +
1215*b**2 - 729)/(32*c) + x**4*(15*b**7 - 315*b**5 + 2025*b**3 - 3645*b)/(
64*c**2) + x**3*(15*b**8 - 420*b**6 + 4050*b**4 - 14580*b**2 + 10935)/(256
*c**3) + x**2*(5*b**9 - 180*b**7 + 2430*b**5 - 14580*b**3 + 32805*b)/(512*
c**4) + x*(b**10 - 45*b**8 + 810*b**6 - 7290*b**4 + 32805*b**2 - 59049)/(1
024*c**5)
```

3.76.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\begin{aligned}
&\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx \\
&= \frac{1}{11}\,c^5x^{11} + \frac{1}{2}\,bc^4x^{10} + \frac{10}{9}\,b^2c^3x^9 + \frac{5}{4}\,b^3c^2x^8 + \frac{5}{7}\,b^4cx^7 + \frac{1}{6}\,b^5x^6 \\
&\quad + \frac{5\,(2\,cx^3+3\,bx^2)(b^2-9)^4}{1536\,c^4} + \frac{(6\,c^2x^5+15\,bcx^4+10\,b^2x^3)(b^2-9)^3}{192\,c^3} \\
&\quad + \frac{(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4)(b^2-9)^2}{224\,c^2} \\
&\quad + \frac{(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5)(b^2-9)}{504\,c} + \frac{(b^2-9)^5x}{1024\,c^5}
\end{aligned}$$

input
```
integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

output
```
1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*
b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 9)^4/c^4 + 1/1
92*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 9)^3/c^3 + 1/224*(20*c^3*x
^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 9)^2/c^2 + 1/504*(70
*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*
(b^2 - 9)/c + 1/1024*(b^2 - 9)^5*x/c^5
```

3.76.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(85) = 170$.

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.06

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{7168\,c^{10}x^{11}+39424\,bc^9x^{10}+98560\,b^2c^8x^9+147840\,b^3c^7x^8+147840\,b^4c^6x^7-98560\,c^8x^9+103488\,b^5c^5x^6}{}$$

input
```
integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

output
```
1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*b^2*c^8*x^9 + 147840*b^
3*c^7*x^8 + 147840*b^4*c^6*x^7 - 98560*c^8*x^9 + 103488*b^5*c^5*x^6 - 4435
20*b*c^7*x^8 + 51744*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 18480*b^7*c^3*x^4
- 1034880*b^3*c^5*x^6 + 4620*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 570240*c^6
*x^7 + 770*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 1995840*b*c^5*x^6 + 77*b^10*x
- 129360*b^6*c^2*x^3 + 2993760*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 2494800*b^3
*c^3*x^4 - 3465*b^8*x + 1247400*b^4*c^2*x^3 - 1796256*c^4*x^5 + 374220*b^5
*c*x^2 - 4490640*b*c^3*x^4 + 62370*b^6*x - 4490640*b^2*c^2*x^3 - 2245320*b
^3*c*x^2 - 561330*b^4*x + 3367980*c^2*x^3 + 5051970*b*c*x^2 + 2525985*b^2*
x - 4546773*x)/c^5
```

3.76.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.61

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{c^5\,x^{11}}{11}+\frac{5\,c^3\,x^9\,(b^2-1)}{4}+\frac{x\,(b^2-9)^5}{1024\,c^5}+\frac{3\,b\,x^6\,(7\,b^4-70\,b^2+135)}{16}+\frac{15\,c\,x^7\,(7\,b^4-42\,b^2+27)}{56}+\frac{b\,c^4\,x^{10}}{2}+\frac{3\,x^5\,(7\,b^6-105\,b^4+405\,b^2-243)}{32\,c}+\frac{15\,b\,c^2\,x^8\,(b^2-3)}{8}+\frac{15\,x^3\,(b^2-1)\,(b^2-9)^3}{256\,c^3}+\frac{5\,b\,x^2\,(b^2-9)^4}{512\,c^4}+\frac{15\,b\,x^4\,(b^2-3)\,(b^2-9)^2}{64\,c^2}$$

input
```
int((b*x + c*x^2 + (b^2/4 - 9/4)/c)^5,x)
```

output
```
(c^5*x^11)/11 + (5*c^3*x^9*(b^2 - 1))/4 + (x*(b^2 - 9)^5)/(1024*c^5) + (3*
b*x^6*(7*b^4 - 70*b^2 + 135))/16 + (15*c*x^7*(7*b^4 - 42*b^2 + 27))/56 + (
b*c^4*x^10)/2 + (3*x^5*(405*b^2 - 105*b^4 + 7*b^6 - 243))/(32*c) + (15*b*c
^2*x^8*(b^2 - 3))/8 + (15*x^3*(b^2 - 1)*(b^2 - 9)^3)/(256*c^3) + (5*b*x^2*
(b^2 - 9)^4)/(512*c^4) + (15*b*x^4*(b^2 - 3)*(b^2 - 9)^2)/(64*c^2)
```

3.76. $\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx$

3.77 $\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$

3.77.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{8(4-b-2cx)^6}{3c^6}-\frac{10(4-b-2cx)^7}{7c^6}+\frac{5(4-b-2cx)^8}{16c^6}-\frac{5(4-b-2cx)^9}{144c^6}+\frac{(4-b-2cx)^{10}}{512c^6}-\frac{(4-b-2cx)^{11}}{22528c^6}$$

output
```
8/3*(-2*c*x-b+4)^6/c^6-10/7*(-2*c*x-b+4)^7/c^6+5/16*(-2*c*x-b+4)^8/c^6-5/1
44*(-2*c*x-b+4)^9/c^6+1/512*(-2*c*x-b+4)^10/c^6-1/22528*(-2*c*x-b+4)^11/c^
6
```

3.77.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.89

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(-16+b^2)^5 x}{1024c^5}+\frac{5b(-16+b^2)^4 x^2}{512c^4} + \frac{5(-16+b^2)^3(-16+9b^2)x^3}{768c^3} + \frac{5b(-16+b^2)^2(-16+3b^2)x^4}{64c^2} + \frac{(-16+b^2)(256-224b^2+21b^4)x^5}{32c} + \frac{1}{48}b(3840-1120b^2+63b^4)x^6 + \frac{5}{56}(256-224b^2+21b^4)cx^7+\frac{5}{8}b(-16+3b^2)c^2x^8 + \frac{5}{36}(-16+9b^2)c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

input
```
Integrate[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((-16 + b^2)^5*x)/(1024*c^5) + (5*b*(-16 + b^2)^4*x^2)/(512*c^4) + (5*(-16
 + b^2)^3*(-16 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-16 + b^2)^2*(-16 + 3*b^2)*
x^4)/(64*c^2) + ((-16 + b^2)*(256 - 224*b^2 + 21*b^4)*x^5)/(32*c) + (b*(38
40 - 1120*b^2 + 63*b^4)*x^6)/48 + (5*(256 - 224*b^2 + 21*b^4)*c*x^7)/56 +
(5*b*(-16 + 3*b^2)*c^2*x^8)/8 + (5*(-16 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10
)/2 + (c^5*x^11)/11
```

3.77.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2-16}{4c}+bx+cx^2\right)^5 dx$$

$$\Big\downarrow \; 1084$$

$$\frac{\int \left(\left(\frac{b-4}{2}+cx\right)^{10}-\frac{5}{128}(-b-2cx+4)^9+\frac{5}{8}(-b-2cx+4)^8-5(-b-2cx+4)^7+20(-b-2cx+4)^6-32(-b-2}{c^5}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{(-b-2cx+4)^{11}}{22528c}+\frac{(-b-2cx+4)^{10}}{512c}-\frac{5(-b-2cx+4)^9}{144c}+\frac{5(-b-2cx+4)^8}{16c}-\frac{10(-b-2cx+4)^7}{7c}+\frac{8(-b-2cx+4)^6}{3c}}{c^5}$$

input
```
Int[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((8*(4 - b - 2*c*x)^6)/(3*c) - (10*(4 - b - 2*c*x)^7)/(7*c) + (5*(4 - b -
2*c*x)^8)/(16*c) - (5*(4 - b - 2*c*x)^9)/(144*c) + (4 - b - 2*c*x)^10/(512
*c) - (4 - b - 2*c*x)^11/(22528*c))/c^5
```

3.77.3.1 Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.77.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(97) = 194.

Time = 2.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.50

method	result
norman	$\frac{\left(\frac{5}{4}b^2c^7-\frac{20}{9}c^7\right)x^9+\left(\frac{15}{8}b^3c^6-10bc^6\right)x^8+\left(\frac{15}{8}b^4c^5-20b^2c^5+\frac{160}{7}c^5\right)x^7+\left(\frac{21}{16}b^5c^4-\frac{70}{3}c^4b^3+80bc^4\right)x^6+\left(\frac{15}{64}b^7c^2-\frac{35}{4}b^5c^2+100c^2b\right.}{}$
gosper	$\frac{x\left(64512c^{10}x^{10}+354816c^9bx^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-1576960x^8c^8+931392x^5b^5c^5-7096320bc^7x^7\right.}{}$
parallelrisch	$\frac{64512c^{10}x^{11}+354816c^9bx^{10}+887040x^9b^2c^8+1330560b^3c^7x^8+1330560x^7b^4c^6-1576960x^9c^8+931392x^6b^5c^5-7096320bc^7x^8+}{}$
risch	$80bx^6-\frac{70b^3x^6}{3}-\frac{1024x}{c^5}+\frac{21b^5x^6}{16}-\frac{128x^5}{c}+\frac{15b^4cx^7}{8}+\frac{21b^6x^5}{32c}+\frac{120b^2x^5}{c}+\frac{5c^3x^9b^2}{4}+\frac{15b^7x^4}{64c^2}-\frac{35b^5x^4}{4c^2}$
default	$\frac{c^5x^{11}}{11}+\frac{bc^4x^{10}}{2}+\frac{\left(256(b^2-16)c^3+4096b^2c^3+4c\left(32(24b^2-128)c^2+1024b^2c^2\right)\right)x^9}{9216}+\frac{\left(1024(b^2-16)c^2b+4b\left(32(24b^2-128\right.\right.}{}$

input
```
int((1/4*(b^2-16)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
((5/4*b^2*c^7-20/9*c^7)*x^9+(15/8*b^3*c^6-10*b*c^6)*x^8+(15/8*b^4*c^5-20*b
^2*c^5+160/7*c^5)*x^7+(21/16*b^5*c^4-70/3*c^4*b^3+80*b*c^4)*x^6+(15/64*b^7
*c^2-35/4*b^5*c^2+100*c^2*b^3-320*b*c^2)*x^4+(21/32*c^3*b^6-35/2*b^4*c^3+1
20*b^2*c^3-128*c^3)*x^5+(5/512*b^9-5/8*b^7+15*b^5-160*b^3+640*b)*x^2+(15/2
56*b^8*c-35/12*b^6*c+50*b^4*c-320*b^2*c+1280/3*c)*x^3+1/11*c^9*x^11+1/2*b*
c^8*x^10+1/1024*(b^10-80*b^8+2560*b^6-40960*b^4+327680*b^2-1048576)/c*x)/c
^4
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(85) = 170$.

Time = 0.38 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.16

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$=\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\,(9\,b^2-16)c^8x^9+443520\,(3\,b^3-16\,b)c^7x^8+63360\,(21\,b^4-224\,b^2+}{}$$

input
```
integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 16)*c^8*x^9
 + 443520*(3*b^3 - 16*b)*c^7*x^8 + 63360*(21*b^4 - 224*b^2 + 256)*c^6*x^7
+ 14784*(63*b^5 - 1120*b^3 + 3840*b)*c^5*x^6 + 22176*(21*b^6 - 560*b^4 + 3
840*b^2 - 4096)*c^4*x^5 + 55440*(3*b^7 - 112*b^5 + 1280*b^3 - 4096*b)*c^3*
x^4 + 4620*(9*b^8 - 448*b^6 + 7680*b^4 - 49152*b^2 + 65536)*c^2*x^3 + 6930
*(b^9 - 64*b^7 + 1536*b^5 - 16384*b^3 + 65536*b)*c*x^2 + 693*(b^10 - 80*b^
8 + 2560*b^6 - 40960*b^4 + 327680*b^2 - 1048576)*x)/c^5
```

3.77.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(97) = 194$.

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.28

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9 \cdot \left(\frac{5b^2c^3}{4} - \frac{20c^3}{9}\right) + x^8 \cdot \left(\frac{15b^3c^2}{8} - 10bc^2\right) + x^7 \cdot \left(\frac{15b^4c}{8} - 20b^2c + \frac{160c}{7}\right) + x^6 \cdot \left(\frac{21b^5}{16} - \frac{70b^3}{3} + 80b\right) + \frac{x^5 \cdot (21b^6 - 560b^4 + 3840b^2 - 4096)}{32c} + \frac{x^4 \cdot (15b^7 - 560b^5 + 6400b^3 - 20480b)}{64c^2} + \frac{x^3 \cdot (45b^8 - 2240b^6 + 38400b^4 - 245760b^2 + 327680)}{768c^3} + \frac{x^2 \cdot (5b^9 - 320b^7 + 7680b^5 - 81920b^3 + 327680b)}{512c^4} + \frac{x(b^{10} - 80b^8 + 2560b^6 - 40960b^4 + 327680b^2 - 1048576)}{1024c^5}$$

input
```
integrate((1/4*(b**2-16)/c+b*x+c*x**2)**5,x)
```

output
```
b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 20*c**3/9) + x**8*(
15*b**3*c**2/8 - 10*b*c**2) + x**7*(15*b**4*c/8 - 20*b**2*c + 160*c/7) + x
**6*(21*b**5/16 - 70*b**3/3 + 80*b) + x**5*(21*b**6 - 560*b**4 + 3840*b**2
 - 4096)/(32*c) + x**4*(15*b**7 - 560*b**5 + 6400*b**3 - 20480*b)/(64*c**2
) + x**3*(45*b**8 - 2240*b**6 + 38400*b**4 - 245760*b**2 + 327680)/(768*c*
*3) + x**2*(5*b**9 - 320*b**7 + 7680*b**5 - 81920*b**3 + 327680*b)/(512*c*
*4) + x*(b**10 - 80*b**8 + 2560*b**6 - 40960*b**4 + 327680*b**2 - 1048576)
/(1024*c**5)
```

3.77.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{1}{11}c^5x^{11} + \frac{1}{2}bc^4x^{10} + \frac{10}{9}b^2c^3x^9 + \frac{5}{4}b^3c^2x^8 + \frac{5}{7}b^4cx^7 + \frac{1}{6}b^5x^6 + \frac{5\,(2\,cx^3+3\,bx^2)(b^2-16)^4}{1536\,c^4} + \frac{(6\,c^2x^5+15\,bcx^4+10\,b^2x^3)(b^2-16)^3}{192\,c^3} + \frac{(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4)(b^2-16)^2}{224\,c^2} + \frac{(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5)(b^2-16)}{504\,c} + \frac{(b^2-16)^5x}{1024\,c^5}$$

input
```
integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

output
```
1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*
b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 16)^4/c^4 + 1/
192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 16)^3/c^3 + 1/224*(20*c^3
*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 16)^2/c^2 + 1/504*
(70*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^
5)*(b^2 - 16)/c + 1/1024*(b^2 - 16)^5*x/c^5
```

3.77.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(85) = 170$.

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.06

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{64512\,c^{10}x^{11} + 354816\,bc^9x^{10} + 887040\,b^2c^8x^9 + 1330560\,b^3c^7x^8 + 1330560\,b^4c^6x^7 - 1576960\,c^8x^9 + 93139}{}$$

input
```
integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 13305
60*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 1576960*c^8*x^9 + 931392*b^5*c^5*x^
6 - 7096320*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 14192640*b^2*c^6*x^7 + 166320
*b^7*c^3*x^4 - 16558080*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 12418560*b^4*c^4
*x^5 + 16220160*c^6*x^7 + 6930*b^9*c*x^2 - 6209280*b^5*c^3*x^4 + 56770560*
b*c^5*x^6 + 693*b^10*x - 2069760*b^6*c^2*x^3 + 85155840*b^2*c^4*x^5 - 4435
20*b^7*c*x^2 + 70963200*b^3*c^3*x^4 - 55440*b^8*x + 35481600*b^4*c^2*x^3 -
 90832896*c^4*x^5 + 10644480*b^5*c*x^2 - 227082240*b*c^3*x^4 + 1774080*b^6
*x - 227082240*b^2*c^2*x^3 - 113541120*b^3*c*x^2 - 28385280*b^4*x + 302776
320*c^2*x^3 + 454164480*b*c*x^2 + 227082240*b^2*x - 726663168*x)/c^5
```

3.77.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.69

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{c^5 x^{11}}{11} + \frac{x (b^2-16)^5}{1024 c^5} + \frac{b x^6 (63 b^4 - 1120 b^2 + 3840)}{48} + \frac{5 c x^7 (21 b^4 - 224 b^2 + 256)}{56} + \frac{b c^4 x^{10}}{2} + \frac{5 c^3 x^9 (9 b^2 - 16)}{36} + \frac{x^5 (21 b^6 - 560 b^4 + 3840 b^2 - 4096)}{32 c} + \frac{5 b c^2 x^8 (3 b^2 - 16)}{8} + \frac{5 b x^2 (b^2-16)^4}{512 c^4} + \frac{5 x^3 (b^2-16)^3 (9 b^2 - 16)}{768 c^3} + \frac{5 b x^4 (b^2-16)^2 (3 b^2 - 16)}{64 c^2}$$

input
```
int((b*x + c*x^2 + (b^2/4 - 4)/c)^5,x)
```

output
```
(c^5*x^11)/11 + (x*(b^2 - 16)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 1120*b^2 +
3840))/48 + (5*c*x^7*(21*b^4 - 224*b^2 + 256))/56 + (b*c^4*x^10)/2 + (5*c^
3*x^9*(9*b^2 - 16))/36 + (x^5*(3840*b^2 - 560*b^4 + 21*b^6 - 4096))/(32*c)
 + (5*b*c^2*x^8*(3*b^2 - 16))/8 + (5*b*x^2*(b^2 - 16)^4)/(512*c^4) + (5*x^
3*(b^2 - 16)^3*(9*b^2 - 16))/(768*c^3) + (5*b*x^4*(b^2 - 16)^2*(3*b^2 - 16
))/(64*c^2)
```

3.78 $\int \frac{1}{2+4x+3x^2} \, dx$

3.78.1 Optimal result

Integrand size = 12, antiderivative size = 18

$$\int \frac{1}{2+4x+3x^2} \, dx = \frac{\arctan\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+4x+3x^2} \, dx = \frac{\arctan\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(2 + 4*x + 3*x^2)^(-1),x]
```

output
```
ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

3.78.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2+4x+2}dx$$
$$\Big\downarrow \ 1083$$
$$-2\int \frac{1}{-(6x+4)^2-8}d(6x+4)$$
$$\Big\downarrow \ 217$$
$$\frac{\arctan\left(\frac{6x+4}{2\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[(2 + 4*x + 3*x^2)^(-1),x]
```

output
```
ArcTan[(4 + 6*x)/(2*Sqrt[2])]/Sqrt[2]
```

3.78.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.78.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{(4+6x)\sqrt{2}}{4}\right)}{2}$	17
risch	$\frac{\arctan\left(\frac{(2+3x)\sqrt{2}}{2}\right)\sqrt{2}}{2}$	17

input
```
int(1/(3*x^2+4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/4*(4+6*x)*2^(1/2))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)$$

input
```
integrate(1/(3*x^2+4*x+2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{\sqrt{2}\,\text{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{2}$$

input
```
integrate(1/(3*x**2+4*x+2),x)
```

output
```
sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/2
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)$$

input
```
integrate(1/(3*x^2+4*x+2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)$$

input
```
integrate(1/(3*x^2+4*x+2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(3\,x+2)}{2}\right)}{2}$$

input
```
int(1/(4*x + 3*x^2 + 2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(3*x + 2))/2))/2
```

3.79 $\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx$

3.79.1 Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx=-\arctan\left(\sqrt{3}-x\right)$$

output
```
arctan(x-3^(1/2))
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx=-\arctan\left(\sqrt{3}-x\right)$$

input
```
Integrate[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
```

output
```
-ArcTan[Sqrt[3] - x]
```

3.79.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-2\sqrt{3}x+4}\,dx$$

$$\Big\downarrow \ 1083$$

$$-2\int \frac{1}{-\left(2x-2\sqrt{3}\right)^2-4}d\left(2x-2\sqrt{3}\right)$$

$$\Big\downarrow \ 217$$

$$\arctan\left(\frac{1}{2}\left(2x-2\sqrt{3}\right)\right)$$

input
```
Int[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
```

output
```
ArcTan[(-2*Sqrt[3] + 2*x)/2]
```

3.79.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.79.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\arctan\left(x-\sqrt{3}\right)$	9
risch	$\arctan\left(x-\sqrt{3}\right)$	9
parallelrisch	$\frac{i\ln\left(x+i-\sqrt{3}\right)}{2}-\frac{i\ln\left(x-\sqrt{3}-i\right)}{2}$	28

input
```
int(1/(4+x^2-2*3^(1/2)*x),x,method=_RETURNVERBOSE)
```

output
```
arctan(x-3^(1/2))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = -\arctan\left(-x+\sqrt{3}\right)$$

input
```
integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="fricas")
```

output
```
-arctan(-x + sqrt(3))
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = \operatorname{atan}\left(x-\sqrt{3}\right)$$

input
```
integrate(1/(4+x**2-2*x*3**(1/2)),x)
```

output
```
atan(x - sqrt(3))
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = \arctan\left(x-\sqrt{3}\right)$$

input
```
integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="maxima")
```

output
```
arctan(x - sqrt(3))
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = \arctan\left(x-\sqrt{3}\right)$$

input
```
integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="giac")
```

output
```
arctan(x - sqrt(3))
```

3.79.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = \operatorname{atan}\left(x-\sqrt{3}\right)$$

input
```
int(1/(x^2 - 2*3^(1/2)*x + 4),x)
```

output
```
atan(x - 3^(1/2))
```

3.80 $\int \frac{1}{2+4x-3x^2}\,dx$

3.80.1 Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{2+4x-3x^2}\,dx = -\frac{\text{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

output
```
-1/10*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{1}{2+4x-3x^2}\,dx = \frac{-\log\left(2+\sqrt{10}-3x\right)+\log\left(-2+\sqrt{10}+3x\right)}{2\sqrt{10}}$$

input
```
Integrate[(2 + 4*x - 3*x^2)^(-1),x]
```

output
```
(-Log[2 + Sqrt[10] - 3*x] + Log[-2 + Sqrt[10] + 3*x])/(2*Sqrt[10])
```

3.80.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3x^2+4x+2}dx$$
$$\Big\downarrow \text{1081}$$
$$-3\int \left(\frac{1}{2\sqrt{10}\left(-3x-\sqrt{10}+2\right)}-\frac{1}{2\sqrt{10}\left(-3x+\sqrt{10}+2\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-3\left(\frac{\log\left(-3x+\sqrt{10}+2\right)}{6\sqrt{10}}-\frac{\log\left(-3x-\sqrt{10}+2\right)}{6\sqrt{10}}\right)$$

input
```
Int[(2 + 4*x - 3*x^2)^(-1),x]
```

output
```
-3*(-1/6*Log[2 - Sqrt[10] - 3*x]/Sqrt[10] + Log[2 + Sqrt[10] - 3*x]/(6*Sqr
t[10]))
```

3.80.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.80.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{10}\,\operatorname{arctanh}\left(\frac{(-4+6x)\sqrt{10}}{20}\right)}{10}$	17
risch	$\frac{\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{20}-\frac{\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{20}$	32

input
```
int(1/(-3*x^2+4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/10*10^(1/2)*arctanh(1/20*(-4+6*x)*10^(1/2))
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{2+4x-3x^2}\,dx = \frac{1}{20}\,\sqrt{10}\log\left(\frac{9\,x^2+2\,\sqrt{10}(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right)$$

input
```
integrate(1/(-3*x^2+4*x+2),x, algorithm="fricas")
```

output
```
1/20*sqrt(10)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x
- 2))
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{2+4x-3x^2}\,dx = \frac{\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{20}-\frac{\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{20}$$

input
```
integrate(1/(-3*x**2+4*x+2),x)
```

output
```
sqrt(10)*log(x - 2/3 + sqrt(10)/3)/20 - sqrt(10)*log(x - sqrt(10)/3 - 2/3)
/20
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{1}{2+4x-3x^2}\,dx = -\frac{1}{20}\sqrt{10}\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right)$$

input
```
integrate(1/(-3*x^2+4*x+2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2))
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{2+4x-3x^2}\,dx = -\frac{1}{20}\sqrt{10}\log\left(\frac{|6\,x-2\sqrt{10}-4|}{|6\,x+2\sqrt{10}-4|}\right)$$

input
```
integrate(1/(-3*x^2+4*x+2),x, algorithm="giac")
```

output
```
-1/20*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4))
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{2+4x-3x^2}\,dx = \frac{\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\,\left(\frac{3\,x}{10}-\frac{1}{5}\right)\right)}{10}$$

input
```
int(1/(4*x - 3*x^2 + 2),x)
```

output
```
(10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/10
```

3.81 $\int \frac{1}{2+5x+3x^2}\,dx$

3.81.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{1}{2+5x+3x^2}\,dx = -\log(1+x)+\log(2+3x)$$

output
```
-ln(1+x)+ln(2+3*x)
```

3.81.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x+3x^2}\,dx = -\log(1+x)+\log(2+3x)$$

input
```
Integrate[(2 + 5*x + 3*x^2)^(-1),x]
```

output
```
-Log[1 + x] + Log[2 + 3*x]
```

3.81.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2+5x+2}\,dx$$

$$\Big\downarrow \text{1081}$$

$$3\int \left(\frac{1}{3x+2}-\frac{1}{3(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{3}\log(3x+2)-\frac{1}{3}\log(x+1)\right)$$

input
```
Int[(2 + 5*x + 3*x^2)^(-1),x]
```

output
```
3*(-1/3*Log[1 + x] + Log[2 + 3*x]/3)
```

3.81.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.81.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$-\ln\left(1+x\right)+\ln\left(\frac{2}{3}+x\right)$	12
default	$-\ln\left(1+x\right)+\ln\left(2+3x\right)$	14
norman	$-\ln\left(1+x\right)+\ln\left(2+3x\right)$	14
risch	$-\ln\left(1+x\right)+\ln\left(2+3x\right)$	14

input
```
int(1/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
-ln(1+x)+ln(2/3+x)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x+3x^2}\,dx = \log\left(3\,x+2\right)-\log\left(x+1\right)$$

input
```
integrate(1/(3*x^2+5*x+2),x, algorithm="fricas")
```

output
```
log(3*x + 2) - log(x + 1)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{2+5x+3x^2}\,dx = \log\left(x+\frac{2}{3}\right)-\log\left(x+1\right)$$

input
```
integrate(1/(3*x**2+5*x+2),x)
```

output
```
log(x + 2/3) - log(x + 1)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x+3x^2} \, dx = \log(3x+2) - \log(x+1)$$

input
```
integrate(1/(3*x^2+5*x+2),x, algorithm="maxima")
```

output
```
log(3*x + 2) - log(x + 1)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{2+5x+3x^2} \, dx = \log(|3x+2|) - \log(|x+1|)$$

input
```
integrate(1/(3*x^2+5*x+2),x, algorithm="giac")
```

output
```
log(abs(3*x + 2)) - log(abs(x + 1))
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{2+5x+3x^2} \, dx = -2 \operatorname{atanh}(6x+5)$$

input
```
int(1/(5*x + 3*x^2 + 2),x)
```

output
```
-2*atanh(6*x + 5)
```

3.82 $\int \frac{1}{2+5x-3x^2} dx$

3.82.1 Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{2+5x-3x^2} dx = -\frac{1}{7}\log(2-x)+\frac{1}{7}\log(1+3x)$$

output
```
-1/7*ln(2-x)+1/7*ln(1+3*x)
```

3.82.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x-3x^2} dx = -\frac{1}{7}\log(2-x)+\frac{1}{7}\log(1+3x)$$

input
```
Integrate[(2 + 5*x - 3*x^2)^(-1),x]
```

output
```
-1/7*Log[2 - x] + Log[1 + 3*x]/7
```

3.82.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3x^2+5x+2}dx$$

$$\Big\downarrow \text{1081}$$

$$-3\int \left(-\frac{1}{7(3x+1)}-\frac{1}{21(2-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{1}{21}\log(2-x)-\frac{1}{21}\log(3x+1)\right)$$

input
```
Int[(2 + 5*x - 3*x^2)^(-1),x]
```

output
```
-3*(Log[2 - x]/21 - Log[1 + 3*x]/21)
```

3.82.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.82.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln(-2+x)}{7}+\frac{\ln\left(x+\frac{1}{3}\right)}{7}$	14
default	$-\frac{\ln(-2+x)}{7}+\frac{\ln(3x+1)}{7}$	16
norman	$-\frac{\ln(-2+x)}{7}+\frac{\ln(3x+1)}{7}$	16
risch	$-\frac{\ln(-2+x)}{7}+\frac{\ln(3x+1)}{7}$	16

input
```
int(1/(-3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*ln(-2+x)+1/7*ln(x+1/3)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2+5x-3x^2}\,dx=\frac{1}{7}\,\log\left(3\,x+1\right)-\frac{1}{7}\,\log\left(x-2\right)$$

input
```
integrate(1/(-3*x^2+5*x+2),x, algorithm="fricas")
```

output
```
1/7*log(3*x + 1) - 1/7*log(x - 2)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1}{2+5x-3x^2}\,dx=-\frac{\log\left(x-2\right)}{7}+\frac{\log\left(x+\frac{1}{3}\right)}{7}$$

input
```
integrate(1/(-3*x**2+5*x+2),x)
```

output
```
-log(x - 2)/7 + log(x + 1/3)/7
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2+5x-3x^2}\,dx = \frac{1}{7}\,\log\left(3\,x+1\right) - \frac{1}{7}\,\log\left(x-2\right)$$

input
```
integrate(1/(-3*x^2+5*x+2),x, algorithm="maxima")
```

output
```
1/7*log(3*x + 1) - 1/7*log(x - 2)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{2+5x-3x^2}\,dx = \frac{1}{7}\,\log\left(|3\,x+1|\right) - \frac{1}{7}\,\log\left(|x-2|\right)$$

input
```
integrate(1/(-3*x^2+5*x+2),x, algorithm="giac")
```

output
```
1/7*log(abs(3*x + 1)) - 1/7*log(abs(x - 2))
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{2+5x-3x^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{7}$$

input
```
int(1/(5*x - 3*x^2 + 2),x)
```

output
```
(2*atanh((6*x)/7 - 5/7))/7
```

3.83 $\int \frac{1}{3+4x+x^2} \, dx$

3.83.1 Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{1}{3+4x+x^2} \, dx = -\operatorname{arctanh}(2+x)$$

output
```
-arctanh(2+x)
```

3.83.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{1}{3+4x+x^2} \, dx = \frac{1}{2}\log(1+x) - \frac{1}{2}\log(3+x)$$

input
```
Integrate[(3 + 4*x + x^2)^(-1),x]
```

output
```
Log[1 + x]/2 - Log[3 + x]/2
```

3.83.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+4x+3}\,dx$$
$$\Big\downarrow \text{1081}$$
$$\int \left(\frac{1}{2(x+1)}-\frac{1}{2(x+3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x+3)$$

input
```
Int[(3 + 4*x + x^2)^(-1),x]
```

output
```
Log[1 + x]/2 - Log[3 + x]/2
```

3.83.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 2.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

method	result	size
default	$-\frac{\ln(3+x)}{2}+\frac{\ln(1+x)}{2}$	14
norman	$-\frac{\ln(3+x)}{2}+\frac{\ln(1+x)}{2}$	14
risch	$-\frac{\ln(3+x)}{2}+\frac{\ln(1+x)}{2}$	14
parallelrisch	$-\frac{\ln(3+x)}{2}+\frac{\ln(1+x)}{2}$	14

input
```
int(1/(x^2+4*x+3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(3+x)+1/2*ln(1+x)
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int \frac{1}{3+4x+x^2}\,dx = -\frac{1}{2}\log\left(x+3\right)+\frac{1}{2}\log\left(x+1\right)$$

input
```
integrate(1/(x^2+4*x+3),x, algorithm="fricas")
```

output
```
-1/2*log(x + 3) + 1/2*log(x + 1)
```

3.83.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{1}{3+4x+x^2}\,dx = \frac{\log\left(x+1\right)}{2}-\frac{\log\left(x+3\right)}{2}$$

input
```
integrate(1/(x**2+4*x+3),x)
```

output
```
log(x + 1)/2 - log(x + 3)/2
```

3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int \frac{1}{3+4x+x^2}\,dx = -\frac{1}{2}\log{(x+3)} + \frac{1}{2}\log{(x+1)}$$

input
```
integrate(1/(x^2+4*x+3),x, algorithm="maxima")
```

output
```
-1/2*log(x + 3) + 1/2*log(x + 1)
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{1}{3+4x+x^2}\,dx = -\frac{1}{2}\log{(|x+3|)} + \frac{1}{2}\log{(|x+1|)}$$

input
```
integrate(1/(x^2+4*x+3),x, algorithm="giac")
```

output
```
-1/2*log(abs(x + 3)) + 1/2*log(abs(x + 1))
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{3+4x+x^2}\,dx = -\mathrm{atanh}(x+2)$$

input
```
int(1/(4*x + x^2 + 3),x)
```

output
```
-atanh(x + 2)
```

3.84 $\int \frac{1}{1+\pi x+2x^2}\,dx$

3.84.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1+\pi x+2x^2}\,dx=-\frac{2\text{arctanh}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

output
```
-2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\pi x+2x^2}\,dx=-\frac{2\text{arctanh}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

input
```
Integrate[(1 + Pi*x + 2*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

3.84.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2x^2+\pi x+1} \, dx$$

$$\Big\downarrow \; 1083$$

$$-2 \int \frac{1}{-(4x+\pi)^2+\pi^2-8} d(4x+\pi)$$

$$\Big\downarrow \; 219$$

$$-\frac{2 \text{arctanh}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

input
```
Int[(1 + Pi*x + 2*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

3.84.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.84.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\pi+4x}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$	24
risch	$\frac{\ln\left(-\pi^2+\pi\sqrt{\pi^2-8}+4x\sqrt{\pi^2-8}+8\right)}{\sqrt{\pi^2-8}}-\frac{\ln\left(\pi^2+\pi\sqrt{\pi^2-8}+4x\sqrt{\pi^2-8}-8\right)}{\sqrt{\pi^2-8}}$	71

input
```
int(1/(Pi*x+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{1}{1+\pi x+2x^2}\,dx=\frac{\log\left(\frac{\pi^2+4\,\pi x+8\,x^2-(\pi+4\,x)\sqrt{\pi^2-8}-4}{\pi x+2\,x^2+1}\right)}{\sqrt{\pi^2-8}}$$

input
```
integrate(1/(pi*x+2*x^2+1),x, algorithm="fricas")
```

output
```
log((pi^2 + 4*pi*x + 8*x^2 - (pi + 4*x)*sqrt(pi^2 - 8) - 4)/(pi*x + 2*x^2
+ 1))/sqrt(pi^2 - 8)
```

3.84.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.81

$$\int \frac{1}{1+\pi x+2x^2}\,dx=\frac{\log\left(x-\frac{\pi^2}{4\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{2}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}-\frac{\log\left(x-\frac{2}{\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{\pi^2}{4\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

input
```
integrate(1/(pi*x+2*x**2+1),x)
```

output
```
log(x - pi**2/(4*sqrt(-8 + pi**2)) + pi/4 + 2/sqrt(-8 + pi**2))/sqrt(-8 +
pi**2) - log(x - 2/sqrt(-8 + pi**2) + pi/4 + pi**2/(4*sqrt(-8 + pi**2)))/s
qrt(-8 + pi**2)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{1+\pi x+2x^2}\,dx = \frac{\log\left(\frac{\pi+4\,x-\sqrt{\pi^2-8}}{\pi+4\,x+\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

input
```
integrate(1/(pi*x+2*x^2+1),x, algorithm="maxima")
```

output
```
log((pi + 4*x - sqrt(pi^2 - 8))/(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^2 - 8
)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1}{1+\pi x+2x^2}\,dx = \frac{\log\left(\frac{\left|\pi+4\,x-\sqrt{\pi^2-8}\right|}{\left|\pi+4\,x+\sqrt{\pi^2-8}\right|}\right)}{\sqrt{\pi^2-8}}$$

input
```
integrate(1/(pi*x+2*x^2+1),x, algorithm="giac")
```

output
```
log(abs(pi + 4*x - sqrt(pi^2 - 8))/abs(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi
^2 - 8)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+\pi x+2x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\Pi+4\,x}{\sqrt{\Pi^2-8}}\right)}{\sqrt{\Pi^2-8}}$$

input
```
int(1/(Pi*x + 2*x^2 + 1),x)
```

output
```
-(2*atanh((Pi + 4*x)/(Pi^2 - 8)^(1/2)))/(Pi^2 - 8)^(1/2)
```

3.85 $\int \frac{1}{1+\pi x-2x^2} \, dx$

3.85.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1+\pi x-2x^2} \, dx = -\frac{2\text{arctanh}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

output
```
-2*arctanh((Pi-4*x)/(Pi^2+8)^(1/2))/(Pi^2+8)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{1+\pi x-2x^2} \, dx = \frac{2\text{arctanh}\left(\frac{-\pi+4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

input
```
Integrate[(1 + Pi*x - 2*x^2)^(-1),x]
```

output
```
(2*ArcTanh[(-Pi + 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

3.85.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-2x^2+\pi x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(\pi-4x)^2+\pi^2+8}d(\pi-4x)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

input
```
Int[(1 + Pi*x - 2*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(Pi - 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

3.85.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.85.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{-\pi+4x}{\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$	26
risch	$\frac{\ln\left(\pi^2-\pi\sqrt{\pi^2+8}+4x\sqrt{\pi^2+8}+8\right)}{\sqrt{\pi^2+8}}-\frac{\ln\left(-\pi^2-\pi\sqrt{\pi^2+8}+4x\sqrt{\pi^2+8}-8\right)}{\sqrt{\pi^2+8}}$	73

input
```
int(1/(Pi*x-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/(Pi^2+8)^(1/2)*arctanh((-Pi+4*x)/(Pi^2+8)^(1/2))
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{1}{1+\pi x-2x^2}\,dx = \frac{\log\left(-\frac{\pi^2-4\,\pi x+8\,x^2-(\pi-4\,x)\sqrt{\pi^2+8}+4}{\pi x-2\,x^2+1}\right)}{\sqrt{\pi^2+8}}$$

input
```
integrate(1/(pi*x-2*x^2+1),x, algorithm="fricas")
```

output
```
log(-(pi^2 - 4*pi*x + 8*x^2 - (pi - 4*x)*sqrt(pi^2 + 8) + 4)/(pi*x - 2*x^2
 + 1))/sqrt(pi^2 + 8)
```

3.85.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.81

$$\int \frac{1}{1+\pi x-2x^2}\,dx = -\frac{\log\left(x-\frac{\pi}{4}-\frac{\pi^2}{4\sqrt{8+\pi^2}}-\frac{2}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}+\frac{\log\left(x-\frac{\pi}{4}+\frac{2}{\sqrt{8+\pi^2}}+\frac{\pi^2}{4\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

input
```
integrate(1/(pi*x-2*x**2+1),x)
```

output
```
-log(x - pi/4 - pi**2/(4*sqrt(8 + pi**2)) - 2/sqrt(8 + pi**2))/sqrt(8 + pi
**2) + log(x - pi/4 + 2/sqrt(8 + pi**2) + pi**2/(4*sqrt(8 + pi**2)))/sqrt(
8 + pi**2)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{1+\pi x-2x^2}\,dx = -\frac{\log\left(\frac{\pi-4\,x+\sqrt{\pi^2+8}}{\pi-4\,x-\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

input
```
integrate(1/(pi*x-2*x^2+1),x, algorithm="maxima")
```

output
```
-log((pi - 4*x + sqrt(pi^2 + 8))/(pi - 4*x - sqrt(pi^2 + 8)))/sqrt(pi^2 +
8)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{1}{1+\pi x-2x^2}\,dx = -\frac{\log\left(\frac{\left|-\pi+4\,x-\sqrt{\pi^2+8}\right|}{\left|-\pi+4\,x+\sqrt{\pi^2+8}\right|}\right)}{\sqrt{\pi^2+8}}$$

input
```
integrate(1/(pi*x-2*x^2+1),x, algorithm="giac")
```

output
```
-log(abs(-pi + 4*x - sqrt(pi^2 + 8))/abs(-pi + 4*x + sqrt(pi^2 + 8)))/sqrt
(pi^2 + 8)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+\pi x-2x^2}\,dx=-\frac{2\,\mathrm{atanh}\left(\frac{\Pi-4\,x}{\sqrt{\Pi^2+8}}\right)}{\sqrt{\Pi^2+8}}$$

input
```
int(1/(Pi*x - 2*x^2 + 1),x)
```

output
```
-(2*atanh((Pi - 4*x)/(Pi^2 + 8)^(1/2)))/(Pi^2 + 8)^(1/2)
```

3.86 $\int \frac{1}{1+\pi x+3x^2} \, dx$

3.86.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{1+\pi x+3x^2} \, dx = \frac{2 \arctan \left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

output
```
2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\pi x+3x^2} \, dx = \frac{2 \arctan \left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

input
```
Integrate[(1 + Pi*x + 3*x^2)^(-1),x]
```

output
```
(2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

3.86.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2+\pi x+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$-2\int \frac{1}{-(6x+\pi)^2+\pi^2-12}d(6x+\pi)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\arctan\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

input
```
Int[(1 + Pi*x + 3*x^2)^(-1),x]
```

output
```
(2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

3.86.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.86.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$	28
risch	$\frac{\ln\left(-\pi^2+\pi\sqrt{\pi^2-12}+6x\sqrt{\pi^2-12}+12\right)}{\sqrt{\pi^2-12}}-\frac{\ln\left(\pi^2+\pi\sqrt{\pi^2-12}+6x\sqrt{\pi^2-12}-12\right)}{\sqrt{\pi^2-12}}$	71

input
```
int(1/(Pi*x+3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{1+\pi x+3x^2}\,dx = \frac{2\sqrt{-\pi^2+12}\arctan\left(\frac{(\pi+6x)\sqrt{-\pi^2+12}}{\pi^2-12}\right)}{\pi^2-12}$$

input
```
integrate(1/(pi*x+3*x^2+1),x, algorithm="fricas")
```

output
```
2*sqrt(-pi^2 + 12)*arctan((pi + 6*x)*sqrt(-pi^2 + 12)/(pi^2 - 12))/(pi^2 -
 12)
```

3.86.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.81

$$\int \frac{1}{1+\pi x+3x^2}\,dx = -\frac{i\log\left(x+\frac{\pi}{6}-\frac{2i}{\sqrt{12-\pi^2}}+\frac{i\pi^2}{6\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}+\frac{i\log\left(x+\frac{\pi}{6}-\frac{i\pi^2}{6\sqrt{12-\pi^2}}+\frac{2i}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

input
```
integrate(1/(pi*x+3*x**2+1),x)
```

output
```
-I*log(x + pi/6 - 2*I/sqrt(12 - pi**2) + I*pi**2/(6*sqrt(12 - pi**2)))/sqr
t(12 - pi**2) + I*log(x + pi/6 - I*pi**2/(6*sqrt(12 - pi**2)) + 2*I/sqrt(1
2 - pi**2))/sqrt(12 - pi**2)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{1+\pi x+3x^2}\,dx = \frac{2\,\arctan\left(\frac{\pi+6\,x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

input
```
integrate(1/(pi*x+3*x^2+1),x, algorithm="maxima")
```

output
```
2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{1+\pi x+3x^2}\,dx = \frac{2\,\arctan\left(\frac{\pi+6\,x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

input
```
integrate(1/(pi*x+3*x^2+1),x, algorithm="giac")
```

output
```
2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{1+\pi x+3x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\Pi+6\,x}{\sqrt{\Pi^2-12}}\right)}{\sqrt{\Pi^2-12}}$$

input
```
int(1/(Pi*x + 3*x^2 + 1),x)
```

output
```
-(2*atanh((Pi + 6*x)/(Pi^2 - 12)^(1/2)))/(Pi^2 - 12)^(1/2)
```

3.87 $\int \frac{1}{1+\pi x-3x^2} dx$

3.87.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1+\pi x-3x^2} dx = -\frac{2\text{arctanh}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

output
```
-2*arctanh((Pi-6*x)/(Pi^2+12)^(1/2))/(Pi^2+12)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{1+\pi x-3x^2} dx = \frac{2\text{arctanh}\left(\frac{-\pi+6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

input
```
Integrate[(1 + Pi*x - 3*x^2)^(-1),x]
```

output
```
(2*ArcTanh[(-Pi + 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

3.87.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3x^2+\pi x+1}dx$$

$$\Big\downarrow \ 1083$$

$$-2\int \frac{1}{-(\pi-6x)^2+\pi^2+12}d(\pi-6x)$$

$$\Big\downarrow \ 219$$

$$-\frac{2\text{arctanh}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

input
```
Int[(1 + Pi*x - 3*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(Pi - 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

3.87.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.87.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{-\pi+6x}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$	26
risch	$\frac{\ln\left(\pi^2-\pi\sqrt{\pi^2+12}+6x\sqrt{\pi^2+12}+12\right)}{\sqrt{\pi^2+12}}-\frac{\ln\left(-\pi^2-\pi\sqrt{\pi^2+12}+6x\sqrt{\pi^2+12}-12\right)}{\sqrt{\pi^2+12}}$	73

input
```
int(1/(Pi*x-3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/(Pi^2+12)^(1/2)*arctanh((-Pi+6*x)/(Pi^2+12)^(1/2))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(23) = 46$.

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{1}{1+\pi x-3x^2}\,dx = \frac{\log\left(-\frac{\pi^2-6\,\pi x+18\,x^2-(\pi-6\,x)\sqrt{\pi^2+12}+6}{\pi x-3\,x^2+1}\right)}{\sqrt{\pi^2+12}}$$

input
```
integrate(1/(pi*x-3*x^2+1),x, algorithm="fricas")
```

output
```
log(-(pi^2 - 6*pi*x + 18*x^2 - (pi - 6*x)*sqrt(pi^2 + 12) + 6)/(pi*x - 3*x
^2 + 1))/sqrt(pi^2 + 12)
```

3.87.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.81

$$\int \frac{1}{1+\pi x-3x^2}\,dx = \frac{\log\left(x-\frac{\pi}{6}+\frac{\pi^2}{6\sqrt{\pi^2+12}}+\frac{2}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}-\frac{\log\left(x-\frac{\pi}{6}-\frac{2}{\sqrt{\pi^2+12}}-\frac{\pi^2}{6\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

input
```
integrate(1/(pi*x-3*x**2+1),x)
```

output
```
log(x - pi/6 + pi**2/(6*sqrt(pi**2 + 12)) + 2/sqrt(pi**2 + 12))/sqrt(pi**2
 + 12) - log(x - pi/6 - 2/sqrt(pi**2 + 12) - pi**2/(6*sqrt(pi**2 + 12)))/s
qrt(pi**2 + 12)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{1+\pi x-3x^2}\,dx = -\frac{\log\left(\frac{\pi-6\,x+\sqrt{\pi^2+12}}{\pi-6\,x-\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

input
```
integrate(1/(pi*x-3*x^2+1),x, algorithm="maxima")
```

output
```
-log((pi - 6*x + sqrt(pi^2 + 12))/(pi - 6*x - sqrt(pi^2 + 12)))/sqrt(pi^2
+ 12)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{1}{1+\pi x-3x^2}\,dx = -\frac{\log\left(\frac{\left|-\pi+6\,x-\sqrt{\pi^2+12}\right|}{\left|-\pi+6\,x+\sqrt{\pi^2+12}\right|}\right)}{\sqrt{\pi^2+12}}$$

input
```
integrate(1/(pi*x-3*x^2+1),x, algorithm="giac")
```

output
```
-log(abs(-pi + 6*x - sqrt(pi^2 + 12))/abs(-pi + 6*x + sqrt(pi^2 + 12)))/sq
rt(pi^2 + 12)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+\pi x-3x^2}\,dx=-\frac{2\,\mathrm{atanh}\left(\frac{\Pi-6\,x}{\sqrt{\Pi^2+12}}\right)}{\sqrt{\Pi^2+12}}$$

input
```
int(1/(Pi*x - 3*x^2 + 1),x)
```

output
```
-(2*atanh((Pi - 6*x)/(Pi^2 + 12)^(1/2)))/(Pi^2 + 12)^(1/2)
```

3.88 $\int \frac{1}{a+cx+bx^2} dx$

3.88.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{1}{a+cx+bx^2} dx = \frac{2\arctan\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

output
```
2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+cx+bx^2} dx = \frac{2\arctan\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

input
```
Integrate[(a + c*x + b*x^2)^(-1),x]
```

output
```
(2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

3.88.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2+cx}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{c^2-(c+2bx)^2-4ab}d(c+2bx)$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

input
```
Int[(a + c*x + b*x^2)^(-1),x]
```

output
```
(2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

3.88.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.88.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{2\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$	35
risch	$-\frac{\ln\left(2bx+\sqrt{-4ab+c^2}+c\right)}{\sqrt{-4ab+c^2}}+\frac{\ln\left(-2bx+\sqrt{-4ab+c^2}-c\right)}{\sqrt{-4ab+c^2}}$	61

input
```
int(1/(b*x^2+c*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.97

$$\int \frac{1}{a+cx+bx^2}\,dx = \left[-\frac{\sqrt{-4\,ab+c^2}\log\left(\frac{2\,b^2x^2+2\,bcx-2\,ab+c^2-\sqrt{-4\,ab+c^2}(2\,bx+c)}{bx^2+cx+a}\right)}{4\,ab-c^2},\right.$$
$$\left.-\frac{2\,\arctan\left(-\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}\right]$$

input
```
integrate(1/(b*x^2+c*x+a),x, algorithm="fricas")
```

output
```
[-sqrt(-4*a*b + c^2)*log((2*b^2*x^2 + 2*b*c*x - 2*a*b + c^2 - sqrt(-4*a*b
+ c^2)*(2*b*x + c))/(b*x^2 + c*x + a))/(4*a*b - c^2), -2*arctan(-(2*b*x +
c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)]
```

3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.26

$$\int \frac{1}{a+cx+bx^2}\,dx = -\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{-4ab\sqrt{-\frac{1}{4ab-c^2}}+c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right) + \sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{4ab\sqrt{-\frac{1}{4ab-c^2}}-c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)$$

input
```
integrate(1/(b*x**2+c*x+a),x)
```

output
```
-sqrt(-1/(4*a*b - c**2))*log(x + (-4*a*b*sqrt(-1/(4*a*b - c**2)) + c**2*sq
rt(-1/(4*a*b - c**2)) + c)/(2*b)) + sqrt(-1/(4*a*b - c**2))*log(x + (4*a*b
*sqrt(-1/(4*a*b - c**2)) - c**2*sqrt(-1/(4*a*b - c**2)) + c)/(2*b))
```

3.88.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+cx+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+c*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for
 more deta
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+cx+bx^2}\,dx = \frac{2\,\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}$$

input
```
integrate(1/(b*x^2+c*x+a),x, algorithm="giac")
```

output
```
2*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{a+cx+bx^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{4\,a\,b-c^2}}+\frac{2\,b\,x}{\sqrt{4\,a\,b-c^2}}\right)}{\sqrt{4\,a\,b-c^2}}$$

input
```
int(1/(a + c*x + b*x^2),x)
```

output
```
(2*atan(c/(4*a*b - c^2)^(1/2) + (2*b*x)/(4*a*b - c^2)^(1/2)))/(4*a*b - c^2
)^(1/2)
```

3.89 $\int \frac{1}{b+2ax+bx^2}\,dx$

3.89.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{b+2ax+bx^2}\,dx=-\frac{\text{arctanh}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
-arctanh((b*x+a)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{b+2ax+bx^2}\,dx=\frac{\arctan\left(\frac{a+bx}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
Integrate[(b + 2*a*x + b*x^2)^(-1),x]
```

output
```
ArcTan[(a + b*x)/Sqrt[-a^2 + b^2]]/Sqrt[-a^2 + b^2]
```

3.89.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2ax+bx^2+b}dx$$

$$\Big\downarrow \; 1083$$

$$-2\int \frac{1}{4(a^2-b^2)-(2a+2bx)^2}d(2a+2bx)$$

$$\Big\downarrow \; 219$$

$$-\frac{\text{arctanh}\left(\frac{2a+2bx}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input

```
Int[(b + 2*a*x + b*x^2)^(-1),x]
```

output

```
-(ArcTanh[(2*a + 2*b*x)/(2*Sqrt[a^2 - b^2])]/Sqrt[a^2 - b^2])
```

3.89.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.89.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	35
risch	$\frac{\ln\left(-bx+\sqrt{a^2-b^2}-a\right)}{2\sqrt{a^2-b^2}}-\frac{\ln\left(bx+\sqrt{a^2-b^2}+a\right)}{2\sqrt{a^2-b^2}}$	65

input
```
int(1/(b*x^2+2*a*x+b),x,method=_RETURNVERBOSE)
```

output
```
1/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.54

$$\int \frac{1}{b+2ax+bx^2}\,dx = \left[\frac{\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}(bx+a)}{bx^2+2\,ax+b}\right)}{2\,\sqrt{a^2-b^2}}, -\frac{\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(bx+a)}{a^2-b^2}\right)}{a^2-b^2}\right]$$

input
```
integrate(1/(b*x^2+2*a*x+b),x, algorithm="fricas")
```

output
```
[1/2*log((b^2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(
b*x^2 + 2*a*x + b))/sqrt(a^2 - b^2), -sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*x + a)/(a^2 - b^2))/(a^2 - b^2)]
```

3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.86

$$\int \frac{1}{b+2ax+bx^2}\,dx = \frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a+b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2} - \frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a-b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}$$

input
```
integrate(1/(b*x**2+2*a*x+b),x)
```

output
```
sqrt(1/((a - b)*(a + b)))*log(x + (-a**2*sqrt(1/((a - b)*(a + b))) + a + b
**2*sqrt(1/((a - b)*(a + b))))/b)/2 - sqrt(1/((a - b)*(a + b)))*log(x + (a
**2*sqrt(1/((a - b)*(a + b))) + a - b**2*sqrt(1/((a - b)*(a + b))))/b)/2
```

3.89.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{b+2ax+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+2*a*x+b),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1}{b+2ax+bx^2}\,dx = \frac{\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
integrate(1/(b*x^2+2*a*x+b),x, algorithm="giac")
```

output
```
arctan((b*x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{b+2ax+bx^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{a+b\,x}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(1/(b + 2*a*x + b*x^2),x)
```

output
```
-atanh((a + b*x)/((a + b)^(1/2)*(a - b)^(1/2)))/((a + b)^(1/2)*(a - b)^(1/
2))
```

3.90 $\int \frac{1}{b+2ax-bx^2}\,dx$

3.90.1 Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{1}{b+2ax-bx^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

output
```
-arctanh((-b*x+a)/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{1}{b+2ax-bx^2}\,dx = -\frac{\arctan\left(\frac{-a+bx}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

input
```
Integrate[(b + 2*a*x - b*x^2)^(-1),x]
```

output
```
-(ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]]/Sqrt[-a^2 - b^2])
```

3.90.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2ax-bx^2+b}dx$$

$$\Big\downarrow \ 1083$$

$$-2\int \frac{1}{4\left(a^2+b^2\right)-(2a-2bx)^2}d(2a-2bx)$$

$$\Big\downarrow \ 219$$

$$-\frac{\text{arctanh}\left(\frac{2a-2bx}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
Int[(b + 2*a*x - b*x^2)^(-1),x]
```

output
```
-(ArcTanh[(2*a - 2*b*x)/(2*Sqrt[a^2 + b^2])]/Sqrt[a^2 + b^2])
```

3.90.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.90.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{-2bx+2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	32
risch	$\frac{\ln\left(bx+\sqrt{a^2+b^2}-a\right)}{2\sqrt{a^2+b^2}}-\frac{\ln\left(-bx+\sqrt{a^2+b^2}+a\right)}{2\sqrt{a^2+b^2}}$	57

input
```
int(1/(-b*x^2+2*a*x+b),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*x+2*a)/(a^2+b^2)^(1/2))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.09

$$\int \frac{1}{b+2ax-bx^2}\,dx = \frac{\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}(bx-a)}{bx^2-2\,ax-b}\right)}{2\,\sqrt{a^2+b^2}}$$

input
```
integrate(1/(-b*x^2+2*a*x+b),x, algorithm="fricas")
```

output
```
1/2*log((b^2*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b
*x^2 - 2*a*x - b))/sqrt(a^2 + b^2)
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(27) = 54.

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.19

$$\int \frac{1}{b+2ax-bx^2}\,dx = -\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{a^2+b^2}}-a-b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2} + \frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{a^2\sqrt{\frac{1}{a^2+b^2}}-a+b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}$$

input
```
integrate(1/(-b*x**2+2*a*x+b),x)
```

output
```
-sqrt(1/(a**2 + b**2))*log(x + (-a**2*sqrt(1/(a**2 + b**2)) - a - b**2*sqr
t(1/(a**2 + b**2)))/b)/2 + sqrt(1/(a**2 + b**2))*log(x + (a**2*sqrt(1/(a**
2 + b**2)) - a + b**2*sqrt(1/(a**2 + b**2)))/b)/2
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

$$\int \frac{1}{b+2ax-bx^2}\,dx=-\frac{\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}$$

input
```
integrate(1/(-b*x^2+2*a*x+b),x, algorithm="maxima")
```

output
```
-1/2*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/sqrt(a^2
 + b^2)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{1}{b+2ax-bx^2}\,dx=-\frac{\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\sqrt{a^2+b^2}}$$

input
```
integrate(1/(-b*x^2+2*a*x+b),x, algorithm="giac")
```

output
```
-1/2*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{b+2ax-bx^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{a-b\,x}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
int(1/(b + 2*a*x - b*x^2),x)
```

output
```
-atanh((a - b*x)/(a^2 + b^2)^(1/2))/(a^2 + b^2)^(1/2)
```

3.91 $\int \frac{1}{(2+4x+3x^2)^2} dx$

3.91.1 Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(2+4x+3x^2)^2} dx = \frac{2+3x}{4(2+4x+3x^2)} + \frac{3\arctan\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
1/4*(2+3*x)/(3*x^2+4*x+2)+3/8*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+4x+3x^2)^2} dx = \frac{2+3x}{4(2+4x+3x^2)} + \frac{3\arctan\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Integrate[(2 + 4*x + 3*x^2)^(-2),x]
```

output
```
(2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2]
)
```

3.91.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+4x+2)^2}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{4}\int \frac{1}{3x^2+4x+2}dx+\frac{3x+2}{4(3x^2+4x+2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3x+2}{4(3x^2+4x+2)}-\frac{3}{2}\int \frac{1}{-(6x+4)^2-8}d(6x+4)$$

$$\Big\downarrow \text{217}$$

$$\frac{3\arctan\left(\frac{6x+4}{2\sqrt{2}}\right)}{4\sqrt{2}}+\frac{3x+2}{4(3x^2+4x+2)}$$

input
```
Int[(2 + 4*x + 3*x^2)^(-2),x]
```

output
```
(2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(4 + 6*x)/(2*Sqrt[2])])/(4*Sqr
t[2])
```

3.91.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.91.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{\frac{x}{4}+\frac{1}{6}}{x^2+\frac{4}{3}x+\frac{2}{3}}+\frac{3\arctan\left(\frac{(2+3x)\sqrt{2}}{2}\right)\sqrt{2}}{8}$	34
default	$\frac{4+6x}{24x^2+32x+16}+\frac{3\sqrt{2}\,\arctan\left(\frac{(4+6x)\sqrt{2}}{4}\right)}{8}$	37

input
```
int(1/(3*x^2+4*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x+1/6)/(x^2+4/3*x+2/3)+3/8*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{3\sqrt{2}(3\,x^2+4\,x+2)\arctan\left(\frac{1}{2}\sqrt{2}(3\,x+2)\right)+6\,x+4}{8\,(3\,x^2+4\,x+2)}$$

input
```
integrate(1/(3*x^2+4*x+2)^2,x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(2)*(3*x^2 + 4*x + 2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 6*x + 4)/
(3*x^2 + 4*x + 2)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{3x+2}{12x^2+16x+8} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{8}$$

input
```
integrate(1/(3*x**2+4*x+2)**2,x)
```

output
```
(3*x + 2)/(12*x**2 + 16*x + 8) + 3*sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/8
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{3x+2}{4(3x^2+4x+2)}$$

input
```
integrate(1/(3*x^2+4*x+2)^2,x, algorithm="maxima")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2
)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{3x+2}{4(3x^2+4x+2)}$$

input
```
integrate(1/(3*x^2+4*x+2)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2
)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{\frac{x}{4}+\frac{1}{6}}{x^2+\frac{4x}{3}+\frac{2}{3}} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}\,x}{2}+\sqrt{2}\right)}{8}$$

input
```
int(1/(4*x + 3*x^2 + 2)^2,x)
```

output
```
(x/4 + 1/6)/((4*x)/3 + x^2 + 2/3) + (3*2^(1/2)*atan((3*2^(1/2)*x)/2 + 2^(1
/2)))/8
```

3.92 $\int \frac{1}{(2+4x-3x^2)^2} dx$

3.92.1 Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(2+4x-3x^2)^2} dx = -\frac{2-3x}{20(2+4x-3x^2)} - \frac{3\text{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

output
```
1/20*(-2+3*x)/(-3*x^2+4*x+2)-3/200*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{1}{(2+4x-3x^2)^2} dx = \frac{2-3x}{20(-2-4x+3x^2)} - \frac{3\log\left(2+\sqrt{10}-3x\right)}{40\sqrt{10}} + \frac{3\log\left(-2+\sqrt{10}+3x\right)}{40\sqrt{10}}$$

input
```
Integrate[(2 + 4*x - 3*x^2)^(-2),x]
```

output
```
(2 - 3*x)/(20*(-2 - 4*x + 3*x^2)) - (3*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[1
0]) + (3*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

3.92.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1086, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2+4x+2)^2}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{20}\int \frac{1}{-3x^2+4x+2}dx-\frac{2-3x}{20(-3x^2+4x+2)}$$

$$\Big\downarrow \text{1081}$$

$$-\frac{9}{20}\int \left(\frac{1}{2\sqrt{10}(-3x-\sqrt{10}+2)}-\frac{1}{2\sqrt{10}(-3x+\sqrt{10}+2)}\right)dx-\frac{2-3x}{20(-3x^2+4x+2)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2-3x}{20(-3x^2+4x+2)}-\frac{9}{20}\left(\frac{\log\left(-3x+\sqrt{10}+2\right)}{6\sqrt{10}}-\frac{\log\left(-3x-\sqrt{10}+2\right)}{6\sqrt{10}}\right)$$

input
```
Int[(2 + 4*x - 3*x^2)^(-2),x]
```

output
```
-1/20*(2 - 3*x)/(2 + 4*x - 3*x^2) - (9*(-1/6*Log[2 - Sqrt[10] - 3*x]/Sqrt[
10] + Log[2 + Sqrt[10] - 3*x]/(6*Sqrt[10])))/20
```

3.92.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.92.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{-4+6x}{40(3x^2-4x-2)}+\frac{3\sqrt{10}\,\operatorname{arctanh}\left(\frac{(-4+6x)\sqrt{10}}{20}\right)}{200}$	37
risch	$\frac{-\frac{x}{20}+\frac{1}{30}}{x^2-\frac{4}{3}x-\frac{2}{3}}+\frac{3\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{400}-\frac{3\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{400}$	48

input
```
int(1/(-3*x^2+4*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(-4+6*x)/(3*x^2-4*x-2)+3/200*10^(1/2)*arctanh(1/20*(-4+6*x)*10^(1/2)
)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx=\frac{3\sqrt{10}(3x^2-4x-2)\log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right)-60x+40}{400\,(3x^2-4x-2)}$$

input
```
integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="fricas")
```

output
```
1/400*(3*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12
*x + 14)/(3*x^2 - 4*x - 2)) - 60*x + 40)/(3*x^2 - 4*x - 2)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = \frac{2-3x}{60x^2-80x-40} + \frac{3\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400} - \frac{3\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

input
```
integrate(1/(-3*x**2+4*x+2)**2,x)
```

output
```
(2 - 3*x)/(60*x**2 - 80*x - 40) + 3*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/400
 - 3*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = -\frac{3}{400}\sqrt{10}\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right) - \frac{3\,x-2}{20\,(3\,x^2-4\,x-2)}$$

input
```
integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="maxima")
```

output
```
-3/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(3*x
 - 2)/(3*x^2 - 4*x - 2)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = -\frac{3}{400}\sqrt{10}\log\left(\frac{|6\,x-2\,\sqrt{10}-4|}{|6\,x+2\,\sqrt{10}-4|}\right) - \frac{3\,x-2}{20\,(3\,x^2-4\,x-2)}$$

input
```
integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="giac")
```

output
```
-3/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
 1/20*(3*x - 2)/(3*x^2 - 4*x - 2)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = \frac{3\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{200} + \frac{\frac{x}{20}-\frac{1}{30}}{-x^2+\frac{4x}{3}+\frac{2}{3}}$$

input
```
int(1/(4*x - 3*x^2 + 2)^2,x)
```

output
```
(3*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + (x/20 - 1/30)/((4*x)/3
 - x^2 + 2/3)
```

3.93 $\int \frac{1}{(2+5x+3x^2)^2} dx$

3.93.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int \frac{1}{(2+5x+3x^2)^2} dx = -\frac{5+6x}{2+5x+3x^2} + 6\log(1+x) - 6\log(2+3x)$$

output
```
(-5-6*x)/(3*x^2+5*x+2)+6*ln(1+x)-6*ln(2+3*x)
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{1}{(2+5x+3x^2)^2} dx = \frac{-5-6x}{2+5x+3x^2} + 6\log(1+x) - 6\log(2+3x)$$

input
```
Integrate[(2 + 5*x + 3*x^2)^(-2),x]
```

output
```
(-5 - 6*x)/(2 + 5*x + 3*x^2) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

3.93.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+5x+2)^2}dx$$
$$\Big\downarrow \text{1084}$$
$$9\int \left(-\frac{2}{3x+2}+\frac{1}{(3x+2)^2}+\frac{2}{3(x+1)}+\frac{1}{9(x+1)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$9\left(-\frac{1}{9(x+1)}-\frac{1}{3(3x+2)}+\frac{2}{3}\log(x+1)-\frac{2}{3}\log(3x+2)\right)$$

input
```
Int[(2 + 5*x + 3*x^2)^(-2),x]
```

output
```
9*(-1/9*1/(1 + x) - 1/(3*(2 + 3*x)) + (2*Log[1 + x])/3 - (2*Log[2 + 3*x])/
3)
```

3.93.3.1 Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.93.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{1+x}+6\ln\left(1+x\right)-\frac{3}{2+3x}-6\ln\left(2+3x\right)$	32
risch	$\frac{-2x-\frac{5}{3}}{x^{2}+\frac{5}{3}x+\frac{2}{3}}+6\ln\left(1+x\right)-6\ln\left(2+3x\right)$	32
norman	$\frac{\frac{15}{2}x^{2}+\frac{13}{2}x}{3x^{2}+5x+2}+6\ln\left(1+x\right)-6\ln\left(2+3x\right)$	38
parallelrisch	$\frac{36\ln(1+x)x^{2}-36\ln\left(\frac{2}{3}+x\right)x^{2}+60\ln(1+x)x-60\ln\left(\frac{2}{3}+x\right)x+15x^{2}+24\ln(1+x)-24\ln\left(\frac{2}{3}+x\right)+13x}{6x^{2}+10x+4}$	68

input
```
int(1/(3*x^2+5*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(1+x)+6*ln(1+x)-3/(2+3*x)-6*ln(2+3*x)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(2+5x+3x^2\right)^2}\,dx = -\frac{6\,\left(3\,x^{2}+5\,x+2\right)\log\left(3\,x+2\right)-6\,\left(3\,x^{2}+5\,x+2\right)\log\left(x+1\right)+6\,x+5}{3\,x^{2}+5\,x+2}$$

input
```
integrate(1/(3*x^2+5*x+2)^2,x, algorithm="fricas")
```

output
```
-(6*(3*x^2 + 5*x + 2)*log(3*x + 2) - 6*(3*x^2 + 5*x + 2)*log(x + 1) + 6*x
+ 5)/(3*x^2 + 5*x + 2)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = \frac{-6x-5}{3x^2+5x+2} - 6\log\left(x+\frac{2}{3}\right) + 6\log(x+1)$$

input
```
integrate(1/(3*x**2+5*x+2)**2,x)
```

output
```
(-6*x - 5)/(3*x**2 + 5*x + 2) - 6*log(x + 2/3) + 6*log(x + 1)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = -\frac{6\,x+5}{3\,x^2+5\,x+2} - 6\,\log(3\,x+2) + 6\,\log(x+1)$$

input
```
integrate(1/(3*x^2+5*x+2)^2,x, algorithm="maxima")
```

output
```
-(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(3*x + 2) + 6*log(x + 1)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = -\frac{6\,x+5}{3\,x^2+5\,x+2} - 6\,\log(|3\,x+2|) + 6\,\log(|x+1|)$$

input
```
integrate(1/(3*x^2+5*x+2)^2,x, algorithm="giac")
```

output
```
-(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(abs(3*x + 2)) + 6*log(abs(x + 1))
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = -6\,\ln\left(\frac{3\,x+2}{x+1}\right) - \frac{2\,\left(3\,x+\frac{5}{2}\right)}{3\,x^2+5\,x+2}$$

input
```
int(1/(5*x + 3*x^2 + 2)^2,x)
```

output
```
- 6*log((3*x + 2)/(x + 1)) - (2*(3*x + 5/2))/(5*x + 3*x^2 + 2)
```

3.94 $\int \frac{1}{(2+5x-3x^2)^2}\,dx$

3.94.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = -\frac{5-6x}{49(2+5x-3x^2)} - \frac{6}{343}\log(2-x) + \frac{6}{343}\log(1+3x)$$

output
```
1/49*(-5+6*x)/(-3*x^2+5*x+2)-6/343*ln(2-x)+6/343*ln(1+3*x)
```

3.94.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = \frac{5-6x}{49(-2-5x+3x^2)} - \frac{6}{343}\log(2-x) + \frac{6}{343}\log(1+3x)$$

input
```
Integrate[(2 + 5*x - 3*x^2)^(-2),x]
```

output
```
(5 - 6*x)/(49*(-2 - 5*x + 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/
343
```

3.94.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2+5x+2)^2} dx$$

$$\Big\downarrow \text{1084}$$

$$9\int \left(\frac{2}{343(3x+1)}+\frac{1}{49(3x+1)^2}+\frac{2}{1029(2-x)}+\frac{1}{441(2-x)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$9\left(\frac{1}{441(2-x)}-\frac{1}{147(3x+1)}-\frac{2\log(2-x)}{1029}+\frac{2\log(3x+1)}{1029}\right)$$

input
```
Int[(2 + 5*x - 3*x^2)^(-2),x]
```

output
```
9*(1/(441*(2 - x)) - 1/(147*(1 + 3*x)) - (2*Log[2 - x])/1029 + (2*Log[1 +
3*x])/1029)
```

3.94.3.1 Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.94.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{1}{49(-2+x)}-\frac{6\ln(-2+x)}{343}-\frac{3}{49(3x+1)}+\frac{6\ln(3x+1)}{343}$	32
risch	$\frac{-\frac{2x}{49}+\frac{5}{147}}{x^2-\frac{5}{3}x-\frac{2}{3}}-\frac{6\ln(-2+x)}{343}+\frac{6\ln(3x+1)}{343}$	32
norman	$\frac{\frac{15}{98}x^2-\frac{37}{98}x}{3x^2-5x-2}-\frac{6\ln(-2+x)}{343}+\frac{6\ln(3x+1)}{343}$	38
parallelrisch	$-\frac{36\ln(-2+x)x^2-36\ln\left(x+\frac{1}{3}\right)x^2-60\ln(-2+x)x+60\ln\left(x+\frac{1}{3}\right)x-105x^2-24\ln(-2+x)+24\ln\left(x+\frac{1}{3}\right)+259x}{686(3x^2-5x-2)}$	68

input
```
int(1/(-3*x^2+5*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/49/(-2+x)-6/343*ln(-2+x)-3/49/(3*x+1)+6/343*ln(3*x+1)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = \frac{6\,(3\,x^2-5\,x-2)\log\left(3\,x+1\right)-6\,(3\,x^2-5\,x-2)\log\left(x-2\right)-42\,x+35}{343\,(3\,x^2-5\,x-2)}$$

input
```
integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="fricas")
```

output
```
1/343*(6*(3*x^2 - 5*x - 2)*log(3*x + 1) - 6*(3*x^2 - 5*x - 2)*log(x - 2) -
 42*x + 35)/(3*x^2 - 5*x - 2)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = \frac{5-6x}{147x^2-245x-98} - \frac{6\log(x-2)}{343} + \frac{6\log\left(x+\frac{1}{3}\right)}{343}$$

input
```
integrate(1/(-3*x**2+5*x+2)**2,x)
```

output
```
(5 - 6*x)/(147*x**2 - 245*x - 98) - 6*log(x - 2)/343 + 6*log(x + 1/3)/343
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = -\frac{6\,x-5}{49\,(3\,x^2-5\,x-2)} + \frac{6}{343}\,\log(3\,x+1) - \frac{6}{343}\,\log(x-2)$$

input
```
integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="maxima")
```

output
```
-1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(3*x + 1) - 6/343*log(x - 2)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = -\frac{6\,x-5}{49\,(3\,x^2-5\,x-2)} + \frac{6}{343}\,\log(|3\,x+1|) - \frac{6}{343}\,\log(|x-2|)$$

input
```
integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="giac")
```

output
```
-1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(abs(3*x + 1)) - 6/343*log(ab
s(x - 2))
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = \frac{6\,\ln\left(\frac{3\,x+1}{x-2}\right)}{343} + \frac{2\,\left(3\,x-\frac{5}{2}\right)}{49\,\left(-3\,x^2+5\,x+2\right)}$$

input
```
int(1/(5*x - 3*x^2 + 2)^2,x)
```

output
```
(6*log((3*x + 1)/(x - 2)))/343 + (2*(3*x - 5/2))/(49*(5*x - 3*x^2 + 2))
```

3.95 $\int \frac{1}{(a+cx+bx^2)^2}\,dx$

3.95.1 Optimal result

Integrand size = 12, antiderivative size = 71

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx=\frac{c+2bx}{(4ab-c^2)(a+cx+bx^2)}+\frac{4b\arctan\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

output
```
(2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2)
)/(4*a*b-c^2)^(3/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx=\frac{c+2bx}{(4ab-c^2)(a+x(c+bx))}+\frac{4b\arctan\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

input
```
Integrate[(a + c*x + b*x^2)^(-2),x]
```

output
```
(c + 2*b*x)/((4*a*b - c^2)*(a + x*(c + b*x))) + (4*b*ArcTan[(c + 2*b*x)/Sq
rt[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

3.95.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx)^2}\,dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{2b\int \frac{1}{bx^2+cx+a}dx}{4ab-c^2}+\frac{2bx+c}{(4ab-c^2)\,(a+bx^2+cx)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2bx+c}{(4ab-c^2)\,(a+bx^2+cx)}-\frac{4b\int \frac{1}{c^2-(c+2bx)^2-4ab}d(c+2bx)}{4ab-c^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{4b\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}+\frac{2bx+c}{(4ab-c^2)\,(a+bx^2+cx)}$$

input
```
Int[(a + c*x + b*x^2)^(-2),x]
```

output
```
(c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*ArcTan[(c + 2*b*x)/Sq
rt[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

3.95.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.95.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

method	result	siz
default	$\frac{2bx+c}{(4ab-c^2)(b\,x^2+cx+a)}+\frac{4b\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2bx}{4ab-c^2}+\frac{c}{4ab-c^2}}{b\,x^2+cx+a}+\frac{2b\ln\left(\left(-8a\,b^2+2b\,c^2\right)x+\left(-4ab+c^2\right)^{\frac{3}{2}}-4abc+c^3\right)}{\left(-4ab+c^2\right)^{\frac{3}{2}}}-\frac{2b\ln\left(\left(8a\,b^2-2b\,c^2\right)x+\left(-4ab+c^2\right)^{\frac{3}{2}}+4abc-c^3\right)}{\left(-4ab+c^2\right)^{\frac{3}{2}}}$	14

input
```
int(1/(b*x^2+c*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2)
)/(4*a*b-c^2)^(3/2)
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(67) = 134.

Time = 0.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.70

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = \left[\frac{4\,abc-c^3+2\,(b^2x^2+bcx+ab)\sqrt{-4\,ab+c^2}\log\left(\frac{2\,b^2x^2+2\,bcx-2\,ab+c^2+\sqrt{-4\,ab+c^2}(2\,bx+c)}{bx^2+cx+a}\right)+2\,(4\,ab^2-bc^2)x}{16\,a^3b^2-8\,a^2bc^2+ac^4+(16\,a^2b^3-8\,ab^2c^2+bc^4)x^2+(16\,a^2b^2c-8\,abc^3+c^5)x},\right.$$

input
```
integrate(1/(b*x^2+c*x+a)^2,x, algorithm="fricas")
```

output
```
[(4*a*b*c - c^3 + 2*(b^2*x^2 + b*c*x + a*b)*sqrt(-4*a*b + c^2)*log((2*b^2*
x^2 + 2*b*c*x - 2*a*b + c^2 + sqrt(-4*a*b + c^2)*(2*b*x + c))/(b*x^2 + c*x
 + a)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c^4 + (16*a^
2*b^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*c^3 + c^5)*x), (4
*a*b*c - c^3 - 4*(b^2*x^2 + b*c*x + a*b)*sqrt(4*a*b - c^2)*arctan(-(2*b*x
+ c)/sqrt(4*a*b - c^2)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2
 + a*c^4 + (16*a^2*b^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*
c^3 + c^5)*x)]
```

3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(60) = 120$.

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.73

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = -2b\sqrt{-\frac{1}{(4ab-c^2)^3}}\log\left(x+\frac{-32a^2b^3\sqrt{-\frac{1}{(4ab-c^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ab-c^2)^3}}-2bc^4\sqrt{-\frac{1}{(4ab-c^2)^3}}+2bc}{4b^2}\right) +2b\sqrt{-\frac{1}{(4ab-c^2)^3}}\log\left(x+\frac{32a^2b^3\sqrt{-\frac{1}{(4ab-c^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ab-c^2)^3}}+2bc^4\sqrt{-\frac{1}{(4ab-c^2)^3}}+2bc}{4b^2}\right) + \frac{2bx+c}{4a^2b-ac^2+x^2\cdot(4ab^2-bc^2)+x\,(4abc-c^3)}$$

input
```
integrate(1/(b*x**2+c*x+a)**2,x)
```

output
```
-2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (-32*a**2*b**3*sqrt(-1/(4*a*b - c*
*2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*b - c**2)**3) - 2*b*c**4*sqrt(-1/(4*
a*b - c**2)**3) + 2*b*c)/(4*b**2)) + 2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x
+ (32*a**2*b**3*sqrt(-1/(4*a*b - c**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*b
 - c**2)**3) + 2*b*c**4*sqrt(-1/(4*a*b - c**2)**3) + 2*b*c)/(4*b**2)) + (2
*b*x + c)/(4*a**2*b - a*c**2 + x**2*(4*a*b**2 - b*c**2) + x*(4*a*b*c - c**
3))
```

3.95.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+c*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for
 more deta
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = \frac{4\,b\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{(4\,ab-c^2)^{\frac{3}{2}}} + \frac{2\,bx+c}{(bx^2+cx+a)(4\,ab-c^2)}$$

input
```
integrate(1/(b*x^2+c*x+a)^2,x, algorithm="giac")
```

output
```
4*b*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/(4*a*b - c^2)^(3/2) + (2*b*x + c
)/((b*x^2 + c*x + a)*(4*a*b - c^2))
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = \frac{\frac{c}{4\,a\,b-c^2}+\frac{2\,b\,x}{4\,a\,b-c^2}}{b\,x^2+c\,x+a} - \frac{4\,b\,\mathrm{atan}\left(\frac{\left(\frac{2\,b\,\left(c^3-4\,a\,b\,c\right)}{\left(4\,a\,b-c^2\right)^{5/2}}-\frac{4\,b^2\,x}{\left(4\,a\,b-c^2\right)^{3/2}}\right)\,\left(4\,a\,b-c^2\right)}{2\,b}\right)}{\left(4\,a\,b-c^2\right)^{3/2}}$$

input
```
int(1/(a + c*x + b*x^2)^2,x)
```

output
```
(c/(4*a*b - c^2) + (2*b*x)/(4*a*b - c^2))/(a + c*x + b*x^2) - (4*b*atan(((
(2*b*(c^3 - 4*a*b*c))/(4*a*b - c^2)^(5/2) - (4*b^2*x)/(4*a*b - c^2)^(3/2))
*(4*a*b - c^2))/(2*b)))/(4*a*b - c^2)^(3/2)
```

3.95. $\int \frac{1}{(a+cx+bx^2)^2}\,dx$

3.96 $\int \frac{1}{(b+2ax+bx^2)^2}\,dx$

3.96.1 Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx=-\frac{a+bx}{2(a^2-b^2)(b+2ax+bx^2)}+\frac{b\,\text{arctanh}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}}$$

output
```
1/2*(-b*x-a)/(a^2-b^2)/(b*x^2+2*a*x+b)+1/2*b*arctanh((b*x+a)/(a^2-b^2)^(1/
2))/(a^2-b^2)^(3/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx=\frac{a+bx}{2(-a^2+b^2)(b+2ax+bx^2)}+\frac{b\arctan\left(\frac{a+bx}{\sqrt{-a^2+b^2}}\right)}{2(-a^2+b^2)^{3/2}}$$

input
```
Integrate[(b + 2*a*x + b*x^2)^(-2),x]
```

output
```
(a + b*x)/(2*(-a^2 + b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTan[(a + b*x)/Sqrt[
-a^2 + b^2]])/(2*(-a^2 + b^2)^(3/2))
```

3.96.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2ax+bx^2+b)^2}dx$$

$$\Big\downarrow \text{1086}$$

$$-\frac{b\int \frac{1}{bx^2+2ax+b}dx}{2(a^2-b^2)}-\frac{a+bx}{2(a^2-b^2)(2ax+bx^2+b)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{b\int \frac{1}{4(a^2-b^2)-(2a+2bx)^2}d(2a+2bx)}{a^2-b^2}-\frac{a+bx}{2(a^2-b^2)(2ax+bx^2+b)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\,\text{arctanh}\left(\frac{2a+2bx}{2\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}}-\frac{a+bx}{2(a^2-b^2)(2ax+bx^2+b)}$$

input
```
Int[(b + 2*a*x + b*x^2)^(-2),x]
```

output
```
-1/2*(a + b*x)/((a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(2*a + 2*b*x
)/(2*Sqrt[a^2 - b^2])])/(2*(a^2 - b^2)^(3/2))
```

3.96.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.96.4 Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{2bx+2a}{(-4a^2+4b^2)(b\,x^2+2ax+b)}+\frac{2b\arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{(-4a^2+4b^2)\sqrt{-a^2+b^2}}$	86
risch	$\frac{-\frac{bx}{4\left(a^2-b^2\right)}-\frac{a}{4\left(a^2-b^2\right)}}{\frac{1}{2}b\,x^2+ax+\frac{1}{2}b}+\frac{b\ln\left(\left(-a^2b+b^3\right)x-\left(a^2-b^2\right)^{\frac{3}{2}}-a^3+a\,b^2\right)}{4\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(a^2b-b^3\right)x-\left(a^2-b^2\right)^{\frac{3}{2}}+a^3-a\,b^2\right)}{4\left(a^2-b^2\right)^{\frac{3}{2}}}$	150

input
```
int(1/(b*x^2+2*a*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*b*x+2*a)/(-4*a^2+4*b^2)/(b*x^2+2*a*x+b)+2*b/(-4*a^2+4*b^2)/(-a^2+b^2)^(
1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(64) = 128.

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 4.40

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = \left[-\frac{2\,a^3-2\,ab^2+(b^2x^2+2\,abx+b^2)\sqrt{a^2-b^2}\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}(bx+a)}{bx^2+2\,ax+b}\right)+2\,(a^2b-b^3)x}{4\,(a^4b-2\,a^2b^3+b^5+(a^4b-2\,a^2b^3+b^5)x^2+2\,(a^5-2\,a^3b^2+ab^4)x)}, -\frac{a^3-ab^2-(b^2x^2+2\,abx+b^2)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(bx+a)}{a^2-b^2}\right)+(a^2b-b^3)x}{2\,(a^4b-2\,a^2b^3+b^5+(a^4b-2\,a^2b^3+b^5)x^2+2\,(a^5-2\,a^3b^2+ab^4)x)}\right]$$

input
```
integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a^3 - 2*a*b^2 + (b^2*x^2 + 2*a*b*x + b^2)*sqrt(a^2 - b^2)*log((b^
2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b*x^2 + 2*a*
x + b)) + 2*(a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3
 + b^5)*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x), -1/2*(a^3 - a*b^2 - (b^2*x^2
 + 2*a*b*x + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*x + a)/(a^2
 - b^2)) + (a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3
+ b^5)*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x)]
```

3.96.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(58) = 116$.

Time = 0.31 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.19

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = -\frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab-b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4} + \frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}-2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab+b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4} + \frac{-a-bx}{2a^2b-2b^3+x^2\cdot(2a^2b-2b^3)+x\,(4a^3-4ab^2)}$$

input
```
integrate(1/(b*x**2+2*a*x+b)**2,x)
```

output
```
-b*sqrt(1/((a - b)**3*(a + b)**3))*log(x + (-a**4*b*sqrt(1/((a - b)**3*(a
+ b)**3)) + 2*a**2*b**3*sqrt(1/((a - b)**3*(a + b)**3)) + a*b - b**5*sqrt(
1/((a - b)**3*(a + b)**3)))/b**2)/4 + b*sqrt(1/((a - b)**3*(a + b)**3))*lo
g(x + (a**4*b*sqrt(1/((a - b)**3*(a + b)**3)) - 2*a**2*b**3*sqrt(1/((a - b
)**3*(a + b)**3)) + a*b + b**5*sqrt(1/((a - b)**3*(a + b)**3)))/b**2)/4 +
(-a - b*x)/(2*a**2*b - 2*b**3 + x**2*(2*a**2*b - 2*b**3) + x*(4*a**3 - 4*a
*b**2))
```

3.96.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = -\frac{b\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{2\,(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{bx+a}{2\,(bx^2+2\,ax+b)(a^2-b^2)}$$

input
```
integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

output
```
-1/2*b*arctan((b*x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) -
 1/2*(b*x + a)/((b*x^2 + 2*a*x + b)*(a^2 - b^2))
```

3.96.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = -\frac{\frac{a}{2\,(a^2-b^2)}+\frac{b\,x}{2\,(a^2-b^2)}}{b\,x^2+2\,a\,x+b} + \frac{b\,\mathrm{atan}\left(\frac{-a^3\,1\mathrm{i}-1\mathrm{i}\,x\,a^2\,b+a\,b^2\,1\mathrm{i}+1\mathrm{i}\,x\,b^3}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,1\mathrm{i}}{2\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(1/(b + 2*a*x + b*x^2)^2,x)
```

output
```
(b*atan((a*b^2*1i + b^3*x*1i - a^3*1i - a^2*b*x*1i)/((a + b)^(3/2)*(a - b)
^(3/2)))*1i)/(2*(a + b)^(3/2)*(a - b)^(3/2)) - (a/(2*(a^2 - b^2)) + (b*x)/
(2*(a^2 - b^2)))/(b + 2*a*x + b*x^2)
```

3.97 $\int \frac{1}{(b+2ax-bx^2)^2} dx$

3.97.1 Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{(b+2ax-bx^2)^2} dx = -\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)} - \frac{b\operatorname{arctanh}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

output
```
1/2*(b*x-a)/(a^2+b^2)/(-b*x^2+2*a*x+b)-1/2*b*arctanh((-b*x+a)/(a^2+b^2)^(1
/2))/(a^2+b^2)^(3/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{1}{(b+2ax-bx^2)^2} dx = \frac{\frac{-a+bx}{b+2ax-bx^2} - \frac{b\arctan\left(\frac{-a+bx}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{2(a^2+b^2)}$$

input
```
Integrate[(b + 2*a*x - b*x^2)^(-2),x]
```

output
```
((-a + b*x)/(b + 2*a*x - b*x^2) - (b*ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2])/(2*(a^2 + b^2))
```

3.97.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2ax-bx^2+b)^2}dx$$
$$\Big\downarrow \text{1086}$$
$$\frac{b\int \frac{1}{-bx^2+2ax+b}dx}{2(a^2+b^2)}-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}$$
$$\Big\downarrow \text{1083}$$
$$-\frac{b\int \frac{1}{4(a^2+b^2)-(2a-2bx)^2}d(2a-2bx)}{a^2+b^2}-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}$$
$$\Big\downarrow \text{219}$$
$$-\frac{b\text{arctanh}\left(\frac{2a-2bx}{2\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}$$

input
```
Int[(b + 2*a*x - b*x^2)^(-2),x]
```

output
```
-1/2*(a - b*x)/((a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(2*a - 2*b*x
)/(2*Sqrt[a^2 + b^2])])/(2*(a^2 + b^2)^(3/2))
```

3.97.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.97.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{-2bx+2a}{(-4a^2-4b^2)(-b\,x^2+2ax+b)}+\frac{2b\,\operatorname{arctanh}\left(\frac{-2bx+2a}{2\sqrt{a^2+b^2}}\right)}{(-4a^2-4b^2)\sqrt{a^2+b^2}}$	83
risch	$\frac{\frac{bx}{4a^2+4b^2}-\frac{a}{4\left(a^2+b^2\right)}}{-\frac{1}{2}b\,x^2+ax+\frac{1}{2}b}+\frac{b\ln\left(\left(a^2b+b^3\right)x+\left(a^2+b^2\right)^{\frac{3}{2}}-a^3-a\,b^2\right)}{4(a^2+b^2)^{\frac{3}{2}}}-\frac{b\ln\left(\left(-a^2b-b^3\right)x+\left(a^2+b^2\right)^{\frac{3}{2}}+a^3+a\,b^2\right)}{4(a^2+b^2)^{\frac{3}{2}}}$	134

input
```
int(1/(-b*x^2+2*a*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*b*x+2*a)/(-4*a^2-4*b^2)/(-b*x^2+2*a*x+b)+2*b/(-4*a^2-4*b^2)/(a^2+b^2)^
(1/2)*arctanh(1/2*(-2*b*x+2*a)/(a^2+b^2)^(1/2))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(65) = 130.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.48

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{2\,a^3+2\,ab^2+(b^2x^2-2\,abx-b^2)\sqrt{a^2+b^2}\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}(bx-a)}{bx^2-2\,ax-b}\right)-2\,(a^2b+b^3)x}{4\,(a^4b+2\,a^2b^3+b^5-(a^4b+2\,a^2b^3+b^5)x^2+2\,(a^5+2\,a^3b^2+ab^4)x)}$$

input
```
integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a^3 + 2*a*b^2 + (b^2*x^2 - 2*a*b*x - b^2)*sqrt(a^2 + b^2)*log((b^2
*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*x^2 - 2*a*x
 - b)) - 2*(a^2*b + b^3)*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3
+ b^5)*x^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*x)
```

3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(56) = 112.

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.16

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{b\sqrt{\frac{1}{(a^2+b^2)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a^2+b^2)^3}}-2a^2b^3\sqrt{\frac{1}{(a^2+b^2)^3}}-ab-b^5\sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4} + \frac{b\sqrt{\frac{1}{(a^2+b^2)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a^2+b^2)^3}}+2a^2b^3\sqrt{\frac{1}{(a^2+b^2)^3}}-ab+b^5\sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4} + \frac{a-bx}{-2a^2b-2b^3+x^2\cdot(2a^2b+2b^3)+x(-4a^3-4ab^2)}$$

input
```
integrate(1/(-b*x**2+2*a*x+b)**2,x)
```

output
```
-b*sqrt((a**2 + b**2)**(-3))*log(x + (-a**4*b*sqrt((a**2 + b**2)**(-3)) -
2*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b - b**5*sqrt((a**2 + b**2)**(-3
)))/b**2)/4 + b*sqrt((a**2 + b**2)**(-3))*log(x + (a**4*b*sqrt((a**2 + b**
2)**(-3)) + 2*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b + b**5*sqrt((a**2
+ b**2)**(-3)))/b**2)/4 + (a - b*x)/(-2*a**2*b - 2*b**3 + x**2*(2*a**2*b +
 2*b**3) + x*(-4*a**3 - 4*a*b**2))
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{b\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{4\,(a^2+b^2)^{\frac{3}{2}}} + \frac{bx-a}{2\,(a^2b+b^3-(a^2b+b^3)x^2+2\,(a^3+ab^2)x)}$$

input
```
integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

output
```
-1/4*b*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/(a^2 +
 b^2)^(3/2) + 1/2*(b*x - a)/(a^2*b + b^3 - (a^2*b + b^3)*x^2 + 2*(a^3 + a*
b^2)*x)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{b\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,(a^2+b^2)^{\frac{3}{2}}} - \frac{bx-a}{2\,(bx^2-2\,ax-b)(a^2+b^2)}$$

input
```
integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

output
```
-1/4*b*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) - 1/2*(b*x - a)/((b*x^2 - 2*a*x - b)*(a^2 +
b^2))
```

3.97.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{\frac{a}{2\,(a^2+b^2)}-\frac{b\,x}{2\,(a^2+b^2)}}{-b\,x^2+2\,a\,x+b} + \frac{b\,\mathrm{atan}\left(\frac{a\,b^2\,1\mathrm{i}+a^3\,1\mathrm{i}-b\,x\,(a^2+b^2)\,1\mathrm{i}}{(a^2+b^2)^{3/2}}\right)\,1\mathrm{i}}{2\,(a^2+b^2)^{3/2}}$$

input
```
int(1/(b + 2*a*x - b*x^2)^2,x)
```

output
```
(b*atan((a*b^2*1i + a^3*1i - b*x*(a^2 + b^2)*1i)/(a^2 + b^2)^(3/2))*1i)/(2
*(a^2 + b^2)^(3/2)) - (a/(2*(a^2 + b^2)) - (b*x)/(2*(a^2 + b^2)))/(b + 2*a
*x - b*x^2)
```

3.98 $\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx$

3.98.1 Optimal result

Integrand size = 40, antiderivative size = 62

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = -\left(\frac{a}{b}\right)^{-1/n}\arctan\left(\cot\left(\frac{\pi-2k\pi}{n}\right)-\left(\frac{a}{b}\right)^{-1/n}x\csc\left(\frac{\pi-2k\pi}{n}\right)\right)\csc\left(\frac{\pi-2k\pi}{n}\right)$$

output
```
arctan(-cot((-2*Pi*k+Pi)/n)+x*csc((-2*Pi*k+Pi)/n)/((a/b)^(1/n)))*csc((-2*P
i*k+Pi)/n)/((a/b)^(1/n))
```

3.98.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = -\left(\frac{a}{b}\right)^{-1/n}\arctan\left(\cot\left(\frac{\pi-2k\pi}{n}\right)-\left(\frac{a}{b}\right)^{-1/n}x\csc\left(\frac{\pi-2k\pi}{n}\right)\right)\csc\left(\frac{\pi-2k\pi}{n}\right)$$

input
```
Integrate[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1)
,x]
```

output
```
-((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)]*Cs
c[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))
```

3.98.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-2x\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2\pi k}{n}\right)+\left(\frac{a}{b}\right)^{2/n}+x^2}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-4\left(1-\cos^2\left(\frac{\pi-2k\pi}{n}\right)\right)\left(\frac{a}{b}\right)^{2/n}-\left(2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2k\pi}{n}\right)\right)^2}d\left(2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2k\pi}{n}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\arctan\left(\frac{1}{2}\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\left(2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2\pi k}{n}\right)\right)\right)$$

input
```
Int[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1),x]
```

output
```
(ArcTan[((2*x - 2*(a/b)^n^(-1)*Cos[(Pi - 2*k*Pi)/n])*Csc[(Pi - 2*k*Pi)/n])
/(2*(a/b)^n^(-1))]*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)
```

3.98.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.98.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.79

method	result	size
default	$\frac{\arctan\left(\frac{2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi(2k-1)}{n}\right)}{2\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}}\left(\cos^2\left(\frac{\pi(2k-1)}{n}\right)\right)+\left(\frac{a}{b}\right)^{\frac{2}{n}}}}\right)}{\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}}\left(\cos^2\left(\frac{\pi(2k-1)}{n}\right)\right)+\left(\frac{a}{b}\right)^{\frac{2}{n}}}}$	111
risch	Expression too large to display	1343

input
```
int(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*Pi*k+Pi)/n)),x,method=_RETU
RNVERBOSE)
```

output
```
1/(-((a/b)^(1/n))^2*cos(Pi*(2*k-1)/n)^2+(a/b)^(2/n))^(1/2)*arctan(1/2*(2*x
-2*(a/b)^(1/n)*cos(Pi*(2*k-1)/n))/(-((a/b)^(1/n))^2*cos(Pi*(2*k-1)/n)^2+(a
/b)^(2/n))^(1/2))
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx=-\frac{\arctan\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}\right)}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}$$

input
```
integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algor
ithm="fricas")
```

output
```
-arctan(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x)/((a/b)^(1/n)*sin(2*pi*k/n -
 pi/n)))/((a/b)^(1/n)*sin(2*pi*k/n - pi/n))
```

3.98.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(46) = 92.

Time = 0.49 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.42

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n} + x^2 - 2\left(\frac{a}{b}\right)^{\frac{1}{n}} x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = -\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^2\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2} + \frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^2\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}$$

input
```
integrate(1/((a/b)**(2/n)+x**2-2*(a/b)**(1/n)*x*cos((-2*pi*k+pi)/n)),x)
```

output
```
-sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*log(x - (a/b)**(1/n)*
cos(2*pi*k/n - pi/n) - sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))
*(-2*(a/b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2 + sqrt(1/
((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*log(x - (a/b)**(1/n)*cos(2*pi
*k/n - pi/n) + sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*(-2*(a/
b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2
```

3.98.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n} + x^2 - 2\left(\frac{a}{b}\right)^{\frac{1}{n}} x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1>0)', see `assume?` for more de
tails)Is 1
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n} + x^2 - 2\left(\frac{a}{b}\right)^{\frac{1}{n}} x \cos\left(\frac{\pi-2k\pi}{n}\right)} \, dx = \frac{\arctan\left(-\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)} \cos\left(-\frac{2\pi k}{n} + \frac{\pi}{n}\right) - x}{\sqrt{-\cos\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right)^2 + 1} \left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}\right)}{\sqrt{-\cos\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right)^2 + 1} \left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algor
ithm="giac")
```

output
```
arctan(-((a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x)/(sqrt(-cos(2*pi*k/n - pi/n
)^2 + 1)*(a/b)^(1/n)))/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/b)^(1/n))
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n} + x^2 - 2\left(\frac{a}{b}\right)^{\frac{1}{n}} x \cos\left(\frac{\pi-2k\pi}{n}\right)} \, dx = -\frac{\operatorname{atanh}\left(\frac{x - \cos\left(\frac{\Pi\,(2\,k-1)}{n}\right) \left(\frac{a}{b}\right)^{1/n}}{\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right) - 1} \sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right) + 1} \left(\frac{a}{b}\right)^{1/n}}\right)}{\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right) - 1} \sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right) + 1} \left(\frac{a}{b}\right)^{1/n}}$$

input
```
int(1/((a/b)^(2/n) + x^2 - 2*x*cos((Pi - 2*Pi*k)/n)*(a/b)^(1/n)),x)
```

output
```
-atanh((x - cos((Pi*(2*k - 1))/n)*(a/b)^(1/n))/((cos((Pi*(2*k - 1))/n) - 1
)^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n)))/((cos((Pi*(2*k - 1
))/n) - 1)^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n))
```

3.99 $\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2} dx$

3.99.1 Optimal result

Integrand size = 30, antiderivative size = 33

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2} dx = \frac{2\text{arctanh}\left(\frac{-\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

output
```
2*arctanh((2*b^2*x-(-4*a*b^3+b^2)^(1/2))/b)/b
```

3.99.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(33) = 66.

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.64

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2} dx = \frac{-2\sqrt{b^2-4ab^3}\arctan\left(\frac{-1+2bx}{\sqrt{-1+4ab}}\right)+2\sqrt{b^2-4ab^3}\arctan\left(\frac{1+2bx}{\sqrt{-1+4ab}}\right)+b\sqrt{-1+4ab}(\log(a+x+bx^2)-\log}{2b^2\sqrt{-1+4ab}}$$

input
```
Integrate[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

output
```
(-2*Sqrt[b^2 - 4*a*b^3]*ArcTan[(-1 + 2*b*x)/Sqrt[-1 + 4*a*b]] + 2*Sqrt[b^2
 - 4*a*b^3]*ArcTan[(1 + 2*b*x)/Sqrt[-1 + 4*a*b]] + b*Sqrt[-1 + 4*a*b]*(Log
[a + x + b*x^2] - Log[a + x*(-1 + b*x)]))/(2*b^2*Sqrt[-1 + 4*a*b])
```

3.99.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{b^2-4ab^3}+ab-b^2x^2}dx$$
$$\Big\downarrow \text{1081}$$
$$-b^2\int \left(-\frac{2}{b\left(2xb^2+b-\sqrt{b^2-4ab^3}\right)}-\frac{2}{b\left(-2xb^2+b+\sqrt{b^2-4ab^3}\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-b^2\left(\frac{\log\left(\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b^3}-\frac{\log\left(-\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b^3}\right)$$

input
```
Int[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

output
```
-(b^2*(Log[b + Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b^3 - Log[b - Sqrt[b^2 - 4*a
*b^3] + 2*b^2*x]/b^3))
```

3.99.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.99.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2\,\operatorname{arctanh}\left(\frac{-2b^2x+\sqrt{-b^2(4ab-1)}}{b}\right)}{b}$	31

input
```
int(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/b*arctanh((-2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(31) = 62.

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = \frac{\log\left(\frac{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

input
```
integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="fricas")
```

output
```
(log((2*b^2*x + b - sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b - sqrt(-4*
a*b^3 + b^2))/b))/b
```

3.99.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(x-\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

input
```
integrate(1/(a*b-b**2*x**2+x*(-4*a*b**3+b**2)**(1/2)),x)
```

output
```
-(log(x - 1/(2*b) - sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) - s
qrt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}\right)}{b}$$

input
```
integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")
```

output
```
-log((2*b^2*x - b - sqrt(-4*a*b^3 + b^2))/(2*b^2*x + b - sqrt(-4*a*b^3 + b
^2)))/b
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(\frac{\left|2\,b^2x-\sqrt{-4\,ab+1}|b|-|b|\right|}{\left|2\,b^2x-\sqrt{-4\,ab+1}|b|+|b|\right|}\right)}{|b|}$$

input
```
integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="giac")
```

output
```
-log(abs(2*b^2*x - sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x - sqrt(-4
*a*b + 1)*abs(b) + abs(b)))/abs(b)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b^2-4\,a\,b^3}}{\sqrt{b^2}}-\frac{2\,b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

input
```
int(1/(a*b + x*(b^2 - 4*a*b^3)^(1/2) - b^2*x^2),x)
```

output
```
-(2*atanh((b^2 - 4*a*b^3)^(1/2)/(b^2)^(1/2) - (2*b^2*x)/(b^2)^(1/2)))/(b^2
)^(1/2)
```

3.100 $\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$

3.100.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

output
```
2*arctanh((2*b^2*x+(-4*a*b^3+b^2)^(1/2))/b)/b
```

3.100.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(31) = 62.

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$$
$$=\frac{2\sqrt{b^2-4ab^3}\arctan\left(\frac{-1+2bx}{\sqrt{-1+4ab}}\right)-2\sqrt{b^2-4ab^3}\arctan\left(\frac{1+2bx}{\sqrt{-1+4ab}}\right)+b\sqrt{-1+4ab}(\log\left(a+x+bx^2\right)-\log(a}{2b^2\sqrt{-1+4ab}}$$

input
```
Integrate[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

output
```
(2*Sqrt[b^2 - 4*a*b^3]*ArcTan[(-1 + 2*b*x)/Sqrt[-1 + 4*a*b]] - 2*Sqrt[b^2
- 4*a*b^3]*ArcTan[(1 + 2*b*x)/Sqrt[-1 + 4*a*b]] + b*Sqrt[-1 + 4*a*b]*(Log[
a + x + b*x^2] - Log[a + x*(-1 + b*x)]))/(2*b^2*Sqrt[-1 + 4*a*b])
```

3.100.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-x\sqrt{b^2-4ab^3}+ab-b^2x^2}\,dx$$

$$\Big\downarrow \text{1081}$$

$$-b^2\int \left(-\frac{2}{b\left(2xb^2+b+\sqrt{b^2-4ab^3}\right)}-\frac{2}{b\left(-2xb^2+b-\sqrt{b^2-4ab^3}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-b^2\left(\frac{\log\left(-\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b^3}-\frac{\log\left(\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b^3}\right)$$

input
```
Int[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

output
```
-(b^2*(Log[b - Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b^3 - Log[b + Sqrt[b^2 - 4*a
*b^3] + 2*b^2*x]/b^3))
```

3.100.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.100.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2b^2x+\sqrt{-b^2(4ab-1)}}{b}\right)}{b}$	31

input
```
int(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/b*arctanh((2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = \frac{\log\left(\frac{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

input
```
integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="fricas")
```

output
```
(log((2*b^2*x + b + sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b + sqrt(-4*
a*b^3 + b^2))/b))/b
```

3.100.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(x-\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

input
```
integrate(1/(a*b-b**2*x**2-x*(-4*a*b**3+b**2)**(1/2)),x)
```

output
```
-(log(x - 1/(2*b) + sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) + s
qrt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}\right)}{b}$$

input
```
integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")
```

output
```
-log((2*b^2*x - b + sqrt(-4*a*b^3 + b^2))/(2*b^2*x + b + sqrt(-4*a*b^3 + b
^2)))/b
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(\frac{|2\,b^2x+\sqrt{-4\,ab+1}|b|-|b||}{|2\,b^2x+\sqrt{-4\,ab+1}|b|+|b||}\right)}{|b|}$$

input
```
integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="giac")
```

output
```
-log(abs(2*b^2*x + sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x + sqrt(-4
*a*b + 1)*abs(b) + abs(b)))/abs(b)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b^2-4\,a\,b^3}}{\sqrt{b^2}}+\frac{2\,b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

input
```
int(-1/(x*(b^2 - 4*a*b^3)^(1/2) - a*b + b^2*x^2),x)
```

output
```
(2*atanh((b^2 - 4*a*b^3)^(1/2)/(b^2)^(1/2) + (2*b^2*x)/(b^2)^(1/2)))/(b^2)
^(1/2)
```

3.101 $\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx$

3.101.1 Optimal result

Integrand size = 14, antiderivative size = 17

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \arctan\left(\left(x+\cos\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)$$

output
```
arctan((x+cos(1/7))*csc(1/7))*csc(1/7)
```

3.101.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \arctan\left(\cot\left(\frac{1}{7}\right)+x\csc\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)$$

input
```
Integrate[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

output
```
ArcTan[Cot[1/7] + x*Csc[1/7]]*Csc[1/7]
```

3.101.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+2x\cos\left(\frac{1}{7}\right)+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-\left(2x+2\cos\left(\frac{1}{7}\right)\right)^2-4\sin^2\left(\frac{1}{7}\right)}d\left(2x+2\cos\left(\frac{1}{7}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\csc\left(\frac{1}{7}\right)\arctan\left(\frac{1}{2}\csc\left(\frac{1}{7}\right)\left(2x+2\cos\left(\frac{1}{7}\right)\right)\right)$$

input
```
Int[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

output
```
ArcTan[((2*x + 2*Cos[1/7])*Csc[1/7])/2]*Csc[1/7]
```

3.101.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(11) = 22$.

Time = 3.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

method	result	size
default	$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{1}{7}\right)}{2\sqrt{-\left(\cos^2\left(\frac{1}{7}\right)\right)+1}}\right)}{\sqrt{-\left(\cos^2\left(\frac{1}{7}\right)\right)+1}}$	33
risch	Expression too large to display	3085

input
```
int(1/(1+x^2+2*x*cos(1/7)),x,method=_RETURNVERBOSE)
```

output
```
1/(-cos(1/7)^2+1)^(1/2)*arctan(1/2*(2*x+2*cos(1/7))/(-cos(1/7)^2+1)^(1/2))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sin\left(\frac{1}{7}\right)}\right)}{\sin\left(\frac{1}{7}\right)}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="fricas")
```

output
```
arctan((x + cos(1/7))/sin(1/7))/sin(1/7)
```

3.101.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.71

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = -\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\cos^2\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}} + \frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i\cos^2\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}$$

input
```
integrate(1/(1+x**2+2*x*cos(1/7)),x)
```

output
```
-I*log(x + cos(1/7) - I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)) + I*cos(1/
7)**2/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos(1/7))*sqrt(
cos(1/7) + 1)) + I*log(x + cos(1/7) - I*cos(1/7)**2/(sqrt(1 - cos(1/7))*sq
rt(cos(1/7) + 1)) + I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 -
 cos(1/7))*sqrt(cos(1/7) + 1))
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="maxima")
```

output
```
arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

3.101.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="giac")
```

output
```
arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \frac{\operatorname{atan}\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^2}}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^2}}$$

input
```
int(1/(2*x*cos(1/7) + x^2 + 1),x)
```

output
```
atan((x + cos(1/7))/(1 - cos(1/7)^2)^(1/2))/(1 - cos(1/7)^2)^(1/2)
```

3.102 $\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx$

3.102.1 Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx=\arctan\left(\cot\left(\frac{\pi}{7}\right)+x\csc\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)$$

output
```
arctan(cot(1/7*Pi)+x*csc(1/7*Pi))*csc(1/7*Pi)
```

3.102.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx=\arctan\left(\cot\left(\frac{\pi}{7}\right)+x\csc\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)$$

input
```
Integrate[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

output
```
ArcTan[Cot[Pi/7] + x*Csc[Pi/7]]*Csc[Pi/7]
```

3.102.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+2x\cos\left(\frac{\pi}{7}\right)+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-\left(2x+2\cos\left(\frac{\pi}{7}\right)\right)^2-4\sin^2\left(\frac{\pi}{7}\right)}d\left(2x+2\cos\left(\frac{\pi}{7}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\csc\left(\frac{\pi}{7}\right)\arctan\left(\frac{1}{2}\csc\left(\frac{\pi}{7}\right)\left(2x+2\cos\left(\frac{\pi}{7}\right)\right)\right)$$

input
```
Int[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

output
```
ArcTan[((2*x + 2*Cos[Pi/7])*Csc[Pi/7])/2]*Csc[Pi/7]
```

3.102.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 3.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

method	result
default	$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{\pi}{7}\right)}{2\sqrt{-\left(\cos^2\left(\frac{\pi}{7}\right)\right)+1}}\right)}{\sqrt{-\left(\cos^2\left(\frac{\pi}{7}\right)\right)+1}}$
norman	$\left(-\frac{4(-1)^{\frac{5}{7}}}{7}+\frac{(-1)^{\frac{4}{7}}}{7}-\frac{5(-1)^{\frac{3}{7}}}{7}+\frac{2(-1)^{\frac{2}{7}}}{7}-\frac{6(-1)^{\frac{1}{7}}}{7}+\frac{3}{7}\right)\ln\left((-1)^{\frac{5}{7}}-(-1)^{\frac{4}{7}}+(-1)^{\frac{3}{7}}-(-1)^{\frac{2}{7}}+(-1$
risch	$\frac{4\ln\left(x+(-1)^{\frac{1}{7}}\right)(-1)^{\frac{5}{7}}}{7}-\frac{\ln\left(x+(-1)^{\frac{1}{7}}\right)(-1)^{\frac{4}{7}}}{7}+\frac{5\ln\left(x+(-1)^{\frac{1}{7}}\right)(-1)^{\frac{3}{7}}}{7}-\frac{2\ln\left(x+(-1)^{\frac{1}{7}}\right)(-1)^{\frac{2}{7}}}{7}+\frac{6\ln\left(x+(-1)^{\frac{1}{7}}\right)(-1)^{\frac{1}{7}}}{7}$

input
```
int(1/(1+x^2+2*x*cos(1/7*Pi)),x,method=_RETURNVERBOSE)
```

output
```
1/(-cos(1/7*Pi)^2+1)^(1/2)*arctan(1/2*(2*x+2*cos(1/7*Pi))/(-cos(1/7*Pi)^2+
1)^(1/2))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sin\left(\frac{1}{7}\pi\right)}\right)}{\sin\left(\frac{1}{7}\pi\right)}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="fricas")
```

output
```
arctan((x + cos(1/7*pi))/sin(1/7*pi))/sin(1/7*pi)
```

3.102.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.04

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = -\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)-\frac{i\left(2-2\cos^2\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)} + \frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)+\frac{i\left(2-2\cos^2\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}$$

input
```
integrate(1/(1+x**2+2*x*cos(1/7*pi)),x)
```

output
```
-I*log(x + cos(pi/7) - I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7))
 + I*log(x + cos(pi/7) + I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7
))
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="maxima")
```

output
```
arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1
)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="giac")
```

output
```
arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1
)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = -\frac{\operatorname{atanh}\left(\frac{x+\cos\left(\frac{\Pi}{7}\right)}{\sqrt{\cos\left(\frac{\Pi}{7}\right)-1}\sqrt{\cos\left(\frac{\Pi}{7}\right)+1}}\right)}{\sqrt{\cos\left(\frac{\Pi}{7}\right)-1}\sqrt{\cos\left(\frac{\Pi}{7}\right)+1}}$$

input
```
int(1/(x^2 + 2*x*cos(Pi/7) + 1),x)
```

output
```
-atanh((x + cos(Pi/7))/((cos(Pi/7) - 1)^(1/2)*(cos(Pi/7) + 1)^(1/2)))/((co
s(Pi/7) - 1)^(1/2)*(cos(Pi/7) + 1)^(1/2))
```

3.103 $\int \sqrt{5-6x+9x^2}\,dx$

3.103.1 Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \sqrt{5-6x+9x^2}\,dx = -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+\frac{2}{3}\text{arcsinh}\left(\frac{1}{2}(-1+3x)\right)$$

output
```
2/3*arcsinh(-1/2+3/2*x)-1/6*(1-3*x)*(9*x^2-6*x+5)^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{1}{6}(-1+3x)\sqrt{5-6x+9x^2}-\frac{2}{3}\log\left(1-3x+\sqrt{5-6x+9x^2}\right)$$

input
```
Integrate[Sqrt[5 - 6*x + 9*x^2],x]
```

output
```
((-1 + 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 - (2*Log[1 - 3*x + Sqrt[5 - 6*x + 9*x
^2]])/3
```

3.103.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^2-6x+5}\,dx$$

$$\Big\downarrow \ 1087$$

$$2\int \frac{1}{\sqrt{9x^2-6x+5}}dx-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{18}\int \frac{1}{\sqrt{\frac{1}{144}(18x-6)^2+1}}d(18x-6)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

$$\Big\downarrow \ 222$$

$$\frac{2}{3}\text{arcsinh}\left(\frac{1}{12}(18x-6)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

input
```
Int[Sqrt[5 - 6*x + 9*x^2],x]
```

output
```
-1/6*((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2]) + (2*ArcSinh[(-6 + 18*x)/12])/3
```

3.103.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.103.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{(18x-6)\sqrt{9x^2-6x+5}}{36}+\frac{2\,\mathrm{arcsinh}\left(-\frac{1}{2}+\frac{3x}{2}\right)}{3}$	29
risch	$\frac{\sqrt{9x^2-6x+5}\,(3x-1)}{6}+\frac{2\,\mathrm{arcsinh}\left(-\frac{1}{2}+\frac{3x}{2}\right)}{3}$	29
trager	$\left(\frac{x}{2}-\frac{1}{6}\right)\sqrt{9x^2-6x+5}+\frac{2\ln\left(-1+3x+\sqrt{9x^2-6x+5}\right)}{3}$	40

input
```
int((9*x^2-6*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/36*(18*x-6)*(9*x^2-6*x+5)^(1/2)+2/3*arcsinh(-1/2+3/2*x)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{1}{6}\sqrt{9\,x^2-6\,x+5}(3\,x-1)-\frac{2}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

input
```
integrate((9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
 + 1)
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \sqrt{5-6x+9x^2}\,dx = \left(\frac{x}{2}-\frac{1}{6}\right)\sqrt{9x^2-6x+5}+\frac{2\operatorname{asinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}$$

input
```
integrate((9*x**2-6*x+5)**(1/2),x)
```

output
```
(x/2 - 1/6)*sqrt(9*x**2 - 6*x + 5) + 2*asinh(3*x/2 - 1/2)/3
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{1}{2}\sqrt{9\,x^2-6\,x+5}x-\frac{1}{6}\sqrt{9\,x^2-6\,x+5}+\frac{2}{3}\operatorname{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

input
```
integrate((9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(9*x^2 - 6*x + 5)*x - 1/6*sqrt(9*x^2 - 6*x + 5) + 2/3*arcsinh(3/2*
x - 1/2)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{1}{6}\sqrt{9\,x^2-6\,x+5}(3\,x-1)-\frac{2}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

input
```
integrate((9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
 + 1)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{2\,\ln\left(3\,x+\sqrt{9\,x^2-6\,x+5}-1\right)}{3}+\left(\frac{x}{2}-\frac{1}{6}\right)\,\sqrt{9\,x^2-6\,x+5}$$

input
```
int((9*x^2 - 6*x + 5)^(1/2),x)
```

output
```
(2*log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1))/3 + (x/2 - 1/6)*(9*x^2 - 6*x +
5)^(1/2)
```

3.104 $\int \sqrt{3-4x-4x^2}\,dx$

3.104.1 Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \sqrt{3-4x-4x^2}\,dx = \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2}+\arcsin\left(\frac{1}{2}+x\right)$$

output
```
arcsin(1/2+x)+1/4*(1+2*x)*(-4*x^2-4*x+3)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \sqrt{3-4x-4x^2}\,dx = \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2}-2\arctan\left(\frac{\sqrt{3-4x-4x^2}}{3+2x}\right)$$

input
```
Integrate[Sqrt[3 - 4*x - 4*x^2],x]
```

output
```
((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 - 2*ArcTan[Sqrt[3 - 4*x - 4*x^2]/(3 +
2*x)]
```

3.104.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-4x^2-4x+3}\,dx$$

$$\Big\downarrow \; 1087$$

$$2\int \frac{1}{\sqrt{-4x^2-4x+3}}dx+\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{4}(2x+1)\sqrt{-4x^2-4x+3}-\frac{1}{8}\int \frac{1}{\sqrt{1-\frac{1}{64}(-8x-4)^2}}d(-8x-4)$$

$$\Big\downarrow \; 223$$

$$\frac{1}{4}(2x+1)\sqrt{-4x^2-4x+3}-\arcsin\left(\frac{1}{8}(-8x-4)\right)$$

input
```
Int[Sqrt[3 - 4*x - 4*x^2],x]
```

output
```
((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 - ArcSin[(-4 - 8*x)/8]
```

3.104.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.104.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
default	$-\frac{(-8x-4)\sqrt{-4x^2-4x+3}}{16}+\arcsin\left(x+\frac{1}{2}\right)$
risch	$-\frac{(4x^2+4x-3)(1+2x)}{4\sqrt{-4x^2-4x+3}}+\arcsin\left(x+\frac{1}{2}\right)$
trager	$\left(\frac{1}{4}+\frac{x}{2}\right)\sqrt{-4x^2-4x+3}+\text{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\text{RootOf}\left(\_Z^2+1\right)x-\text{RootOf}\left(\_Z^2+1\right)\right.$

input
```
int((-4*x^2-4*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-8*x-4)*(-4*x^2-4*x+3)^(1/2)+arcsin(x+1/2)
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int\sqrt{3-4x-4x^2}\,dx=\frac{1}{4}\sqrt{-4\,x^2-4\,x+3}(2\,x+1)-\arctan\left(\frac{\sqrt{-4\,x^2-4\,x+3}(2\,x+1)}{4\,x^2+4\,x-3}\right)$$

input
```
integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) - arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x
+ 1)/(4*x^2 + 4*x - 3))
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \sqrt{3-4x-4x^2}\,dx = \left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{-4x^2-4x+3} + \operatorname{asin}\left(x+\frac{1}{2}\right)$$

input
```
integrate((-4*x**2-4*x+3)**(1/2),x)
```

output
```
(x/2 + 1/4)*sqrt(-4*x**2 - 4*x + 3) + asin(x + 1/2)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \sqrt{3-4x-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2-4\,x+3}x + \frac{1}{4}\sqrt{-4\,x^2-4\,x+3} - \arcsin\left(-x-\frac{1}{2}\right)$$

input
```
integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*x^2 - 4*x + 3)*x + 1/4*sqrt(-4*x^2 - 4*x + 3) - arcsin(-x - 1/
2)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \sqrt{3-4x-4x^2}\,dx = \frac{1}{4}\sqrt{-4\,x^2-4\,x+3}(2\,x+1) + \arcsin\left(x+\frac{1}{2}\right)$$

input
```
integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \sqrt{3-4x-4x^2}\,dx = \operatorname{asin}\left(x+\frac{1}{2}\right)+\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{-4\,x^2-4\,x+3}$$

input
```
int((3 - 4*x^2 - 4*x)^(1/2),x)
```

output
```
asin(x + 1/2) + (x/2 + 1/4)*(3 - 4*x^2 - 4*x)^(1/2)
```

3.105 $\int \sqrt{-8+6x+9x^2}\,dx$

3.105.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \sqrt{-8+6x+9x^2}\,dx=\frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2}-\frac{3}{2}\text{arctanh}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)$$

output
```
-3/2*arctanh((1+3*x)/(9*x^2+6*x-8)^(1/2))+1/6*(1+3*x)*(9*x^2+6*x-8)^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \sqrt{-8+6x+9x^2}\,dx=\frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2}-3\text{arctanh}\left(\frac{\sqrt{-8+6x+9x^2}}{-2+3x}\right)$$

input
```
Integrate[Sqrt[-8 + 6*x + 9*x^2],x]
```

output
```
((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - 3*ArcTanh[Sqrt[-8 + 6*x + 9*x^2]/(-
2 + 3*x)]
```

3.105.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^2+6x-8}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{9}{2}\int \frac{1}{\sqrt{9x^2+6x-8}}dx$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-9\int \frac{1}{36-\frac{36(3x+1)^2}{9x^2+6x-8}}d\frac{6(3x+1)}{\sqrt{9x^2+6x-8}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\text{arctanh}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

input
```
Int[Sqrt[-8 + 6*x + 9*x^2],x]
```

output
```
((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - (3*ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x
+ 9*x^2]])/2
```

3.105.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.105.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
trager	$\left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8}-\frac{3\ln\left(\sqrt{9x^2+6x-8}+1+3x\right)}{2}$	40
default	$\frac{(18x+6)\sqrt{9x^2+6x-8}}{36}-\frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{2}$	50
risch	$\frac{(3x+1)\sqrt{9x^2+6x-8}}{6}-\frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{2}$	50

input
```
int((9*x^2+6*x-8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2*x+1/6)*(9*x^2+6*x-8)^(1/2)-3/2*ln((9*x^2+6*x-8)^(1/2)+1+3*x)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int\sqrt{-8+6x+9x^2}\,dx=\frac{1}{6}\sqrt{9\,x^2+6\,x-8}(3\,x+1)+\frac{3}{2}\log\left(-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right)$$

input
```
integrate((9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(-3*x + sqrt(9*x^2 + 6*x - 8)
 - 1)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \sqrt{-8+6x+9x^2}\,dx = \left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8} - \frac{3\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)}{2}$$

input
```
integrate((9*x**2+6*x-8)**(1/2),x)
```

output
```
(x/2 + 1/6)*sqrt(9*x**2 + 6*x - 8) - 3*log(18*x + 6*sqrt(9*x**2 + 6*x - 8)
 + 6)/2
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{1}{2}\sqrt{9x^2+6x-8}x + \frac{1}{6}\sqrt{9x^2+6x-8} - \frac{3}{2}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

input
```
integrate((9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(9*x^2 + 6*x - 8)*x + 1/6*sqrt(9*x^2 + 6*x - 8) - 3/2*log(18*x + 6
*sqrt(9*x^2 + 6*x - 8) + 6)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{1}{6}\sqrt{9x^2+6x-8}(3x+1) + \frac{3}{2}\log\left(\left|-3x+\sqrt{9x^2+6x-8}-1\right|\right)$$

input
```
integrate((9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x
- 8) - 1))
```

3.105.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \sqrt{-8+6x+9x^2}\,dx=\left(\frac{x}{2}+\frac{1}{6}\right)\,\sqrt{9\,x^2+6\,x-8}-\frac{3\,\ln\left(3\,x+\sqrt{9\,x^2+6\,x-8}+1\right)}{2}$$

input
```
int((6*x + 9*x^2 - 8)^(1/2),x)
```

output
```
(x/2 + 1/6)*(6*x + 9*x^2 - 8)^(1/2) - (3*log(3*x + (6*x + 9*x^2 - 8)^(1/2)
 + 1))/2
```

3.106 $\int \sqrt{2+4x+3x^2}\,dx$

3.106.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{\operatorname{arcsinh}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}$$

output
```
1/9*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)+1/6*(2+3*x)*(3*x^2+4*x+2)^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} - \frac{\log\left(-2-3x+\sqrt{6+12x+9x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 - Log[-2 - 3*x + Sqrt[6 + 12*x + 9*x^2
]]/(3*Sqrt[3])
```

3.106.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x^2+4x+2}\,dx$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{3}\int \frac{1}{\sqrt{3x^2+4x+2}}dx+\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)$$

$$\Big\downarrow \ 1090$$

$$\frac{\int \frac{1}{\sqrt{\frac{1}{8}(6x+4)^2+1}}d(6x+4)}{6\sqrt{6}}+\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)$$

$$\Big\downarrow \ 222$$

$$\frac{\operatorname{arcsinh}\left(\frac{6x+4}{2\sqrt{2}}\right)}{3\sqrt{3}}+\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)$$

input
```
Int[Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(4 + 6*x)/(2*Sqrt[2])]/(3*Sq
rt[3])
```

3.106.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.106.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{(4+6x)\sqrt{3x^2+4x+2}}{12}+\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\left(\frac{2}{3}+x\right)}{2}\right)}{9}$	35
risch	$\frac{(2+3x)\sqrt{3x^2+4x+2}}{6}+\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\left(\frac{2}{3}+x\right)}{2}\right)}{9}$	35
trager	$\left(\frac{1}{3}+\frac{x}{2}\right)\sqrt{3x^2+4x+2}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+2\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x+2}\right)}{9}$	61

input
```
int((3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(4+6*x)*(3*x^2+4*x+2)^(1/2)+1/9*3^(1/2)*arcsinh(3/2*2^(1/2)*(2/3+x))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \sqrt{2+4x+3x^2}\,dx=\frac{1}{6}\sqrt{3\,x^2+4\,x+2}(3\,x+2)+\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x+2}(3\,x+2)-9\,x^2-12\,x-5\right)$$

input
```
integrate((3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) + 1/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \sqrt{2+4x+3x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3x^2+4x+2}+\frac{\sqrt{3}\operatorname{asinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{9}$$

input
```
integrate((3*x**2+4*x+2)**(1/2),x)
```

output
```
(x/2 + 1/3)*sqrt(3*x**2 + 4*x + 2) + sqrt(3)*asinh(3*sqrt(2)*(x + 2/3)/2)/
9
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \sqrt{2+4x+3x^2}\,dx$$
$$= \frac{1}{2}\sqrt{3x^2+4x+2}x+\frac{1}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right)+\frac{1}{3}\sqrt{3x^2+4x+2}$$

input
```
integrate((3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 + 4*x + 2)*x + 1/9*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2)) +
 1/3*sqrt(3*x^2 + 4*x + 2)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{1}{6}\sqrt{3x^2+4x+2}(3x+2) - \frac{1}{9}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x+2}\right)-2\right)$$

input
```
integrate((3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 4*x + 2)) - 2)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3\,x^2+4\,x+2}+\frac{\sqrt{3}\,(3\,x+2)}{3}\right)}{9}+\left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3\,x^2+4\,x+2}$$

input
```
int((4*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*log((4*x + 3*x^2 + 2)^(1/2) + (3^(1/2)*(3*x + 2))/3))/9 + (x/2 +
1/3)*(4*x + 3*x^2 + 2)^(1/2)
```

3.107 $\int \sqrt{2+4x-3x^2}\,dx$

3.107.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \sqrt{2+4x-3x^2}\,dx=-\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2}-\frac{5\arcsin\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

output
```
-5/9*arcsin(1/10*(2-3*x)*10^(1/2))*3^(1/2)-1/6*(2-3*x)*(-3*x^2+4*x+2)^(1/2
)
```

3.107.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \sqrt{2+4x-3x^2}\,dx=\frac{1}{6}(-2+3x)\sqrt{2+4x-3x^2}+\frac{10\arctan\left(\frac{-2-\sqrt{10}+3x}{\sqrt{6+12x-9x^2}}\right)}{3\sqrt{3}}$$

input
```
Integrate[Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
((-2 + 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 + (10*ArcTan[(-2 - Sqrt[10] + 3*x)/Sq
rt[6 + 12*x - 9*x^2]])/(3*Sqrt[3])
```

3.107.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x^2+4x+2}\,dx$$

$$\Big\downarrow \; 1087$$

$$\frac{5}{3}\int \frac{1}{\sqrt{-3x^2+4x+2}}dx-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x+2}$$

$$\Big\downarrow \; 1090$$

$$-\frac{1}{6}\sqrt{\frac{5}{6}}\int \frac{1}{\sqrt{1-\frac{1}{40}(4-6x)^2}}d(4-6x)-\frac{1}{6}\sqrt{-3x^2+4x+2}(2-3x)$$

$$\Big\downarrow \; 223$$

$$-\frac{5\arcsin\left(\frac{4-6x}{2\sqrt{10}}\right)}{3\sqrt{3}}-\frac{1}{6}\sqrt{-3x^2+4x+2}(2-3x)$$

input
```
Int[Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
-1/6*((2 - 3*x)*Sqrt[2 + 4*x - 3*x^2]) - (5*ArcSin[(4 - 6*x)/(2*Sqrt[10])]
)/(3*Sqrt[3])
```

3.107.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.107.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result
default	$-\frac{(4-6x)\sqrt{-3x^2+4x+2}}{12}+\frac{5\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\left(-\frac{2}{3}+x\right)}{10}\right)}{9}$
risch	$-\frac{(3x^2-4x-2)(-2+3x)}{6\sqrt{-3x^2+4x+2}}+\frac{5\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\left(-\frac{2}{3}+x\right)}{10}\right)}{9}$
trager	$\left(-\frac{1}{3}+\frac{x}{2}\right)\sqrt{-3x^2+4x+2}-\frac{5\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(3x\,\mathrm{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x+2}-2\,\mathrm{RootOf}\left(\_Z^2+3\right)\right)}{9}$

input
```
int((-3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(4-6*x)*(-3*x^2+4*x+2)^(1/2)+5/9*3^(1/2)*arcsin(3/10*10^(1/2)*(-2/3+
x))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \sqrt{2+4x-3x^2}\,dx = \frac{1}{6}\sqrt{-3\,x^2+4\,x+2}(3\,x-2) - \frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)}{3\,(3\,x^2-4\,x-2)}\right)$$

input
```
integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt
(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4*x - 2))
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \sqrt{2+4x-3x^2}\,dx = \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3x^2+4x+2}+\frac{5\sqrt{3}\operatorname{asin}\left(\frac{3\sqrt{10}\left(x-\frac{2}{3}\right)}{10}\right)}{9}$$

input
```
integrate((-3*x**2+4*x+2)**(1/2),x)
```

output
```
(x/2 - 1/3)*sqrt(-3*x**2 + 4*x + 2) + 5*sqrt(3)*asin(3*sqrt(10)*(x - 2/3)/
10)/9
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \sqrt{2+4x-3x^2}\,dx = \frac{1}{2}\sqrt{-3x^2+4x+2}x-\frac{5}{9}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x-2)\right) - \frac{1}{3}\sqrt{-3x^2+4x+2}$$

input
```
integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 4*x + 2)*x - 5/9*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2)
) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sqrt{2+4x-3x^2}\,dx = \frac{1}{6}\sqrt{-3x^2+4x+2}(3x-2)+\frac{5}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x-2)\right)$$

input
```
integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3
*x - 2))
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \sqrt{2+4x-3x^2}\,dx = \frac{5\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{10}\,(3\,x-2)}{10}\right)}{9} + \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3\,x^2+4\,x+2}$$

input
```
int((4*x - 3*x^2 + 2)^(1/2),x)
```

output
```
(5*3^(1/2)*asin((10^(1/2)*(3*x - 2))/10))/9 + (x/2 - 1/3)*(4*x - 3*x^2 + 2
)^(1/2)
```

3.108 $\int \sqrt{2+5x+3x^2}\,dx$

3.108.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \sqrt{2+5x+3x^2}\,dx=\frac{1}{12}(5+6x)\sqrt{2+5x+3x^2}-\frac{\operatorname{arctanh}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{24\sqrt{3}}$$

output
```
-1/72*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x+2)^(1/2))*3^(1/2)+1/12*(5+6*x
)*(3*x^2+5*x+2)^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \sqrt{2+5x+3x^2}\,dx=\frac{1}{36}\left(3(5+6x)\sqrt{2+5x+3x^2}-\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}+\frac{5x}{3}+x^2}}{1+x}\right)\right)$$

input
```
Integrate[Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(3*(5 + 6*x)*Sqrt[2 + 5*x + 3*x^2] - Sqrt[3]*ArcTanh[Sqrt[2/3 + (5*x)/3 +
x^2]/(1 + x)])/36
```

3.108.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x^2+5x+2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{1}{24}\int \frac{1}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{1}{12}\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x+2}}d\frac{6x+5}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

input
```
Int[Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2
 + 5*x + 3*x^2])]/(24*Sqrt[3])
```

3.108.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.108.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{(5+6x)\sqrt{3x^2+5x+2}}{12}-\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{72}$	50
risch	$\frac{(5+6x)\sqrt{3x^2+5x+2}}{12}-\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{72}$	50
trager	$\left(\frac{5}{12}+\frac{x}{2}\right)\sqrt{3x^2+5x+2}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(6\operatorname{RootOf}\left(\_Z^2-3\right)x+5\operatorname{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2+5x+2}\right)}{72}$	61

input
```
int((3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(5+6*x)*(3*x^2+5*x+2)^(1/2)-1/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+
2)^(1/2))*3^(1/2)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{1}{12}\sqrt{3\,x^2+5\,x+2}(6\,x+5) + \frac{1}{144}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2+5\,x+2}(6\,x+5)+72\,x^2+120\,x+49\right)$$

input
```
integrate((3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3
*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x + 49)
```

3.108.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \sqrt{2+5x+3x^2}\,dx = \left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3x^2+5x+2}-\frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x+2}+5\right)}{72}$$

input
```
integrate((3*x**2+5*x+2)**(1/2),x)
```

output
```
(x/2 + 5/12)*sqrt(3*x**2 + 5*x + 2) - sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x
**2 + 5*x + 2) + 5)/72
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{1}{2}\sqrt{3x^2+5x+2}x-\frac{1}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\frac{5}{12}\sqrt{3x^2+5x+2}$$

input
```
integrate((3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 + 5*x + 2)*x - 1/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x
+ 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x + 2)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{1}{12}\sqrt{3x^2+5x+2}(6x+5)+\frac{1}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x+2}\right)-5\right|\right)$$

input
```
integrate((3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \sqrt{2+5x+3x^2}\,dx = \left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3\,x^2+5\,x+2}-\frac{\sqrt{3}\,\ln\left(\sqrt{3\,x^2+5\,x+2}+\frac{\sqrt{3}\,\left(3\,x+\frac{5}{2}\right)}{3}\right)}{72}$$

input

```
int((5*x + 3*x^2 + 2)^(1/2),x)
```

output

```
(x/2 + 5/12)*(5*x + 3*x^2 + 2)^(1/2) - (3^(1/2)*log((5*x + 3*x^2 + 2)^(1/2
) + (3^(1/2)*(3*x + 5/2))/3))/72
```

3.109 $\int \sqrt{2+5x-3x^2}\,dx$

3.109.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \sqrt{2+5x-3x^2}\,dx = -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} - \frac{49\arcsin\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

output
```
49/72*arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{1}{36}\left(3(-5+6x)\sqrt{2+5x-3x^2} - 49\sqrt{3}\arctan\left(\frac{\sqrt{6+15x-9x^2}}{1+3x}\right)\right)$$

input
```
Integrate[Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
(3*(-5 + 6*x)*Sqrt[2 + 5*x - 3*x^2] - 49*Sqrt[3]*ArcTan[Sqrt[6 + 15*x - 9*
x^2]/(1 + 3*x)])/36
```

3.109.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x^2+5x+2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{49}{24}\int \frac{1}{\sqrt{-3x^2+5x+2}}dx-\frac{1}{12}(5-6x)\sqrt{-3x^2+5x+2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{7\int \frac{1}{\sqrt{1-\frac{1}{49}(5-6x)^2}}d(5-6x)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x+2}(5-6x)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{49\arcsin\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x+2}(5-6x)$$

input
```
Int[Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
-1/12*((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2]) - (49*ArcSin[(5 - 6*x)/7])/(24*Sqr
t[3])
```

3.109.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.109.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result
default	$\frac{49\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{72}-\frac{(5-6x)\sqrt{-3x^2+5x+2}}{12}$
risch	$-\frac{(3x^2-5x-2)(-5+6x)}{12\sqrt{-3x^2+5x+2}}+\frac{49\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{72}$
trager	$\left(-\frac{5}{12}+\frac{x}{2}\right)\sqrt{-3x^2+5x+2}+\frac{49\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-6x\,\mathrm{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x+2}+5\,\mathrm{RootOf}\left(\_Z^2+3\right)\right.}{72}$

input
```
int((-3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
49/72*arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{1}{12}\sqrt{-3\,x^2+5\,x+2}(6\,x-5) - \frac{49}{72}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x+2}(6\,x-5)}{6\,(3\,x^2-5\,x-2)}\right)$$

input
```
integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) - 49/72*sqrt(3)*arctan(1/6*sqrt(3)*s
qrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5*x - 2))
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \sqrt{2+5x-3x^2}\,dx = \left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x+2}+\frac{49\sqrt{3}\operatorname{asin}\left(\frac{6x}{7}-\frac{5}{7}\right)}{72}$$

input
```
integrate((-3*x**2+5*x+2)**(1/2),x)
```

output
```
(x/2 - 5/12)*sqrt(-3*x**2 + 5*x + 2) + 49*sqrt(3)*asin(6*x/7 - 5/7)/72
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{1}{2}\sqrt{-3\,x^2+5\,x+2}x-\frac{49}{72}\sqrt{3}\arcsin\left(-\frac{6}{7}\,x+\frac{5}{7}\right) -\frac{5}{12}\sqrt{-3\,x^2+5\,x+2}$$

input
```
integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 5*x + 2)*x - 49/72*sqrt(3)*arcsin(-6/7*x + 5/7) - 5/12*s
qrt(-3*x^2 + 5*x + 2)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{1}{12}\sqrt{-3\,x^2+5\,x+2}(6\,x-5)+\frac{49}{72}\sqrt{3}\arcsin\left(\frac{6}{7}\,x-\frac{5}{7}\right)$$

input
```
integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*x - 5/7)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{49\sqrt{3}\,\mathrm{asin}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{72}+\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3\,x^2+5\,x+2}$$

input
```
int((5*x - 3*x^2 + 2)^(1/2),x)
```

output
```
(49*3^(1/2)*asin((6*x)/7 - 5/7))/72 + (x/2 - 5/12)*(5*x - 3*x^2 + 2)^(1/2)
```

3.110 $\int \sqrt{-2+4x+3x^2}\,dx$

3.110.1 Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \sqrt{-2+4x+3x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2}-\frac{5\text{arctanh}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}$$

output
```
-5/9*arctanh(1/3*(2+3*x)*3^(1/2)/(3*x^2+4*x-2)^(1/2))*3^(1/2)+1/6*(2+3*x)*
(3*x^2+4*x-2)^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \sqrt{-2+4x+3x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2}-\frac{10\text{arctanh}\left(\frac{\sqrt{-6+12x+9x^2}}{2+\sqrt{10}+3x}\right)}{3\sqrt{3}}$$

input
```
Integrate[Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (10*ArcTanh[Sqrt[-6 + 12*x + 9*x^2]
/(2 + Sqrt[10] + 3*x)])/(3*Sqrt[3])
```

3.110.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x^2+4x-2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5}{3}\int \frac{1}{\sqrt{3x^2+4x-2}}dx$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{10}{3}\int \frac{1}{12-\frac{4(3x+2)^2}{3x^2+4x-2}}d\frac{2(3x+2)}{\sqrt{3x^2+4x-2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\text{arctanh}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

input
```
Int[Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[
-2 + 4*x + 3*x^2])])/(3*Sqrt[3])
```

3.110.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.110.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{(4+6x)\sqrt{3x^2+4x-2}}{12}-\frac{5\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{9}$	50
risch	$\frac{(2+3x)\sqrt{3x^2+4x-2}}{6}-\frac{5\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{9}$	50
trager	$\left(\frac{1}{3}+\frac{x}{2}\right)\sqrt{3x^2+4x-2}-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(3\,\mathrm{RootOf}\left(\_Z^2-3\right)x+2\,\mathrm{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x-2}\right)}{9}$	61

input
```
int((3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(4+6*x)*(3*x^2+4*x-2)^(1/2)-5/9*ln(1/3*(2+3*x)*3^(1/2)+(3*x^2+4*x-2)^
(1/2))*3^(1/2)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{1}{6}\sqrt{3\,x^2+4\,x-2}(3\,x+2) + \frac{5}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x-2}(3\,x+2)+9\,x^2+12\,x-1\right)$$

input
```
integrate((3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \sqrt{-2+4x+3x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3x^2+4x-2} - \frac{5\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+4x-2}+4\right)}{9}$$

input
```
integrate((3*x**2+4*x-2)**(1/2),x)
```

output
```
(x/2 + 1/3)*sqrt(3*x**2 + 4*x - 2) - 5*sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*
x**2 + 4*x - 2) + 4)/9
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{1}{2}\sqrt{3x^2+4x-2}x - \frac{5}{9}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right) + \frac{1}{3}\sqrt{3x^2+4x-2}$$

input
```
integrate((3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 + 4*x - 2)*x - 5/9*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x -
 2) + 6*x + 4) + 1/3*sqrt(3*x^2 + 4*x - 2)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{1}{6}\sqrt{3x^2+4x-2}(3x+2) + \frac{5}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)-2\right|\right)$$

input
```
integrate((3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

3.110.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \sqrt{-2+4x+3x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3\,x^2+4\,x-2}-\frac{5\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2+4\,x-2}+\frac{\sqrt{3}\,(3\,x+2)}{3}\right)}{9}$$

input
```
int((4*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(x/2 + 1/3)*(4*x + 3*x^2 - 2)^(1/2) - (5*3^(1/2)*log((4*x + 3*x^2 - 2)^(1/
2) + (3^(1/2)*(3*x + 2))/3))/9
```

3.111 $\int \sqrt{-2+4x-3x^2}\,dx$

3.111.1 Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \sqrt{-2+4x-3x^2}\,dx=-\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2}+\frac{\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{3\sqrt{3}}$$

output
```
1/9*arctan(1/3*(2-3*x)*3^(1/2)/(-3*x^2+4*x-2)^(1/2))*3^(1/2)-1/6*(2-3*x)*(
-3*x^2+4*x-2)^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \sqrt{-2+4x-3x^2}\,dx=\frac{1}{6}(-2+3x)\sqrt{-2+4x-3x^2}-\frac{\arctan\left(\frac{-2+3x}{\sqrt{-6+12x-9x^2}}\right)}{3\sqrt{3}}$$

input
```
Integrate[Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
((-2 + 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 - ArcTan[(-2 + 3*x)/Sqrt[-6 + 12*x -
 9*x^2]]/(3*Sqrt[3])
```

3.111.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x^2+4x-2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{-3x^2+4x-2}}dx-\frac{1}{6}\sqrt{-3x^2+4x-2}(2-3x)$$

$$\Big\downarrow \text{1092}$$

$$-\frac{2}{3}\int \frac{1}{-\frac{4(2-3x)^2}{-3x^2+4x-2}-12}d\frac{2(2-3x)}{\sqrt{-3x^2+4x-2}}-\frac{1}{6}\sqrt{-3x^2+4x-2}(2-3x)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

input
```
Int[Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
-1/6*((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2]) + ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-
2 + 4*x - 3*x^2])]/(3*Sqrt[3])
```

3.111.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.111.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

method	result
default	$-\frac{(4-6x)\sqrt{-3x^2+4x-2}}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-\frac{2}{3}+x\right)}{\sqrt{-3x^2+4x-2}}\right)}{9}$
risch	$-\frac{(3x^2-4x+2)(-2+3x)}{6\sqrt{-3x^2+4x-2}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-\frac{2}{3}+x\right)}{\sqrt{-3x^2+4x-2}}\right)}{9}$
trager	$\left(-\frac{1}{3}+\frac{x}{2}\right)\sqrt{-3x^2+4x-2}+\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(3x\,\mathrm{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x-2}-2\,\mathrm{RootOf}\left(\_Z^2+3\right)\right)}{9}$

input
```
int((-3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(4-6*x)*(-3*x^2+4*x-2)^(1/2)-1/9*3^(1/2)*arctan(3^(1/2)*(-2/3+x)/(-3
*x^2+4*x-2)^(1/2))
```

3.111.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int\sqrt{-2+4x-3x^2}\,dx=\frac{1}{6}\sqrt{-3\,x^2+4\,x-2}(3\,x-2)-\frac{1}{18}i\,\sqrt{3}\log\left(-\frac{2\left(i\,\sqrt{3}\sqrt{-3\,x^2+4\,x-2}+3\,x-2\right)}{x}\right)+\frac{1}{18}i\,\sqrt{3}\log\left(-\frac{2\left(-i\,\sqrt{3}\sqrt{-3\,x^2+4\,x-2}+3\,x-2\right)}{x}\right)$$

input
```
integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(-3*x^2 + 4*x - 2)*(3*x - 2) - 1/18*I*sqrt(3)*log(-2*(I*sqrt(3)*sq
rt(-3*x^2 + 4*x - 2) + 3*x - 2)/x) + 1/18*I*sqrt(3)*log(-2*(-I*sqrt(3)*sqr
t(-3*x^2 + 4*x - 2) + 3*x - 2)/x)
```

3.111.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \sqrt{-2+4x-3x^2}\,dx = \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3x^2+4x-2} + \frac{\sqrt{3}i\log\left(-6x+2\sqrt{3}i\sqrt{-3x^2+4x-2}+4\right)}{9}$$

input
```
integrate((-3*x**2+4*x-2)**(1/2),x)
```

output
```
(x/2 - 1/3)*sqrt(-3*x**2 + 4*x - 2) + sqrt(3)*I*log(-6*x + 2*sqrt(3)*I*sqr
t(-3*x**2 + 4*x - 2) + 4)/9
```

3.111.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \sqrt{-2+4x-3x^2}\,dx = \frac{1}{2}\sqrt{-3\,x^2+4\,x-2}x + \frac{1}{9}i\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{2}(3\,x-2)\right) - \frac{1}{3}\sqrt{-3\,x^2+4\,x-2}$$

input
```
integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 4*x - 2)*x + 1/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2)
) - 1/3*sqrt(-3*x^2 + 4*x - 2)
```

3.111.8 Giac [F]

$$\int \sqrt{-2+4x-3x^2}\,dx = \int \sqrt{-3\,x^2+4\,x-2}\,dx$$

input
```
integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-3*x^2 + 4*x - 2), x)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \sqrt{-2+4x-3x^2}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{2}\,(3\,x-2)\,1\mathrm{i}}{2}\right)}{9} + \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3\,x^2+4\,x-2}$$

input
```
int((4*x - 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*asin((2^(1/2)*(3*x - 2)*1i)/2))/9 + (x/2 - 1/3)*(4*x - 3*x^2 - 2)
^(1/2)
```

3.112 $\int \sqrt{-2+5x+3x^2}\,dx$

3.112.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \sqrt{-2+5x+3x^2}\,dx=\frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49\text{arctanh}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{24\sqrt{3}}$$

output
```
-49/72*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x-2)^(1/2))*3^(1/2)+1/12*(5+6*
x)*(3*x^2+5*x-2)^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \sqrt{-2+5x+3x^2}\,dx=\frac{1}{36}\left(3(5+6x)\sqrt{-2+5x+3x^2}-49\sqrt{3}\text{arctanh}\left(\frac{\sqrt{-\frac{2}{3}+\frac{5x}{3}+x^2}}{2+x}\right)\right)$$

input
```
Integrate[Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
(3*(5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2] - 49*Sqrt[3]*ArcTanh[Sqrt[-2/3 + (5*x)
/3 + x^2]/(2 + x)])/36
```

3.112.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x^2+5x-2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49}{24}\int \frac{1}{\sqrt{3x^2+5x-2}}dx$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49}{12}\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x-2}}d\frac{6x+5}{\sqrt{3x^2+5x-2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

input
```
Int[Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*S
qrt[-2 + 5*x + 3*x^2])])/(24*Sqrt[3])
```

3.112.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.112.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

method	result	s
default	$\frac{(5+6x)\sqrt{3x^2+5x-2}}{12}-\frac{49\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{72}$	5
risch	$\frac{(5+6x)\sqrt{3x^2+5x-2}}{12}-\frac{49\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{72}$	5
trager	$\left(\frac{5}{12}+\frac{x}{2}\right)\sqrt{3x^2+5x-2}+\frac{49\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x-5\,\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2+5x-2}\right)}{72}$	6

input
```
int((3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(5+6*x)*(3*x^2+5*x-2)^(1/2)-49/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x
-2)^(1/2))*3^(1/2)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int\sqrt{-2+5x+3x^2}\,dx=\frac{1}{12}\sqrt{3\,x^2+5\,x-2}(6\,x+5)+\frac{49}{144}\sqrt{3}\log\left(-4\,\sqrt{3}\sqrt{3\,x^2+5\,x-2}(6\,x+5)+72\,x^2+120\,x+1\right)$$

input
```
integrate((3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/144*sqrt(3)*log(-4*sqrt(3)*sqrt(
3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x + 1)
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \sqrt{-2+5x+3x^2}\,dx$$
$$= \left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3x^2+5x-2} - \frac{49\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x-2}+5\right)}{72}$$

input
```
integrate((3*x**2+5*x-2)**(1/2),x)
```

output
```
(x/2 + 5/12)*sqrt(3*x**2 + 5*x - 2) - 49*sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(
3*x**2 + 5*x - 2) + 5)/72
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \sqrt{-2+5x+3x^2}\,dx = \frac{1}{2}\sqrt{3\,x^2+5\,x-2}x - \frac{49}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3\,x^2+5\,x-2}+6\,x+5\right) + \frac{5}{12}\sqrt{3\,x^2+5\,x-2}$$

input
```
integrate((3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 + 5*x - 2)*x - 49/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x
 - 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x - 2)
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \sqrt{-2+5x+3x^2}\,dx = \frac{1}{12}\sqrt{3\,x^2+5\,x-2}(6\,x+5) + \frac{49}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x-2}\right)-5\right|\right)$$

input
```
integrate((3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(s
qrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \sqrt{-2+5x+3x^2}\,dx$$
$$= \left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3\,x^2+5\,x-2}-\frac{49\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2+5\,x-2}+\frac{\sqrt{3}\,\left(3\,x+\frac{5}{2}\right)}{3}\right)}{72}$$

input
```
int((5*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(x/2 + 5/12)*(5*x + 3*x^2 - 2)^(1/2) - (49*3^(1/2)*log((5*x + 3*x^2 - 2)^(
1/2) + (3^(1/2)*(3*x + 5/2))/3))/72
```

3.113 $\int \sqrt{-2+5x-3x^2}\,dx$

3.113.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \sqrt{-2+5x-3x^2}\,dx = -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2} - \frac{\arcsin(5-6x)}{24\sqrt{3}}$$

output
```
1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{1}{36}\left(3(-5+6x)\sqrt{-2+5x-3x^2} - \sqrt{3}\arctan\left(\frac{\sqrt{-6+15x-9x^2}}{-2+3x}\right)\right)$$

input
```
Integrate[Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
(3*(-5 + 6*x)*Sqrt[-2 + 5*x - 3*x^2] - Sqrt[3]*ArcTan[Sqrt[-6 + 15*x - 9*x
^2]/(-2 + 3*x)])/36
```

3.113.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x^2+5x-2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{24}\int \frac{1}{\sqrt{-3x^2+5x-2}}dx-\frac{1}{12}(5-6x)\sqrt{-3x^2+5x-2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-(5-6x)^2}}d(5-6x)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x-2}(5-6x)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{\arcsin(5-6x)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x-2}(5-6x)$$

input
```
Int[Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
-1/12*((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2]) - ArcSin[5 - 6*x]/(24*Sqrt[3])
```

3.113.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.113.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result
default	$\frac{\arcsin(-5+6x)\sqrt{3}}{72}-\frac{(5-6x)\sqrt{-3x^2+5x-2}}{12}$
risch	$-\frac{(3x^2-5x+2)(-5+6x)}{12\sqrt{-3x^2+5x-2}}+\frac{\arcsin(-5+6x)\sqrt{3}}{72}$
trager	$\left(-\frac{5}{12}+\frac{x}{2}\right)\sqrt{-3x^2+5x-2}+\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(-6x\operatorname{RootOf}\left(\_Z^2+3\right)+5\operatorname{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x-2}\right)}{72}$

input
```
int((-3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{1}{12}\sqrt{-3\,x^2+5\,x-2}(6\,x-5) - \frac{1}{72}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x-2}(6\,x-5)}{6\,(3\,x^2-5\,x+2)}\right)$$

input
```
integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) - 1/72*sqrt(3)*arctan(1/6*sqrt(3)*sq
rt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5*x + 2))
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \sqrt{-2+5x-3x^2}\,dx = \left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x-2}+\frac{\sqrt{3}\operatorname{asin}(6x-5)}{72}$$

input
```
integrate((-3*x**2+5*x-2)**(1/2),x)
```

output
```
(x/2 - 5/12)*sqrt(-3*x**2 + 5*x - 2) + sqrt(3)*asin(6*x - 5)/72
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{1}{2}\sqrt{-3x^2+5x-2}x+\frac{1}{72}\sqrt{3}\arcsin(6x-5)-\frac{5}{12}\sqrt{-3x^2+5x-2}$$

input
```
integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 5*x - 2)*x + 1/72*sqrt(3)*arcsin(6*x - 5) - 5/12*sqrt(-3
*x^2 + 5*x - 2)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{1}{12}\sqrt{-3x^2+5x-2}(6x-5)+\frac{1}{72}\sqrt{3}\arcsin(6x-5)$$

input
```
integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(6\,x-5)}{72} + \left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3\,x^2+5\,x-2}$$

input
```
int((5*x - 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*asin(6*x - 5))/72 + (x/2 - 5/12)*(5*x - 3*x^2 - 2)^(1/2)
```

3.114 $\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx$

3.114.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{1}{3}\text{arcsinh}\left(\frac{1}{2}(-1+3x)\right)$$

output
```
1/3*arcsinh(-1/2+3/2*x)
```

3.114.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = -\frac{1}{3}\log\left(1-3x+\sqrt{5-6x+9x^2}\right)$$

input
```
Integrate[1/Sqrt[5 - 6*x + 9*x^2],x]
```

output
```
-1/3*Log[1 - 3*x + Sqrt[5 - 6*x + 9*x^2]]
```

3.114.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9x^2-6x+5}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{36}\int \frac{1}{\sqrt{\frac{1}{144}(18x-6)^2+1}}d(18x-6)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{3}\text{arcsinh}\left(\frac{1}{12}(18x-6)\right)$$

input
```
Int[1/Sqrt[5 - 6*x + 9*x^2],x]
```

output
```
ArcSinh[(-6 + 18*x)/12]/3
```

3.114.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.114.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\operatorname{arcsinh}\left(-\frac{1}{2}+\frac{3x}{2}\right)}{3}$	9
trager	$\frac{\ln\left(-1+3x+\sqrt{9x^2-6x+5}\right)}{3}$	21

input
```
int(1/(9*x^2-6*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsinh(-1/2+3/2*x)
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = -\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

input
```
integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

3.114.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}$$

input
```
integrate(1/(9*x**2-6*x+5)**(1/2),x)
```

output
```
asinh(3*x/2 - 1/2)/3
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{1}{3}\,\text{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

input
```
integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsinh(3/2*x - 1/2)
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{1}{6}\sqrt{9\,x^2-6\,x+5}(3\,x-1)-\frac{2}{3}\,\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

input
```
integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
 + 1)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{\ln\left(3\,x+\sqrt{9\,x^2-6\,x+5}-1\right)}{3}$$

input
```
int(1/(9*x^2 - 6*x + 5)^(1/2),x)
```

output
```
log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1)/3
```

3.115 $\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx$

3.115.1 Optimal result

Integrand size = 14, antiderivative size = 10

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{1}{2}+x\right)$$

output
```
1/2*arcsin(1/2+x)
```

3.115.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = -\arctan\left(\frac{\sqrt{3-4x-4x^2}}{3+2x}\right)$$

input
```
Integrate[1/Sqrt[3 - 4*x - 4*x^2],x]
```

output
```
-ArcTan[Sqrt[3 - 4*x - 4*x^2]/(3 + 2*x)]
```

3.115.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-4x^2-4x+3}}\,dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{16}\int \frac{1}{\sqrt{1-\frac{1}{64}(-8x-4)^2}}d(-8x-4)$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{2}\arcsin\left(\frac{1}{8}(-8x-4)\right)$$

input
```
Int[1/Sqrt[3 - 4*x - 4*x^2],x]
```

output
```
-1/2*ArcSin[(-4 - 8*x)/8]
```

3.115.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.115.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arcsin\left(x+\frac{1}{2}\right)}{2}$	7
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(2\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-4x^2-4x+3}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{2}$	38

input
```
int(1/(-4*x^2-4*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x+1/2)
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.30

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = -\frac{1}{2}\arctan\left(\frac{\sqrt{-4\,x^2-4\,x+3}(2\,x+1)}{4\,x^2+4\,x-3}\right)$$

input
```
integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x + 1)/(4*x^2 + 4*x - 3))
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = \frac{\operatorname{asin}\left(x+\frac{1}{2}\right)}{2}$$

input
```
integrate(1/(-4*x**2-4*x+3)**(1/2),x)
```

output
```
asin(x + 1/2)/2
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = -\frac{1}{2}\arcsin\left(-x-\frac{1}{2}\right)$$

input
```
integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arcsin(-x - 1/2)
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(6) = 12.

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = \frac{1}{4}\sqrt{-4\,x^2-4\,x+3}(2\,x+1)+\arcsin\left(x+\frac{1}{2}\right)$$

input
```
integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = \frac{\mathrm{asin}\left(x+\frac{1}{2}\right)}{2}$$

input
```
int(1/(3 - 4*x^2 - 4*x)^(1/2),x)
```

output
```
asin(x + 1/2)/2
```

3.116 $\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$

3.116.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{1}{3}\text{arctanh}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)$$

output
```
1/3*arctanh((1+3*x)/(9*x^2+6*x-8)^(1/2))
```

3.116.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = -\frac{1}{3}\log\left(-1-3x+\sqrt{-8+6x+9x^2}\right)$$

input
```
Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

output
```
-1/3*Log[-1 - 3*x + Sqrt[-8 + 6*x + 9*x^2]]
```

3.116.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9x^2+6x-8}}dx$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{36-\frac{36(3x+1)^2}{9x^2+6x-8}}d\frac{6(3x+1)}{\sqrt{9x^2+6x-8}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\text{arctanh}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

input
```
Int[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

output
```
ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

3.116.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.116.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
trager	$\frac{\ln\left(\sqrt{9x^2+6x-8}+1+3x\right)}{3}$	21
default	$\frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{9}$	30

input
```
int(1/(9*x^2+6*x-8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln((9*x^2+6*x-8)^(1/2)+1+3*x)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = -\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right)$$

input
```
integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)}{3}$$

input
```
integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

output
```
log(18*x + 6*sqrt(9*x**2 + 6*x - 8) + 6)/3
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{1}{3}\log\left(18\,x+6\,\sqrt{9\,x^2+6\,x-8}+6\right)$$

input
```
integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{1}{6}\,\sqrt{9\,x^2+6\,x-8}(3\,x+1)+\frac{3}{2}\,\log\left(\left|-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right|\right)$$

input
```
integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x
- 8) - 1))
```

3.116.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{\ln\left(3\,x+\sqrt{9\,x^2+6\,x-8}+1\right)}{3}$$

input
```
int(1/(6*x + 9*x^2 - 8)^(1/2),x)
```

output
```
log(3*x + (6*x + 9*x^2 - 8)^(1/2) + 1)/3
```

3.117 $\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx$

3.117.1 Optimal result

Integrand size = 14, antiderivative size = 18

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx=\frac{\operatorname{arcsinh}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{3}}$$

output
```
1/3*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx=-\frac{\log\left(-2-3x+\sqrt{6+12x+9x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
-(Log[-2 - 3*x + Sqrt[6 + 12*x + 9*x^2]]/Sqrt[3])
```

3.117.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+4x+2}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{\int \frac{1}{\sqrt{\frac{1}{8}(6x+4)^2+1}}d(6x+4)}{2\sqrt{6}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}\left(\frac{6x+4}{2\sqrt{2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
ArcSinh[(4 + 6*x)/(2*Sqrt[2])]/Sqrt[3]
```

3.117.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.117.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\,\left(\frac{2}{3}+x\right)}{2}\right)}{3}$	15
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^{2}-3\right)x+2\operatorname{RootOf}\left(\_Z^{2}-3\right)+3\sqrt{3x^{2}+4x+2}\right)}{3}$	42

input
```
int(1/(3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsinh(3/2*2^(1/2)*(2/3+x))
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x+2}(3\,x+2)-9\,x^2-12\,x-5\right)$$

input
```
integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x -
5)
```

3.117.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{3}$$

input
```
integrate(1/(3*x**2+4*x+2)**(1/2),x)
```

output
```
sqrt(3)*asinh(3*sqrt(2)*(x + 2/3)/2)/3
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

input
```
integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2))
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(16) = 32.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{1}{6}\sqrt{3x^2+4x+2}(3x+2) - \frac{1}{9}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x+2}\right)-2\right)$$

input
```
integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 4*x + 2)) - 2)
```

3.117.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{3}\ln\left(\sqrt{3}\left(x+\frac{2}{3}\right)+\sqrt{3x^2+4x+2}\right)}{3}$$

input
```
int(1/(4*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 + 2)^(1/2)))/3
```

3.118 $\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx$

3.118.1 Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx=-\frac{\arcsin\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

output
```
-1/3*arcsin(1/10*(2-3*x)*10^(1/2))*3^(1/2)
```

3.118.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3}x}{\sqrt{2}-\sqrt{2+4x-3x^2}}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
(-2*ArcTan[(Sqrt[3]*x)/(Sqrt[2] - Sqrt[2 + 4*x - 3*x^2])])/Sqrt[3]
```

3.118.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2+4x+2}}dx$$

$$\Big\downarrow \ 1090$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{40}(4-6x)^2}}d(4-6x)}{2\sqrt{30}}$$

$$\Big\downarrow \ 223$$

$$-\frac{\arcsin\left(\frac{4-6x}{2\sqrt{10}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
-(ArcSin[(4 - 6*x)/(2*Sqrt[10])]/Sqrt[3])
```

3.118.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.118.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\sqrt{3}\, \arcsin\left(\frac{3\sqrt{10}\,\left(-\frac{2}{3}+x\right)}{10}\right)}{3}$	15
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+3\right) \ln\left(3x \operatorname{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x+2}-2 \operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

input
```
int(1/(-3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsin(3/10*10^(1/2)*(-2/3+x))
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)}{3\,(3\,x^2-4\,x-2)}\right)$$

input
```
integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 -
4*x - 2))
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{3\sqrt{10}(x-\frac{2}{3})}{10}\right)}{3}$$

input
```
integrate(1/(-3*x**2+4*x+2)**(1/2),x)
```

output
```
sqrt(3)*asin(3*sqrt(10)*(x - 2/3)/10)/3
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x-2)\right)$$

input
```
integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = \frac{1}{6}\sqrt{-3x^2+4x+2}(3x-2) + \frac{5}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x-2)\right)$$

input
```
integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3
*x - 2))
```

3.118.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{40}\,(6x-4)}{40}\right)}{3}$$

input
```
int(1/(4*x - 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*asin((40^(1/2)*(6*x - 4))/40))/3
```

3.119 $\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$

3.119.1 Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}}$$

output
```
1/3*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}+\frac{5x}{3}+x^2}}{1+x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(2*ArcTanh[Sqrt[2/3 + (5*x)/3 + x^2]/(1 + x)])/Sqrt[3]
```

3.119.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x+2}}d\frac{6x+5}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

3.119.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.119.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}}{3}$	30
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(6\operatorname{RootOf}\left(\_Z^{2}-3\right)x+5\operatorname{RootOf}\left(\_Z^{2}-3\right)+6\sqrt{3x^{2}+5x+2}\right)}{3}$	42

input
```
int(1/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2+5x+2}(6x+5)+72x^2+120x+49\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x
 + 49)
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x+2}+5\right)}{3}$$

input
```
integrate(1/(3*x**2+5*x+2)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 5*x + 2) + 5)/3
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{12}\sqrt{3x^2+5x+2}(6x+5) + \frac{1}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x+2}\right)-5\right|\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

3.119.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3}\ln\left(\sqrt{3}\left(x+\frac{5}{6}\right)+\sqrt{3x^2+5x+2}\right)}{3}$$

input
```
int(1/(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 + 2)^(1/2)))/3
```

3.120 $\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx$

3.120.1 Optimal result

Integrand size = 14, antiderivative size = 17

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx=-\frac{\arcsin\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

output
```
1/3*arcsin(-5/7+6/7*x)*3^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{6+15x-9x^2}}{1+3x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
(-2*ArcTan[Sqrt[6 + 15*x - 9*x^2]/(1 + 3*x)])/Sqrt[3]
```

3.120.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2+5x+2}}dx$$

$$\Big\downarrow \ 1090$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{49}(5-6x)^2}}d(5-6x)}{7\sqrt{3}}$$

$$\Big\downarrow \ 223$$

$$-\frac{\arcsin\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
-(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

3.120.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.120.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{3}$	12
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(-6x\operatorname{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x+2}+5\operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

input
```
int(1/(-3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(-5/7+6/7*x)*3^(1/2)
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}(6x-5)}{6(3x^2-5x-2)}\right)$$

input
```
integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 -
5*x - 2))
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asin}\left(\frac{6x}{7}-\frac{5}{7}\right)}{3}$$

input
```
integrate(1/(-3*x**2+5*x+2)**(1/2),x)
```

output
```
sqrt(3)*asin(6*x/7 - 5/7)/3
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)$$

input
```
integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arcsin(-6/7*x + 5/7)
```

3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = \frac{1}{12}\sqrt{-3x^2+5x+2}(6x-5)+\frac{49}{72}\sqrt{3}\arcsin\left(\frac{6}{7}x-\frac{5}{7}\right)$$

input
```
integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*x - 5/7)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{6x}{7}-\frac{5}{7}\right)}{3}$$

input
```
int(1/(5*x - 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*asin((6*x)/7 - 5/7))/3
```

3.121 $\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx$

3.121.1 Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{\sqrt{3}}$$

output
```
1/3*arctanh(1/3*(2+3*x)*3^(1/2)/(3*x^2+4*x-2)^(1/2))*3^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = -\frac{\log\left(-2-3x+\sqrt{-6+12x+9x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
-(Log[-2 - 3*x + Sqrt[-6 + 12*x + 9*x^2]]/Sqrt[3])
```

3.121.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+4x-2}}dx$$
$$\Big\downarrow \text{1092}$$
$$2\int \frac{1}{12-\frac{4(3x+2)^2}{3x^2+4x-2}}d\frac{2(3x+2)}{\sqrt{3x^2+4x-2}}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]
```

3.121.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.121.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{3}$	30
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+2\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x-2}\right)}{3}$	42

input
```
int(1/(3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1/3*(2+3*x)*3^(1/2)+(3*x^2+4*x-2)^(1/2))*3^(1/2)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+4\,x-2}(3\,x+2)+9\,x^2+12\,x-1\right)$$

input
```
integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1
)
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+4x-2}+4\right)}{3}$$

input
```
integrate(1/(3*x**2+4*x-2)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 4*x - 2) + 4)/3
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right)$$

input
```
integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{6}\sqrt{3x^2+4x-2}(3x+2) + \frac{5}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)-2\right|\right)$$

input
```
integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

3.121.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(x+\frac{2}{3}\right)+\sqrt{3x^2+4x-2}\right)}{3}$$

input
```
int(1/(4*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 - 2)^(1/2)))/3
```

3.122 $\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx$

3.122.1 Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{\sqrt{3}}$$

output
```
-1/3*arctan(1/3*(2-3*x)*3^(1/2)/(-3*x^2+4*x-2)^(1/2))*3^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\arctan\left(\frac{2-3x}{\sqrt{-6+12x-9x^2}}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
-(ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9*x^2]]/Sqrt[3])
```

3.122.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2+4x-2}}dx$$
$$\Big\downarrow \text{1092}$$
$$2\int \frac{1}{-\frac{4(2-3x)^2}{-3x^2+4x-2}-12}d\frac{2(2-3x)}{\sqrt{-3x^2+4x-2}}$$
$$\Big\downarrow \text{217}$$
$$-\frac{\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
-(ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/Sqrt[3])
```

3.122.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.122.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-\frac{2}{3}+x\right)}{\sqrt{-3x^{2}+4x-2}}\right)}{3}$	26
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}+3\right)\ln\left(-3x\operatorname{RootOf}\left(\_Z^{2}+3\right)+3\sqrt{-3x^{2}+4x-2}+2\operatorname{RootOf}\left(\_Z^{2}+3\right)\right)}{3}$	42

input
```
int(1/(-3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctan(3^(1/2)*(-2/3+x)/(-3*x^2+4*x-2)^(1/2))
```

3.122.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = \frac{1}{6}i\sqrt{3}\log\left(-\frac{2\left(i\sqrt{3}\sqrt{-3\,x^2+4\,x-2}+3\,x-2\right)}{x}\right) - \frac{1}{6}i\sqrt{3}\log\left(-\frac{2\left(-i\sqrt{3}\sqrt{-3\,x^2+4\,x-2}+3\,x-2\right)}{x}\right)$$

input
```
integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*I*sqrt(3)*log(-2*(I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) + 3*x - 2)/x) - 1/6
*I*sqrt(3)*log(-2*(-I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) + 3*x - 2)/x)
```

3.122.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\sqrt{3}i\log\left(-6x+2\sqrt{3}i\sqrt{-3x^2+4x-2}+4\right)}{3}$$

input
```
integrate(1/(-3*x**2+4*x-2)**(1/2),x)
```

output
```
-sqrt(3)*I*log(-6*x + 2*sqrt(3)*I*sqrt(-3*x**2 + 4*x - 2) + 4)/3
```

3.122.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{1}{3}i\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{2}(3\,x-2)\right)$$

input
```
integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
```

3.122.8 Giac [F]

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = \int \frac{1}{\sqrt{-3\,x^2+4\,x-2}}\,dx$$

input
```
integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^2 + 4*x - 2), x)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\sqrt{3}\,\mathrm{asin}\left(\sqrt{2}\,\left(\frac{3\,x}{2}-1\right)\,1\mathrm{i}\right)}{3}$$

input
```
int(1/(4*x - 3*x^2 - 2)^(1/2),x)
```

output
```
-(3^(1/2)*asin(2^(1/2)*((3*x)/2 - 1)*1i))/3
```

3.123 $\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx$

3.123.1 Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{\sqrt{3}}$$

output
```
1/3*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x-2)^(1/2))*3^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2}{3}+\frac{5x}{3}+x^2}}{2+x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
(2*ArcTanh[Sqrt[-2/3 + (5*x)/3 + x^2]/(2 + x)])/Sqrt[3]
```

3.123.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+5x-2}}\,dx$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x-2}}d\frac{6x+5}{\sqrt{3x^2+5x-2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]
```

3.123.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.123.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{3}$	30
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2+5x-2}+5\,\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{3}$	42

input
```
int(1/(3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-2)^(1/2))*3^(1/2)
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2+5x-2}(6x+5)+72x^2+120x+1\right)$$

input
```
integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x
 + 1)
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x-2}+5\right)}{3}$$

input
```
integrate(1/(3*x**2+5*x-2)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 5*x - 2) + 5)/3
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^2+5\,x-2}+6\,x+5\right)$$

input
```
integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{12}\,\sqrt{3\,x^2+5\,x-2}(6\,x+5) + \frac{49}{72}\,\sqrt{3}\log\left(\left|-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x-2}\right)-5\right|\right)$$

input
```
integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(s
qrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

3.123.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x+\frac{5}{6}\right)+\sqrt{3\,x^2+5\,x-2}\right)}{3}$$

input
```
int(1/(5*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 - 2)^(1/2)))/3
```

3.124 $\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx$

3.124.1 Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = -\frac{\arcsin(5-6x)}{\sqrt{3}}$$

output
```
1/3*arcsin(-5+6*x)*3^(1/2)
```

3.124.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-6+15x-9x^2}}{-2+3x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
(-2*ArcTan[Sqrt[-6 + 15*x - 9*x^2]/(-2 + 3*x)])/Sqrt[3]
```

3.124.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2+5x-2}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-(5-6x)^2}}d(5-6x)}{\sqrt{3}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\arcsin(5-6x)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
-(ArcSin[5 - 6*x]/Sqrt[3])
```

3.124.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.124.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\arcsin(-5+6x)\sqrt{3}}{3}$	12
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-6x\,\mathrm{RootOf}\left(\_Z^2+3\right)+5\,\mathrm{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x-2}\right)}{3}$	42

input
```
int(1/(-3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(-5+6*x)*3^(1/2)
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.08

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x-2}(6\,x-5)}{6\,(3\,x^2-5\,x+2)}\right)$$

input
```
integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 -
5*x + 2))
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\,(6x-5)}{3}$$

input
```
integrate(1/(-3*x**2+5*x-2)**(1/2),x)
```

output
```
sqrt(3)*asin(6*x - 5)/3
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{1}{3}\sqrt{3}\arcsin\left(6\,x-5\right)$$

input
```
integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsin(6*x - 5)
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{1}{12}\sqrt{-3\,x^2+5\,x-2}(6\,x-5) + \frac{1}{72}\sqrt{3}\arcsin\left(6\,x-5\right)$$

input
```
integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)
```

3.124.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(6\,x-5)}{3}$$

input
```
int(1/(5*x - 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*asin(6*x - 5))/3
```

3.125 $\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx$

3.125.1 Optimal result

Integrand size = 27, antiderivative size = 22

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx=\frac{\operatorname{arcsinh}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
arcsinh(1/2*(2*c*x+b)/c^(1/2))/c^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx=-\frac{\log\left(b+2cx-\sqrt{c}\sqrt{4+\frac{b^2}{c}+4bx+4cx^2}\right)}{\sqrt{c}}$$

input
```
Integrate[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

output
```
-(Log[b + 2*c*x - Sqrt[c]*Sqrt[4 + b^2/c + 4*b*x + 4*c*x^2]]/Sqrt[c])
```

3.125.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{\int \frac{1}{\sqrt{\frac{(b+2cx)^2}{4c}+1}}d(b+2cx)}{2c}$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

output
```
ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

3.125.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(16) = 32$.

Time = 2.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

method	result	size
default	$\frac{\ln\left(\frac{(4cx+2b)\sqrt{4}}{4\sqrt{c}}+\sqrt{\frac{b^2+4c}{c}+4bx+4c\,x^2}\right)\sqrt{4}}{2\sqrt{c}}$	51

input
```
int(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1/4*(4*c*x+2*b)*4^(1/2)/c^(1/2)+((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2))*
4^(1/2)/c^(1/2)
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 6.23

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = \left[\frac{\log\left(-4\,c^2x^2-4\,bcx-b^2-(2\,cx+b)\sqrt{c}\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}-2\,c\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{(2\,cx+b)\sqrt{-c}\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}}{4\,c^2x^2+4\,bcx+b^2+4\,c}\right)}{c}\right]$$

input
```
integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-4*c^2*x^2 - 4*b*c*x - b^2 - (2*c*x + b)*sqrt(c)*sqrt((4*c^2*x^2
+ 4*b*c*x + b^2 + 4*c)/c) - 2*c)/sqrt(c), -sqrt(-c)*arctan((2*c*x + b)*sqr
t(-c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c)/(4*c^2*x^2 + 4*b*c*x + b^2
 + 4*c))/c]
```

3.125.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.64

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = 2\left(\begin{cases} \frac{\log\left(4b+4\sqrt{c}\sqrt{4bx+4cx^2+\frac{b^2+4c}{c}}+8cx\right)}{2\sqrt{c}} & \text{for } c\neq 0 \\ \frac{\sqrt{4bx+\frac{b^2+4c}{c}}}{2b} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{\frac{b^2+4c}{c}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(2/((b**2+4*c)/c+4*b*x+4*c*x**2)**(1/2),x)
```

output
```
2*Piecewise((log(4*b + 4*sqrt(c)*sqrt(4*b*x + 4*c*x**2 + (b**2 + 4*c)/c) +
 8*c*x)/(2*sqrt(c)), Ne(c, 0)), (sqrt(4*b*x + (b**2 + 4*c)/c)/(2*b), Ne(b,
 0)), (x/sqrt((b**2 + 4*c)/c), True))
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{2\,cx+b}{2\,\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/2*(2*c*x + b)/sqrt(c))/sqrt(c)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(16) = 32.

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = -\frac{\log\left(\left|-bc^2-\left(2\,\sqrt{c^3}x-\sqrt{4\,c^3x^2+4\,bc^2x+b^2c+4\,c^2}\right)\sqrt{c}|c|\right|\right)}{\sqrt{c}}$$

input
```
integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-b*c^2 - (2*sqrt(c^3)*x - sqrt(4*c^3*x^2 + 4*b*c^2*x + b^2*c + 4*
c^2))*sqrt(c)*abs(c)))/sqrt(c)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx=\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+\sqrt{\frac{b^2+4\,c}{c}+4\,b\,x+4\,c\,x^2}\right)}{\sqrt{c}}$$

input
```
int(2/((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2),x)
```

output
```
log((b + 2*c*x)/c^(1/2) + ((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2))/c^(1/2)
```

3.126 $\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx$

3.126.1 Optimal result

Integrand size = 30, antiderivative size = 23

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx=-\frac{\arcsin\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
-arcsin(1/2*(-2*c*x+b)/c^(1/2))/c^(1/2)
```

3.126.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(23) = 46.

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 5.35

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx = 2\left(-\frac{\arctan\left(\frac{2\sqrt{-c^2}x-\sqrt{c}\sqrt{4-\frac{b^2}{c}+4bx-4cx^2}}{b}\right)}{2\sqrt{c}} - \frac{\log\left(2c^2x^2+c\left(-1-bx+\sqrt{-c}x\sqrt{4-\frac{b^2}{c}+4bx-4cx^2}\right)\right)}{4\sqrt{-c}}\right)$$

input
```
Integrate[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

output
```
2*(-1/2*ArcTan[(2*Sqrt[-c^2]*x - Sqrt[c]*Sqrt[4 - b^2/c + 4*b*x - 4*c*x^2]
)/b]/Sqrt[c] - Log[2*c^2*x^2 + c*(-1 - b*x + Sqrt[-c]*x*Sqrt[4 - b^2/c + 4
*b*x - 4*c*x^2])]/(4*Sqrt[-c]))
```

3.126.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{4c-b^2}{4c}+bx-cx^2}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{(b-2cx)^2}{4c}}}d(b-2cx)}{2c}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\arcsin\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

output
```
-(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

3.126.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.

Time = 2.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{\arctan\left(\frac{2\sqrt{c}\left(x-\frac{b}{2c}\right)}{\sqrt{-4c\,x^{2}+4bx-\frac{b^{2}-4c}{c}}}\right)}{\sqrt{c}}$	44

input
```
int(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2/c*b)/(-4*c*x^2+4*b*x-(b^2-4*c)/c)^(1/2))
```

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 6.13

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx = \left[-\frac{\sqrt{-c}\log\left(4\,c^2x^2-4\,bcx+b^2-(2\,cx-b)\sqrt{-c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}-2\,c\right)}{2\,c}, -\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}}{4\,c^2x^2-4\,bcx+b^2-4\,c}\right)}{\sqrt{c}}\right]$$

input
```
integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-c)*log(4*c^2*x^2 - 4*b*c*x + b^2 - (2*c*x - b)*sqrt(-c)*sqrt(-
(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c) - 2*c)/c, -arctan((2*c*x - b)*sqrt(c)
*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - 4
*c))/sqrt(c)]
```

3.126.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.61

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx = 2\left(\begin{cases} \frac{\log\left(4b-8cx+4\sqrt{-c}\sqrt{4bx-4cx^2+\frac{-b^2+4c}{c}}\right)}{2\sqrt{-c}} & \text{for } c\neq 0 \\ \frac{\sqrt{4bx+\frac{-b^2+4c}{c}}}{2b} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{\frac{-b^2+4c}{c}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(2/((-b**2+4*c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

output
```
2*Piecewise((log(4*b - 8*c*x + 4*sqrt(-c)*sqrt(4*b*x - 4*c*x**2 + (-b**2 +
 4*c)/c))/(2*sqrt(-c)), Ne(c, 0)), (sqrt(4*b*x + (-b**2 + 4*c)/c)/(2*b), N
e(b, 0)), (x/sqrt((-b**2 + 4*c)/c), True))
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx = -\frac{\arcsin\left(-\frac{2\,cx-b}{2\,\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/2*(2*c*x - b)/sqrt(c))/sqrt(c)
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx = -\frac{\log\left(b\sqrt{-c}c-\left(2\,\sqrt{-c^3}x-\sqrt{-4\,c^3x^2+4\,bc^2x-b^2c+4\,c^2}\right)|c|\right)}{\sqrt{-c}}$$

input
```
integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

output
```
-log(b*sqrt(-c)*c - (2*sqrt(-c^3)*x - sqrt(-4*c^3*x^2 + 4*b*c^2*x - b^2*c
+ 4*c^2))*abs(c))/sqrt(-c)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx = \frac{\ln\left(\frac{b-2\,c\,x}{\sqrt{-c}}+\sqrt{4\,b\,x+\frac{4\,c-b^2}{c}-4\,c\,x^2}\right)}{\sqrt{-c}}$$

input
```
int(2/(4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2),x)
```

output
```
log((b - 2*c*x)/(-c)^(1/2) + (4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2))/(-c)
^(1/2)
```

3.126. $\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx$

3.127 $\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}} \, dx$

3.127.1 Optimal result

Integrand size = 28, antiderivative size = 20

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}} \, dx = -\frac{\arcsin\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
-arcsin((-2*c*x+b)/c^(1/2))/c^(1/2)
```

3.127.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(20) = 40.

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 6.15

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}} \, dx = -\frac{-2\sqrt{-c}\arctan\left(\frac{\sqrt{c}\left(-2\sqrt{-c}x+\sqrt{1-\frac{b^2}{c}+4bx-4cx^2}\right)}{b}\right)+\sqrt{c}\log\left(c\left(-1-4bx+8cx^2+4\sqrt{-c}x\sqrt{1-\frac{b^2}{c}+4bx\right.\right.}{2\sqrt{-c^2}}$$

input
```
Integrate[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

output
```
-1/2*(-2*Sqrt[-c]*ArcTan[(Sqrt[c]*(-2*Sqrt[-c]*x + Sqrt[1 - b^2/c + 4*b*x
- 4*c*x^2]))/b] + Sqrt[c]*Log[c*(-1 - 4*b*x + 8*c*x^2 + 4*Sqrt[-c]*x*Sqrt[
1 - b^2/c + 4*b*x - 4*c*x^2]])])/Sqrt[-c^2]
```

3.127.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{c-b^2}{4c}+bx-cx^2}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{(b-2cx)^2}{c}}}d(b-2cx)}{c}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{\arcsin\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

output
```
-(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

3.127.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.127.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 2.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

method	result	size
default	$\frac{\arctan\left(\frac{2\sqrt{c}\,\left(x-\frac{b}{2c}\right)}{\sqrt{-4c\,x^{2}+4bx-\frac{b^{2}-c}{c}}}\right)}{\sqrt{c}}$	44

input
```
int(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2/c*b)/(-4*c*x^2+4*b*x-(b^2-c)/c)^(1/2))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(19) = 38$.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 7.15

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx$$
$$=\left[-\frac{\sqrt{-c}\log\left(8\,c^2x^2-8\,bcx+2\,b^2-2\,(2\,cx-b)\sqrt{-c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}-c\right)}{2\,c},\ -\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}}{4\,c^2x^2-4\,bcx+b^2-c}\right)}{\sqrt{c}}\right]$$

input
```
integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + 2*b^2 - 2*(2*c*x - b)*sqrt(-c)*sq
rt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c) - c)/c, -arctan((2*c*x - b)*sqrt(c)
*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - c))
/sqrt(c)]
```

3.127.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.90

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx = 2\left(\begin{cases} \frac{\log\left(4b-8cx+4\sqrt{-c}\sqrt{4bx-4cx^2+\frac{-b^2+c}{c}}\right)}{2\sqrt{-c}} & \text{for } c \neq 0 \\ \frac{\sqrt{4bx+\frac{-b^2+c}{c}}}{2b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{\frac{-b^2+c}{c}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(2/((-b**2+c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

output
```
2*Piecewise((log(4*b - 8*c*x + 4*sqrt(-c)*sqrt(4*b*x - 4*c*x**2 + (-b**2 +
 c)/c))/(2*sqrt(-c)), Ne(c, 0)), (sqrt(4*b*x + (-b**2 + c)/c)/(2*b), Ne(b,
 0)), (x/sqrt((-b**2 + c)/c), True))
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx = -\frac{\arcsin\left(-\frac{2\,cx-b}{\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-(2*c*x - b)/sqrt(c))/sqrt(c)
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(19) = 38.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx = -\frac{\log\left(b\sqrt{-c}c - \left(2\sqrt{-c^3}x - \sqrt{-4\,c^3x^2+4\,bc^2x-b^2c+c^2}\right)|c|\right)}{\sqrt{-c}}$$

input
```
integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

output
```
-log(b*sqrt(-c)*c - (2*sqrt(-c^3)*x - sqrt(-4*c^3*x^2 + 4*b*c^2*x - b^2*c
+ c^2))*abs(c))/sqrt(-c)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx=\frac{\ln\left(\frac{b-2\,c\,x}{\sqrt{-c}}+\sqrt{\frac{c-b^2}{c}+4\,b\,x-4\,c\,x^2}\right)}{\sqrt{-c}}$$

input
```
int(2/((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2),x)
```

output
```
log((b - 2*c*x)/(-c)^(1/2) + ((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2))/(-c)^(
1/2)
```

3.128 $\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx$

3.128.1 Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

output
```
-2*(3+2*x)/(x^2+3*x+2)^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

input
```
Integrate[(2 + 3*x + x^2)^(-3/2),x]
```

output
```
(-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

3.128.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+3x+2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1088$$

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

input
```
Int[(2 + 3*x + x^2)^(-3/2),x]
```

output
```
(-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

3.128.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && 
 NeQ[b^2 - 4*a*c, 0]
```

3.128.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
trager	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
risch	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
gosper	$-\frac{2(2+x)(1+x)(2x+3)}{(x^2+3x+2)^{\frac{3}{2}}}$	24

input
```
int(1/(x^2+3*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*x+3)/(x^2+3*x+2)^(1/2)
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2\left(2\,x^2+\sqrt{x^2+3\,x+2}(2\,x+3)+6\,x+4\right)}{x^2+3\,x+2}$$

input
```
integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*x^2 + sqrt(x^2 + 3*x + 2)*(2*x + 3) + 6*x + 4)/(x^2 + 3*x + 2)
```

3.128.6 Sympy [F]

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+3x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**2+3*x+2)**(3/2),x)
```

output
```
Integral((x**2 + 3*x + 2)**(-3/2), x)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{4\,x}{\sqrt{x^2+3\,x+2}} - \frac{6}{\sqrt{x^2+3\,x+2}}$$

input
```
integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-4*x/sqrt(x^2 + 3*x + 2) - 6/sqrt(x^2 + 3*x + 2)
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2\,(2\,x+3)}{\sqrt{x^2+3\,x+2}}$$

input
```
integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*x + 3)/sqrt(x^2 + 3*x + 2)
```

3.128.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{4\,\left(x+\frac{3}{2}\right)}{\sqrt{x^2+3\,x+2}}$$

input
```
int(1/(3*x + x^2 + 2)^(3/2),x)
```

output
```
-(4*(x + 3/2))/(3*x + x^2 + 2)^(1/2)
```

3.129 $\int \frac{1}{(27-24x+4x^2)^{3/2}} dx$

3.129.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{1}{(27-24x+4x^2)^{3/2}} dx = \frac{3-x}{9\sqrt{27-24x+4x^2}}$$

output
```
1/9*(3-x)/(4*x^2-24*x+27)^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(27-24x+4x^2)^{3/2}} dx = \frac{3-x}{9\sqrt{27-24x+4x^2}}$$

input
```
Integrate[(27 - 24*x + 4*x^2)^(-3/2),x]
```

output
```
(3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

3.129.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^2-24x+27)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 1088}$$
$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

input
```
Int[(27 - 24*x + 4*x^2)^(-3/2),x]
```

output
```
(3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

3.129.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

3.129.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
trager	$-\frac{-3+x}{9\sqrt{4x^2-24x+27}}$	18
risch	$-\frac{-3+x}{9\sqrt{4x^2-24x+27}}$	18
default	$-\frac{8x-24}{72\sqrt{4x^2-24x+27}}$	20
gosper	$-\frac{(-3+2x)(2x-9)(-3+x)}{9(4x^2-24x+27)^{\frac{3}{2}}}$	28

input
```
int(1/(4*x^2-24*x+27)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3+x)/(4*x^2-24*x+27)^(1/2)
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = -\frac{4\,x^2+2\,\sqrt{4\,x^2-24\,x+27}(x-3)-24\,x+27}{18\,(4\,x^2-24\,x+27)}$$

input
```
integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="fricas")
```

output
```
-1/18*(4*x^2 + 2*sqrt(4*x^2 - 24*x + 27)*(x - 3) - 24*x + 27)/(4*x^2 - 24*
x + 27)
```

3.129.6 Sympy [F]

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = \int \frac{1}{(4x^2-24x+27)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x**2-24*x+27)**(3/2),x)
```

output
```
Integral((4*x**2 - 24*x + 27)**(-3/2), x)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = -\frac{x}{9\,\sqrt{4\,x^2-24\,x+27}}+\frac{1}{3\,\sqrt{4\,x^2-24\,x+27}}$$

input
```
integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="maxima")
```

output
```
-1/9*x/sqrt(4*x^2 - 24*x + 27) + 1/3/sqrt(4*x^2 - 24*x + 27)
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = -\frac{x-3}{9\sqrt{4\,x^2-24\,x+27}}$$

input
```
integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="giac")
```

output
```
-1/9*(x - 3)/sqrt(4*x^2 - 24*x + 27)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = -\frac{x-3}{9\sqrt{4\,x^2-24\,x+27}}$$

input
```
int(1/(4*x^2 - 24*x + 27)^(3/2),x)
```

output
```
-(x - 3)/(9*(4*x^2 - 24*x + 27)^(1/2))
```

3.130 $\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx$

3.130.1 Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx=\frac{5-2x}{9\sqrt{5-4x-x^2}}$$

output
```
1/9*(5-2*x)/(-x^2-4*x+5)^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx=\frac{(-5+2x)\sqrt{5-4x-x^2}}{9(-1+x)(5+x)}$$

input
```
Integrate[x/(5 - 4*x - x^2)^(3/2),x]
```

output
```
((-5 + 2*x)*Sqrt[5 - 4*x - x^2])/(9*(-1 + x)*(5 + x))
```

3.130.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(-x^2-4x+5)^{3/2}}\,dx$$

$$\Big\downarrow \text{1158}$$

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

input
```
Int[x/(5 - 4*x - x^2)^(3/2),x]
```

output
```
(5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

3.130.3.1 Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

3.130.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{-5+2x}{9\sqrt{-x^2-4x+5}}$	20
gosper	$\frac{(x+5)(-1+x)(-5+2x)}{9(-x^2-4x+5)^{\frac{3}{2}}}$	26
trager	$\frac{(-5+2x)\sqrt{-x^2-4x+5}}{9x^2+36x-45}$	30
default	$\frac{1}{\sqrt{-x^2-4x+5}}+\frac{-2x-4}{9\sqrt{-x^2-4x+5}}$	33

input
```
int(x/(-x^2-4*x+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-5+2*x)/(-x^2-4*x+5)^(1/2)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = \frac{\sqrt{-x^2-4\,x+5}(2\,x-5)}{9\,(x^2+4\,x-5)}$$

input
```
integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

3.130.6 Sympy [F]

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = \int \frac{x}{(-(x-1)(x+5))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x**2-4*x+5)**(3/2),x)
```

output
```
Integral(x/(-(x - 1)*(x + 5))**(3/2), x)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = -\frac{2\,x}{9\,\sqrt{-x^2-4\,x+5}} + \frac{5}{9\,\sqrt{-x^2-4\,x+5}}$$

input
```
integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="maxima")
```

output
```
-2/9*x/sqrt(-x^2 - 4*x + 5) + 5/9/sqrt(-x^2 - 4*x + 5)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = \frac{\sqrt{-x^2-4\,x+5}(2\,x-5)}{9\,(x^2+4\,x-5)}$$

input
```
integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="giac")
```

output
```
1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = -\frac{2\,x-5}{9\,\sqrt{-x^2-4\,x+5}}$$

input
```
int(x/(5 - x^2 - 4*x)^(3/2),x)
```

output
```
-(2*x - 5)/(9*(5 - x^2 - 4*x)^(1/2))
```

3.131 $\int \frac{1}{(5-4x-x^2)^{5/2}} \, dx$

3.131.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{1}{(5-4x-x^2)^{5/2}} \, dx = \frac{2+x}{27(5-4x-x^2)^{3/2}} + \frac{2(2+x)}{243\sqrt{5-4x-x^2}}$$

output
```
1/27*(2+x)/(-x^2-4*x+5)^(3/2)+2/243*(2+x)/(-x^2-4*x+5)^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(5-4x-x^2)^{5/2}} \, dx = \frac{\sqrt{5-4x-x^2}(38+3x-12x^2-2x^3)}{243(-1+x)^2(5+x)^2}$$

input
```
Integrate[(5 - 4*x - x^2)^(-5/2),x]
```

output
```
(Sqrt[5 - 4*x - x^2]*(38 + 3*x - 12*x^2 - 2*x^3))/(243*(-1 + x)^2*(5 + x)^
2)
```

3.131.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^2-4x+5)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2}{27}\int \frac{1}{(-x^2-4x+5)^{3/2}}dx+\frac{x+2}{27(-x^2-4x+5)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27(-x^2-4x+5)^{3/2}}$$

input
```
Int[(5 - 4*x - x^2)^(-5/2),x]
```

output
```
(2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

3.131.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.131.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{(x+5)(-1+x)(2x^3+12x^2-3x-38)}{243(-x^2-4x+5)^{\frac{5}{2}}}$	36
default	$-\frac{-2x-4}{54(-x^2-4x+5)^{\frac{3}{2}}}-\frac{-2x-4}{243\sqrt{-x^2-4x+5}}$	40
trager	$-\frac{(2x^3+12x^2-3x-38)\sqrt{-x^2-4x+5}}{243(x^2+4x-5)^2}$	40
risch	$\frac{2x^3+12x^2-3x-38}{243(x^2+4x-5)\sqrt{-x^2-4x+5}}$	40

input
```
int(1/(-x^2-4*x+5)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/243*(x+5)*(-1+x)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{(2\,x^3+12\,x^2-3\,x-38)\sqrt{-x^2-4\,x+5}}{243\,(x^4+8\,x^3+6\,x^2-40\,x+25)}$$

input
```
integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="fricas")
```

output
```
-1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x
^2 - 40*x + 25)
```

3.131.6 Sympy [F]

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = \int \frac{1}{(-x^2-4x+5)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**2-4*x+5)**(5/2),x)
```

output
```
Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = \frac{2\,x}{243\,\sqrt{-x^2-4\,x+5}} + \frac{4}{243\,\sqrt{-x^2-4\,x+5}} + \frac{x}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}} + \frac{2}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}}$$

input
```
integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")
```

output
```
2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 -
 4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{((2\,(x+6)x-3)x-38)\sqrt{-x^2-4\,x+5}}{243\,(x^2+4\,x-5)^2}$$

input
```
integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="giac")
```

output
```
-1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x - 5)^2
```

3.131.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{(4\,x+8)\,(8\,x^2+32\,x-76)}{3888\,(-x^2-4\,x+5)^{3/2}}$$

input
```
int(1/(5 - x^2 - 4*x)^(5/2),x)
```

output
```
-((4*x + 8)*(32*x + 8*x^2 - 76))/(3888*(5 - x^2 - 4*x)^(3/2))
```

3.132 $\int (a+bx+cx^2)^p \, dx$

3.132.1 Optimal result

Integrand size = 12, antiderivative size = 122

$$\int (a+bx+cx^2)^p \, dx = -\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx+cx^2)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}(1+p)}$$

output
```
-2^(p+1)*(c*x^2+b*x+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(-1-p)/(p+1)/(-4*a*c+b^2)^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int (a+bx+cx^2)^p \, dx = \frac{2^{-1+p}\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{-}{}\right.}{c(1+p)}$$

input
```
Integrate[(a + b*x + c*x^2)^p,x]
```

output
```
(2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(a + x*(b + c*x))^p*Hypergeome
tric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4
*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)
```

3.132.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \; 1096$$

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x + c*x^2)^p,x]
```

output
```
-((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 -
p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p))
)
```

3.132.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

3.132.4 Maple [F]

$$\int \left(c\,x^2+bx+a\right)^p dx$$

input
```
int((c*x^2+b*x+a)^p,x)
```

output
```
int((c*x^2+b*x+a)^p,x)
```

3.132.5 Fricas [F]

$$\int \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p, x)
```

3.132.6 Sympy [F]

$$\int \left(a+bx+cx^2\right)^p \, dx = \int \left(a+bx+cx^2\right)^p \, dx$$

input
```
integrate((c*x**2+b*x+a)**p,x)
```

output
```
Integral((a + b*x + c*x**2)**p, x)
```

3.132.7 Maxima [F]

$$\int (a+bx+cx^2)^p \, dx = \int (cx^2+bx+a)^p \, dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p, x)
```

3.132.8 Giac [F]

$$\int (a+bx+cx^2)^p \, dx = \int (cx^2+bx+a)^p \, dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^p \, dx = \int (c\,x^2+b\,x+a)^p \, dx$$

input
```
int((a + b*x + c*x^2)^p,x)
```

output
```
int((a + b*x + c*x^2)^p, x)
```

3.133 $\int (3+4x+5x^2)^p \, dx$

3.133.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int (3+4x+5x^2)^p \, dx = 5^{-1-p}11^p(2+5x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{11}(2+5x)^2\right)$$

output
```
5^(-1-p)*11^p*(2+5*x)*hypergeom([1/2, -p],[3/2],-1/11*(2+5*x)^2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (3+4x+5x^2)^p \, dx = 5^{-1-p}11^p(2+5x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{11}(2+5x)^2\right)$$

input
```
Integrate[(3 + 4*x + 5*x^2)^p,x]
```

output
```
5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/11*(2 + 5*x)^
2]
```

3.133.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+4x+3\right)^p dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}5^{-p-1}11^p\int \left(\frac{1}{44}(10x+4)^2+1\right)^p d(10x+4)$$

$$\Big\downarrow \text{237}$$

$$\frac{1}{2}5^{-p-1}11^p(10x+4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{44}(10x+4)^2\right)$$

input
```
Int[(3 + 4*x + 5*x^2)^p,x]
```

output
```
(5^(-1 - p)*11^p*(4 + 10*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/44*(4 + 10*
x)^2])/2
```

3.133.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.133.4 Maple [F]

$$\int \left(5x^2+4x+3\right)^p dx$$

input
```
int((5*x^2+4*x+3)^p,x)
```

output
```
int((5*x^2+4*x+3)^p,x)
```

3.133.5 Fricas [F]

$$\int \left(3+4x+5x^2\right)^p \, dx = \int \left(5\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((5*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((5*x^2 + 4*x + 3)^p, x)
```

3.133.6 Sympy [F]

$$\int \left(3+4x+5x^2\right)^p \, dx = \int \left(5x^2+4x+3\right)^p \, dx$$

input
```
integrate((5*x**2+4*x+3)**p,x)
```

output
```
Integral((5*x**2 + 4*x + 3)**p, x)
```

3.133.7 Maxima [F]

$$\int \left(3+4x+5x^2\right)^p\,dx=\int \left(5\,x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((5*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 4*x + 3)^p, x)
```

3.133.8 Giac [F]

$$\int \left(3+4x+5x^2\right)^p\,dx=\int \left(5\,x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((5*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((5*x^2 + 4*x + 3)^p, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \left(3+4x+5x^2\right)^p\,dx=\int \left(5\,x^2+4\,x+3\right)^p\,dx$$

input
```
int((4*x + 5*x^2 + 3)^p,x)
```

output
```
int((4*x + 5*x^2 + 3)^p, x)
```

3.134 $\int (3+4x+4x^2)^p \, dx$

3.134.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int (3+4x+4x^2)^p \, dx = 2^{-1+p}(1+2x) \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{1}{2}(1+2x)^2\right)$$

output
```
2^(-1+p)*(1+2*x)*hypergeom([1/2, -p],[3/2],-1/2*(1+2*x)^2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (3+4x+4x^2)^p \, dx = 2^{-3+p}(4+8x) \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{1}{32}(4+8x)^2\right)$$

input
```
Integrate[(3 + 4*x + 4*x^2)^p,x]
```

output
```
2^(-3 + p)*(4 + 8*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/32*(4 + 8*x)^2]
```

3.134.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^2+4x+3\right)^p \, dx$$

$$\Big\downarrow \ 1090$$

$$2^{p-3}\int \left(\frac{1}{32}(8x+4)^2+1\right)^p d(8x+4)$$

$$\Big\downarrow \ 237$$

$$2^{p-3}(8x+4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{32}(8x+4)^2\right)$$

input
```
Int[(3 + 4*x + 4*x^2)^p,x]
```

output
```
2^(-3 + p)*(4 + 8*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/32*(4 + 8*x)^2]
```

3.134.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.134.4 Maple [F]

$$\int (4x^2 + 4x + 3)^p \, dx$$

input
```
int((4*x^2+4*x+3)^p,x)
```

output
```
int((4*x^2+4*x+3)^p,x)
```

3.134.5 Fricas [F]

$$\int (3 + 4x + 4x^2)^p \, dx = \int (4\,x^2 + 4\,x + 3)^p \, dx$$

input
```
integrate((4*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((4*x^2 + 4*x + 3)^p, x)
```

3.134.6 Sympy [F]

$$\int (3 + 4x + 4x^2)^p \, dx = \int (4x^2 + 4x + 3)^p \, dx$$

input
```
integrate((4*x**2+4*x+3)**p,x)
```

output
```
Integral((4*x**2 + 4*x + 3)**p, x)
```

3.134.7 Maxima [F]

$$\int \left(3+4x+4x^2\right)^p \, dx = \int \left(4\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((4*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((4*x^2 + 4*x + 3)^p, x)
```

3.134.8 Giac [F]

$$\int \left(3+4x+4x^2\right)^p \, dx = \int \left(4\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((4*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((4*x^2 + 4*x + 3)^p, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \left(3+4x+4x^2\right)^p \, dx = \int \left(4\,x^2+4\,x+3\right)^p \, dx$$

input
```
int((4*x + 4*x^2 + 3)^p,x)
```

output
```
int((4*x + 4*x^2 + 3)^p, x)
```

3.135 $\int (3+4x+3x^2)^p \, dx$

3.135.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int (3+4x+3x^2)^p \, dx = 3^{-1-p}5^p(2+3x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{5}(2+3x)^2\right)$$

output
```
3^(-1-p)*5^p*(2+3*x)*hypergeom([1/2, -p],[3/2],-1/5*(2+3*x)^2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (3+4x+3x^2)^p \, dx = 3^{-1-p}5^p(2+3x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{5}(2+3x)^2\right)$$

input
```
Integrate[(3 + 4*x + 3*x^2)^p,x]
```

output
```
3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/5*(2 + 3*x)^2]
```

3.135.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+4x+3)^p\,dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}3^{-p-1}5^p\int \left(\frac{1}{20}(6x+4)^2+1\right)^p d(6x+4)$$

$$\Big\downarrow \text{237}$$

$$\frac{1}{2}3^{-p-1}5^p(6x+4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{20}(6x+4)^2\right)$$

input
```
Int[(3 + 4*x + 3*x^2)^p,x]
```

output
```
(3^(-1 - p)*5^p*(4 + 6*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/20*(4 + 6*x)^
2])/2
```

3.135.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.135.4 Maple [F]

$$\int \left(3x^2+4x+3\right)^p dx$$

input
```
int((3*x^2+4*x+3)^p,x)
```

output
```
int((3*x^2+4*x+3)^p,x)
```

3.135.5 Fricas [F]

$$\int \left(3+4x+3x^2\right)^p \, dx = \int \left(3\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((3*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((3*x^2 + 4*x + 3)^p, x)
```

3.135.6 Sympy [F]

$$\int \left(3+4x+3x^2\right)^p \, dx = \int \left(3x^2+4x+3\right)^p \, dx$$

input
```
integrate((3*x**2+4*x+3)**p,x)
```

output
```
Integral((3*x**2 + 4*x + 3)**p, x)
```

3.135.7 Maxima [F]

$$\int (3+4x+3x^2)^p\,dx = \int (3\,x^2+4\,x+3)^p\,dx$$

input
```
integrate((3*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 4*x + 3)^p, x)
```

3.135.8 Giac [F]

$$\int (3+4x+3x^2)^p\,dx = \int (3\,x^2+4\,x+3)^p\,dx$$

input
```
integrate((3*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((3*x^2 + 4*x + 3)^p, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int (3+4x+3x^2)^p\,dx = \int (3\,x^2+4\,x+3)^p\,dx$$

input
```
int((4*x + 3*x^2 + 3)^p,x)
```

output
```
int((4*x + 3*x^2 + 3)^p, x)
```

3.136 $\int (3+4x+2x^2)^p \, dx$

3.136.1 Optimal result

Integrand size = 12, antiderivative size = 21

$$\int (3+4x+2x^2)^p \, dx = (1+x) \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-2(1+x)^2\right)$$

output
```
(1+x)*hypergeom([1/2, -p],[3/2],-2*(1+x)^2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (3+4x+2x^2)^p \, dx = (1+x) \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-2(1+x)^2\right)$$

input
```
Integrate[(3 + 4*x + 2*x^2)^p,x]
```

output
```
(1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

3.136.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2+4x+3)^p \, dx$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{4}\int \left(\frac{1}{8}(4x+4)^2+1\right)^p d(4x+4)$$

$$\Big\downarrow \; 237$$

$$\frac{1}{4}(4x+4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{8}(4x+4)^2\right)$$

input
```
Int[(3 + 4*x + 2*x^2)^p,x]
```

output
```
((4 + 4*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/8*(4 + 4*x)^2])/4
```

3.136.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.136.4 Maple [F]

$$\int \left(2x^2+4x+3\right)^p dx$$

input
```
int((2*x^2+4*x+3)^p,x)
```

output
```
int((2*x^2+4*x+3)^p,x)
```

3.136.5 Fricas [F]

$$\int \left(3+4x+2x^2\right)^p \, dx = \int \left(2\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((2*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((2*x^2 + 4*x + 3)^p, x)
```

3.136.6 Sympy [F]

$$\int \left(3+4x+2x^2\right)^p \, dx = \int \left(2x^2+4x+3\right)^p \, dx$$

input
```
integrate((2*x**2+4*x+3)**p,x)
```

output
```
Integral((2*x**2 + 4*x + 3)**p, x)
```

3.136.7 Maxima [F]

$$\int (3+4x+2x^2)^p \, dx = \int (2\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((2*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)^p, x)
```

3.136.8 Giac [F]

$$\int (3+4x+2x^2)^p \, dx = \int (2\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((2*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((2*x^2 + 4*x + 3)^p, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int (3+4x+2x^2)^p \, dx = \int (2\,x^2+4\,x+3)^p \, dx$$

input
```
int((4*x + 2*x^2 + 3)^p,x)
```

output
```
int((4*x + 2*x^2 + 3)^p, x)
```

3.137 $\int (3+4x+x^2)^p \, dx$

3.137.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int (3+4x+x^2)^p \, dx = -\frac{2^{1+2p}(-2-2x)^{-1-p}\left(3+4x+x^2\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{3+x}{2}\right)}{1+p}$$

output
```
-2^(1+2*p)*(-2-2*x)^(-1-p)*(x^2+4*x+3)^(p+1)*hypergeom([-p, p+1],[2+p],3/2
+1/2*x)/(p+1)
```

3.137.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int (3+4x+x^2)^p \, dx = \frac{2^p(1+x)(3+x)^{-p}\left(3+4x+x^2\right)^p\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{1}{2}(-1-x)\right)}{1+p}$$

input
```
Integrate[(3 + 4*x + x^2)^p,x]
```

output
```
(2^p*(1 + x)*(3 + 4*x + x^2)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-1 - x
)/2])/((1 + p)*(3 + x)^p)
```

3.137.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2+4x+3)^p \, dx$$

$$\Big\downarrow \text{1096}$$

$$-\frac{2^{2p+1}(-2x-2)^{-p-1}\left(x^2+4x+3\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{x+3}{2}\right)}{p+1}$$

input
```
Int[(3 + 4*x + x^2)^p,x]
```

output
```
-((2^(1 + 2*p)*(-2 - 2*x)^(-1 - p)*(3 + 4*x + x^2)^(1 + p)*Hypergeometric2
F1[-p, 1 + p, 2 + p, (3 + x)/2])/(1 + p))
```

3.137.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

3.137.4 Maple [F]

$$\int (x^2+4x+3)^p \, dx$$

input
```
int((x^2+4*x+3)^p,x)
```

output
```
int((x^2+4*x+3)^p,x)
```

3.137.5 Fricas [F]

$$\int (3+4x+x^2)^p \, dx = \int (x^2+4\,x+3)^p \, dx$$

input
```
integrate((x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((x^2 + 4*x + 3)^p, x)
```

3.137.6 Sympy [F]

$$\int (3+4x+x^2)^p \, dx = \int (x^2+4x+3)^p \, dx$$

input
```
integrate((x**2+4*x+3)**p,x)
```

output
```
Integral((x**2 + 4*x + 3)**p, x)
```

3.137.7 Maxima [F]

$$\int (3+4x+x^2)^p \, dx = \int (x^2+4\,x+3)^p \, dx$$

input
```
integrate((x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((x^2 + 4*x + 3)^p, x)
```

3.137.8 Giac [F]

$$\int (3+4x+x^2)^p \, dx = \int (x^2+4x+3)^p \, dx$$

input
```
integrate((x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((x^2 + 4*x + 3)^p, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int (3+4x+x^2)^p \, dx = \int (x^2+4x+3)^p \, dx$$

input
```
int((4*x + x^2 + 3)^p,x)
```

output
```
int((4*x + x^2 + 3)^p, x)
```

3.138 $\int (3+4x)^p \, dx$

3.138.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (3+4x)^p \, dx = \frac{(3+4x)^{1+p}}{4(1+p)}$$

output
```
1/4*(3+4*x)^(p+1)/(p+1)
```

3.138.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (3+4x)^p \, dx = \frac{(3+4x)^{1+p}}{4(1+p)}$$

input
```
Integrate[(3 + 4*x)^p,x]
```

output
```
(3 + 4*x)^(1 + p)/(4*(1 + p))
```

3.138.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4x+3)^p\,dx$$

$$\Big\downarrow \, 17$$

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

input
```
Int[(3 + 4*x)^p,x]
```

output
```
(3 + 4*x)^(1 + p)/(4*(1 + p))
```

3.138.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.138.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{(4x+3)^{1+p}}{4p+4}$	17
default	$\frac{(4x+3)^{1+p}}{4p+4}$	17
meijerg	$3^p x {}_2F_1\left(1,-p;2;-\frac{4x}{3}\right)$	17
risch	$\frac{(4x+3)(4x+3)^p}{4p+4}$	20
parallelrisch	$\frac{12(4x+3)^p x+9(4x+3)^p}{12+12p}$	28
norman	$\frac{x\,e^{p\ln(4x+3)}}{1+p}+\frac{3\,e^{p\ln(4x+3)}}{4(1+p)}$	34

input
```
int((4*x+3)^p,x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*x+3)^(1+p)/(1+p)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int (3+4x)^p\,dx = \frac{(4\,x+3)^p(4\,x+3)}{4\,(p+1)}$$

input
```
integrate((3+4*x)^p,x, algorithm="fricas")
```

output
```
1/4*(4*x + 3)^p*(4*x + 3)/(p + 1)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (3+4x)^p\,dx = \frac{\begin{cases} \frac{(4x+3)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(4x+3) & \text{otherwise} \end{cases}}{4}$$

input
```
integrate((3+4*x)**p,x)
```

output
```
Piecewise(((4*x + 3)**(p + 1)/(p + 1), Ne(p, -1)), (log(4*x + 3), True))/4
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (3+4x)^p\,dx = \frac{(4\,x+3)^{p+1}}{4\,(p+1)}$$

input
```
integrate((3+4*x)^p,x, algorithm="maxima")
```

output
```
1/4*(4*x + 3)^(p + 1)/(p + 1)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (3+4x)^p\,dx = \frac{(4\,x+3)^{p+1}}{4\,(p+1)}$$

input
```
integrate((3+4*x)^p,x, algorithm="giac")
```

output
```
1/4*(4*x + 3)^(p + 1)/(p + 1)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int (3+4x)^p\,dx = \begin{cases} \frac{\ln(4\,x+3)}{4} & \text{if} \;\; p=-1 \\ \frac{(4\,x+3)^{p+1}}{4\,(p+1)} & \text{if} \;\; p\neq -1 \end{cases}$$

input
```
int((4*x + 3)^p,x)
```

output
```
piecewise(p == -1, log(4*x + 3)/4, p ~= -1, (4*x + 3)^(p + 1)/(4*(p + 1)))
```

3.139 $\int (3+4x-x^2)^p \, dx$

3.139.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int (3+4x-x^2)^p \, dx = -7^p(2-x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{7}(2-x)^2\right)$$

output
```
-7^p*(2-x)*hypergeom([1/2, -p],[3/2],1/7*(2-x)^2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (3+4x-x^2)^p \, dx = 7^p(-2+x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{7}(-2+x)^2\right)$$

input
```
Integrate[(3 + 4*x - x^2)^p,x]
```

output
```
7^p*(-2 + x)*Hypergeometric2F1[1/2, -p, 3/2, (-2 + x)^2/7]
```

3.139.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^2+4x+3\right)^p dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{2}7^p \int \left(1-\frac{1}{28}(4-2x)^2\right)^p d(4-2x)$$

$$\Big\downarrow \text{237}$$

$$-\frac{1}{2}7^p(4-2x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{28}(4-2x)^2\right)$$

input
```
Int[(3 + 4*x - x^2)^p,x]
```

output
```
-1/2*(7^p*(4 - 2*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 2*x)^2/28])
```

3.139.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.139.4 Maple [F]

$$\int \left(-x^2+4x+3\right)^p dx$$

input
```
int((-x^2+4*x+3)^p,x)
```

output
```
int((-x^2+4*x+3)^p,x)
```

3.139.5 Fricas [F]

$$\int \left(3+4x-x^2\right)^p \, dx = \int \left(-x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((-x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-x^2 + 4*x + 3)^p, x)
```

3.139.6 Sympy [F]

$$\int \left(3+4x-x^2\right)^p \, dx = \int \left(-x^2+4x+3\right)^p \, dx$$

input
```
integrate((-x**2+4*x+3)**p,x)
```

output
```
Integral((-x**2 + 4*x + 3)**p, x)
```

3.139.7 Maxima [F]

$$\int (3+4x-x^2)^p \, dx = \int (-x^2+4x+3)^p \, dx$$

input
```
integrate((-x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-x^2 + 4*x + 3)^p, x)
```

3.139.8 Giac [F]

$$\int (3+4x-x^2)^p \, dx = \int (-x^2+4x+3)^p \, dx$$

input
```
integrate((-x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-x^2 + 4*x + 3)^p, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int (3+4x-x^2)^p \, dx = \int (-x^2+4x+3)^p \, dx$$

input
```
int((4*x - x^2 + 3)^p,x)
```

output
```
int((4*x - x^2 + 3)^p, x)
```

3.140 $\int (3+4x-2x^2)^p \, dx$

3.140.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int (3+4x-2x^2)^p \, dx = -5^p(1-x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{2}{5}(1-x)^2\right)$$

output
```
-5^p*(1-x)*hypergeom([1/2, -p],[3/2],2/5*(1-x)^2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (3+4x-2x^2)^p \, dx = 5^p(-1+x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{2}{5}(-1+x)^2\right)$$

input
```
Integrate[(3 + 4*x - 2*x^2)^p,x]
```

output
```
5^p*(-1 + x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(-1 + x)^2)/5]
```

3.140.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-2x^2+4x+3\right)^p\,dx$$

$$\Big\downarrow \ 1090$$

$$-\frac{1}{4}5^p\int \left(1-\frac{1}{40}(4-4x)^2\right)^p d(4-4x)$$

$$\Big\downarrow \ 237$$

$$-\frac{1}{4}5^p(4-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{40}(4-4x)^2\right)$$

input
```
Int[(3 + 4*x - 2*x^2)^p,x]
```

output
```
-1/4*(5^p*(4 - 4*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 4*x)^2/40])
```

3.140.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.140.4 Maple [F]

$$\int \left(-2x^2+4x+3\right)^p dx$$

input
```
int((-2*x^2+4*x+3)^p,x)
```

output
```
int((-2*x^2+4*x+3)^p,x)
```

3.140.5 Fricas [F]

$$\int \left(3+4x-2x^2\right)^p \, dx = \int \left(-2\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((-2*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-2*x^2 + 4*x + 3)^p, x)
```

3.140.6 Sympy [F]

$$\int \left(3+4x-2x^2\right)^p \, dx = \int \left(-2x^2+4x+3\right)^p \, dx$$

input
```
integrate((-2*x**2+4*x+3)**p,x)
```

output
```
Integral((-2*x**2 + 4*x + 3)**p, x)
```

3.140.7 Maxima [F]

$$\int \left(3+4x-2x^2\right)^p\,dx=\int \left(-2\,x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((-2*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-2*x^2 + 4*x + 3)^p, x)
```

3.140.8 Giac [F]

$$\int \left(3+4x-2x^2\right)^p\,dx=\int \left(-2\,x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((-2*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-2*x^2 + 4*x + 3)^p, x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \left(3+4x-2x^2\right)^p\,dx=\int \left(-2\,x^2+4\,x+3\right)^p\,dx$$

input
```
int((4*x - 2*x^2 + 3)^p,x)
```

output
```
int((4*x - 2*x^2 + 3)^p, x)
```

3.141 $\int (3+4x-3x^2)^p \, dx$

3.141.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int (3+4x-3x^2)^p \, dx = -3^{-1-p}13^p(2-3x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{13}(2-3x)^2\right)$$

output
```
-3^(-1-p)*13^p*(2-3*x)*hypergeom([1/2, -p],[3/2],1/13*(2-3*x)^2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int (3+4x-3x^2)^p \, dx = 3^{-1-p}13^p(-2+3x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{13}(2-3x)^2\right)$$

input
```
Integrate[(3 + 4*x - 3*x^2)^p,x]
```

output
```
3^(-1 - p)*13^p*(-2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 3*x)^2/13]
```

3.141.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+4x+3\right)^p \, dx$$

$$\Big\downarrow \; 1090$$

$$-\frac{1}{2}3^{-p-1}13^p \int \left(1-\frac{1}{52}(4-6x)^2\right)^p d(4-6x)$$

$$\Big\downarrow \; 237$$

$$-\frac{1}{2}3^{-p-1}13^p(4-6x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{52}(4-6x)^2\right)$$

input
```
Int[(3 + 4*x - 3*x^2)^p,x]
```

output
```
-1/2*(3^(-1 - p)*13^p*(4 - 6*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 6*x)^
2/52])
```

3.141.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.141.4 Maple [F]

$$\int \left(-3x^2+4x+3\right)^p dx$$

input
```
int((-3*x^2+4*x+3)^p,x)
```

output
```
int((-3*x^2+4*x+3)^p,x)
```

3.141.5 Fricas [F]

$$\int \left(3+4x-3x^2\right)^p dx = \int \left(-3\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((-3*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 4*x + 3)^p, x)
```

3.141.6 Sympy [F]

$$\int \left(3+4x-3x^2\right)^p dx = \int \left(-3x^2+4x+3\right)^p dx$$

input
```
integrate((-3*x**2+4*x+3)**p,x)
```

output
```
Integral((-3*x**2 + 4*x + 3)**p, x)
```

3.141.7 Maxima [F]

$$\int \left(3+4x-3x^2\right)^p\,dx = \int \left(-3\,x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((-3*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 4*x + 3)^p, x)
```

3.141.8 Giac [F]

$$\int \left(3+4x-3x^2\right)^p\,dx = \int \left(-3\,x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((-3*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 4*x + 3)^p, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \left(3+4x-3x^2\right)^p\,dx = \int \left(-3\,x^2+4\,x+3\right)^p\,dx$$

input
```
int((4*x - 3*x^2 + 3)^p,x)
```

output
```
int((4*x - 3*x^2 + 3)^p, x)
```

3.142 $\int (3+4x-4x^2)^p \, dx$

3.142.1 Optimal result

Integrand size = 12, antiderivative size = 35

$$\int (3+4x-4x^2)^p \, dx = -2^{-1+2p}(1-2x) \, \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{4}(1-2x)^2\right)$$

output
```
-2^(-1+2*p)*(1-2*x)*hypergeom([1/2, -p],[3/2],1/4*(1-2*x)^2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (3+4x-4x^2)^p \, dx = -2^{-3+2p}(4-8x) \, \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{64}(4-8x)^2\right)$$

input
```
Integrate[(3 + 4*x - 4*x^2)^p,x]
```

output
```
-(2^(-3 + 2*p)*(4 - 8*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 8*x)^2/64])
```

3.142.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-4x^2+4x+3\right)^p dx$$

$$\Big\downarrow \text{1090}$$

$$-2^{2p-3}\int \left(1-\frac{1}{64}(4-8x)^2\right)^p d(4-8x)$$

$$\Big\downarrow \text{237}$$

$$-2^{2p-3}(4-8x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{64}(4-8x)^2\right)$$

input
```
Int[(3 + 4*x - 4*x^2)^p,x]
```

output
```
-(2^(-3 + 2*p)*(4 - 8*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 8*x)^2/64])
```

3.142.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.142.4 Maple [F]

$$\int \left(-4x^2+4x+3\right)^p dx$$

input
```
int((-4*x^2+4*x+3)^p,x)
```

output
```
int((-4*x^2+4*x+3)^p,x)
```

3.142.5 Fricas [F]

$$\int \left(3+4x-4x^2\right)^p \, dx = \int \left(-4\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((-4*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-4*x^2 + 4*x + 3)^p, x)
```

3.142.6 Sympy [F]

$$\int \left(3+4x-4x^2\right)^p \, dx = \int \left(-4x^2+4x+3\right)^p \, dx$$

input
```
integrate((-4*x**2+4*x+3)**p,x)
```

output
```
Integral((-4*x**2 + 4*x + 3)**p, x)
```

3.142.7 Maxima [F]

$$\int (3+4x-4x^2)^p \, dx = \int (-4\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((-4*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-4*x^2 + 4*x + 3)^p, x)
```

3.142.8 Giac [F]

$$\int (3+4x-4x^2)^p \, dx = \int (-4\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((-4*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-4*x^2 + 4*x + 3)^p, x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int (3+4x-4x^2)^p \, dx = \int (-4\,x^2+4\,x+3)^p \, dx$$

input
```
int((4*x - 4*x^2 + 3)^p,x)
```

output
```
int((4*x - 4*x^2 + 3)^p, x)
```

3.143 $\int (3+4x-5x^2)^p \, dx$

3.143.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int (3+4x-5x^2)^p \, dx = -5^{-1-p}19^p(2-5x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{19}(2-5x)^2\right)$$

output
```
-5^(-1-p)*19^p*(2-5*x)*hypergeom([1/2, -p],[3/2],1/19*(2-5*x)^2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int (3+4x-5x^2)^p \, dx = 5^{-1-p}19^p(-2+5x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{19}(2-5x)^2\right)$$

input
```
Integrate[(3 + 4*x - 5*x^2)^p,x]
```

output
```
5^(-1 - p)*19^p*(-2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 5*x)^2/19]
```

3.143.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-5x^2+4x+3\right)^p \, dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{2}5^{-p-1}19^p \int \left(1-\frac{1}{76}(4-10x)^2\right)^p d(4-10x)$$

$$\Big\downarrow \text{237}$$

$$-\frac{1}{2}5^{-p-1}19^p(4-10x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{76}(4-10x)^2\right)$$

input
```
Int[(3 + 4*x - 5*x^2)^p,x]
```

output
```
-1/2*(5^(-1 - p)*19^p*(4 - 10*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 10*x
)^2/76])
```

3.143.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.143.4 Maple [F]

$$\int \left(-5x^2+4x+3\right)^p dx$$

input
```
int((-5*x^2+4*x+3)^p,x)
```

output
```
int((-5*x^2+4*x+3)^p,x)
```

3.143.5 Fricas [F]

$$\int \left(3+4x-5x^2\right)^p \, dx = \int \left(-5\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((-5*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-5*x^2 + 4*x + 3)^p, x)
```

3.143.6 Sympy [F]

$$\int \left(3+4x-5x^2\right)^p \, dx = \int \left(-5x^2+4x+3\right)^p \, dx$$

input
```
integrate((-5*x**2+4*x+3)**p,x)
```

output
```
Integral((-5*x**2 + 4*x + 3)**p, x)
```

3.143.7 Maxima [F]

$$\int (3+4x-5x^2)^p \, dx = \int (-5\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((-5*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-5*x^2 + 4*x + 3)^p, x)
```

3.143.8 Giac [F]

$$\int (3+4x-5x^2)^p \, dx = \int (-5\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((-5*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-5*x^2 + 4*x + 3)^p, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int (3+4x-5x^2)^p \, dx = \int (-5\,x^2+4\,x+3)^p \, dx$$

input
```
int((4*x - 5*x^2 + 3)^p,x)
```

output
```
int((4*x - 5*x^2 + 3)^p, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```